Multivariate GARCH Approaches: case of major sectorial Tunisian stock markets

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Abstract

The objectif in this paper is to proposes multivariate GARCH volatility models to assess the dynamic interdependence among volatility of returns for 5 tunisian sectorial stock index series (namely : Bank, FINancial service, AUTOmobile, INDustry, and Materials (MATB)) and TUNindex series. The Monthly returns of stock indices have been considered from 2010M02 to 2019M07. Two systems are considered. The first System, with Constant Conditional (C) mean, allows for market interaction. Results from DVECH model reveals that some sectorial stock markets are **interdependent,** the presence of a significance and **positive effect** of **cross shock** of Finance and Bank stock returns on Tunindex return, and volatility is **predictable**. C Correlation, ρ_{ij}, have **decreasing** evolution for full period or for recent years for almost all i and j except CC between Tunindex return and R_FIN (and R_BANK) and CC between R_FIN and R_IND (and R_MATB). The tests for volatility spillovers effects suggests significant **volatility spillovers** from MATB and AUTO sectors to IND sector and from AUTO sector to MATB sector. The second system, with macroeconomic factor **instability effects** as Conditional mean**,** examine the CCC and DCC between different sectors. The main result supports the hypotheses of DCC. The DCC provides evidence of cross border relationship between sectors and macro economic instability factors have significant effect on the mean of returns evolutions (at 5% or 10% level). **Volatility of exchange rate** has significant **positive** effect on R, R_FIN, and R_MATB, while **volatility of inflation** has significant **negative** effect on R_Fin and **volatility of oil price** has significant **negative** effect on R_AUTO.

Keywords: Sectorial stock return, MGARCH model, DVECH and DBEKK models, Conditional Correlations (CC), Dynamic CC (DCC) and Constant CC models (CCC).

JEL classification: C32, G11, G14.

Table of content

I. Introduction

The first two moments respectively called mean and variance of return series have been investigated extensively in the univariate finance literature to understand the trading dynamics of risk and returns in the financial asset markets.

In developing dynamic volatility models, there are two strands of modelling conditional volatility : the univariate and multivariate volatility modelling. (Engle R. , 1982) first introduced univariate autoregressive conditional heteroskedasticity (ARCH) model for predicting asset return volatility. This model is useful because it captures some stylized facts such as volatility clustering and thick-tail distribution of return series.[1]

In recent years, the information technology revolution has had a tremendous impact on the structure of financial markets. The dynamic dependence of multivariate financial assets provides rich sources of volatility transmission that helps the investors to play active role in financial transactions. This leads to spillovers from one market to other markets and causes the linkages between stock markets. There are extensive literatures on volatility spillover between financial markets. Indeed, the multivariate GARCH and its various extensions have been widely used to examine the co-movement and the transmission of the volatility between index financial sectorial markets. There is a large body of literature which focuses on the volatility spillover of different markets over time, using a multivariate GARCH model. Table 1 (a) gives a selected review (see Appendice I).

The multivariate extension to univariate model was first introduced by (Engle & Granger, 1987) in the ARCH context, and (Bollerslev, Engle, & Wooldridge, 1988) in the GARCH context. This multivariate GARCH is known as VECH model because of its form. The general MGARCH model is so flexible that not all the parameters can be estimated. For this reason, we consider 4 MGARCH models that reparameterize the model to be more parsimonious: the diagonal VECH model (DVECH), the BEKK model [proposed by (Baba, 1992) and (Engle & Kroner, 1995)],[2] the constant conditional correlation (CCC) model [proposed by (Bollerslev, 1990)], and the dynamic conditional correlation (DCC) model

[1] ARCH models were introduced by (Engle R. , 1982) in a study of inflation rates. Overviews of the literature can found in (Bollerslev, Engle, & Nelson, ARCH Models, 1994) and (Bollerslev, Chou, & Kroner, 1992). Introductions to basic ARCH models appear in many *general econometrics texts*, including (Davidson & MacKinnon, 1993), (Greene, 2018), (Stock & Watson, 2015), and (Wooldridge, 2016). In the larger context of *econometric time-series modeling,* (Harvey A. C., 1989) and (Enders, 2004) provide introductions to ARCH, and considerably more *detail* in the same context are given in (Hamilton, 1994).

[2] BEKK is from Baba-Engle-Kraft-Kroner.

[proposed by (Engle R. F., 2002)] or the time-varying conditional correlation (VCC) model [proposed by (Tse & Tsui, 2002)].

Up to this point, not much attention has been given on the detection of the volatility spillover between sector indexes among Tunisian sectorial stock markets. This paper aims at examining the volatility experiences of multivariate framework for 5 major sectorial (namely: Bank, Financial service, Automobile, Industry, and Materials) and TUNindex Tunisian stock markets. Within this framework the shocks to volatility from one market is allowed to affect both the risk and return of the other markets. The Monthly stock indices of 5 major sectorial Tunisian stock markets and TUNindex have been considered from 2010M02 to 2019M07.

Since, within multivariate framework the shocks to volatility from one market is allowed to affect both the risk and return of the other markets, the volatility modelling is emerged from modelling volatilities of returns within the six-dimensional volatility model in 2 systems. System (I) is with a constant as conditional mean and with different conditional covariance specifications given by DVECH, DBEKK, and CCC. And, System (II) is with expected mean depending on several macroeconomic variables reflecting instability effect as volatility of oil price in log, volatility of consumer price index in log (LCPI), and volatolity of excghange rate in log and with conditional correlation specifications given by DCC and CCC.

Because the normality assumption of unconditional volatility of innovation do not hold, one might not want to perform a maximum likelihood estimation using normal distribution. Joint estimation of the Multivariate mean-variance models in this paper uses then t-distribution. Finally, model checking will be performed in each case to ensure the adequacy of the fitted models.

This paper is organized as follows. Section II describes the sources and statistical properties of the data. In section III, model and methodology in multivariate framework is discussed [subsection A for conditional covariance analysiqs and subsection B for conditional correlation study]. Real application of the proposed models is reported in section IV [subsection C for system (I) and subsection D for system (II)]. Five tunisian major sectorial stock market returns are considered (namely: Bank, FINancial service, AUTOmobile, INDustry, and Materials (MATB)) with Tunindex stock return as benchmark. Finally, section V concludes the paper.

II. Data Analysis : Properties and Preliminary Results

This study covers a total of k = 5 sectoral tunisian stock index series (Finance, Bank, Automobile, Indutrie, and Material) and TUNindex as benchmark. The monthly data are collected from investing.com for the period from 2010M02 to 2019M07 (T = 114 observations). All observations are monthly. The data are expressed in Logarithms and then taken in first difference. Retuns of stock indices are calculated ;[3]

$$R_{it} = \log(\frac{SP_{it}}{SP_{it-1}})$$

SP_{it}: stock price for Market $i = 1, \ldots, k = 6$ at time $t = 1, \ldots, T = 114$.

Figure 1 illustrates the development of 5 Tunisian stock index and TUNindex. We could see from the graph that there is strong similarity between TUNindex, and Finance, Bank stock index series evolution from 2010M02 to 2019M07. While an other ressemblance is present between Automobile, Indutrie, and Material stock index series trend.

In inspecting the returns series how they evolved from 2010M02 to 2019M07 Period, Figure 2 (a) exposes that at most of the time there are large swings in all returns. Figure 2 (a) show also that the movement of stock returns is both positive and negative. It can be noted that the returns fluctuate around the mean value, but close to zero. Larger fluctuations tend to cluster together followed by periods of calmness.[4] The volatility seems to be larger during Yesamin 2011 revolution and at the end for 2017 for all returns except Auto' one. Therefore, the six Tunisian stock markets are affected simultaneously with some variation.

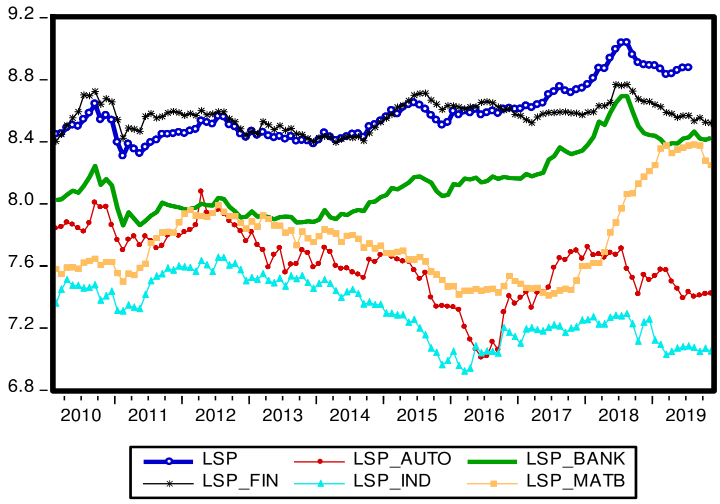

Figure 1: The movements of TUNindex (LSP) and 5 sectors' stock indices.

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[3] The software used for the analysis is Eviews 10 and Stata 15.

[4] (Fama, 1990) noted that stock returns tend to fluctuate thereby exhibiting volatility clustering, where large returns are usually complimented by small returns.

Table1 presents some sample distributional statistics for the stock market indices included in this paper. Statistics consist of the Monthly sample mean returns, standard deviation, minimum returns and maximum returns, and Normality JB test results. It presents also ARCH-LM test results as well as test BDS (Brock, Dechert, Scheinkman, & LeBaron, 1996) independance results.[5]

Mean returns are positive (except R_AUTO and R_IND) and apparently the higher is for R_MATB. But the standard deviations of these returns have the higher value for R_AUTO. While R and R_FIN have the smaller volatility of returns.

To test the normality of sample data, we practice two methods. In the first stage, the skewness and kurtosis were utilized to test by Jaque Bera (JB) test null hypothesis H_0 : The sample data are normally distributed. The result is presented at Table1. We reject the null hypothesis that the sample is normally distributed at 5% significant level except for R_MATB. In the second stage, we get quantiles of normal distribution plot. Figure A 1 (see Appendice) presents the quantiles of normal distribution plot.[6] Apparently, all six Tunisian indices do follow normal distribution in the long run. There are more points falling on the 45 degree lines, implying that they are perhaps more close to normal distributions. The skewness parameters are negative for R and R_Bank, indicating that these stock market returns were not symmetrically distributed and the distribution has a long left tail. In addition, coefficients of kurtosis (are way over 3) are almost equal to 5 for R, R_Bank, and (R_Fin), referring that for these stock market return volatility exist suggesting that the underlying time series data are heavily tailed (not) and sharply peaked when compared to a normal distribution.[7]

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[5] The BDS test, named after the three authors who first developed it, is a portmanteau test for time based dependence in a series. It can be used for testing against a variety of possible deviations from independence including linear dependence, non-linear dependence, or chaos.

[6] If the sample is perfectly normally distributed the points should all fall on the 45 degree line. The more the points diverge from this line, the less data will be approximate to a Normal Distribution.

[7] The reason for standardized returns have excess of kurtosis could be that returns would be not necessary conditionally Gaussian.

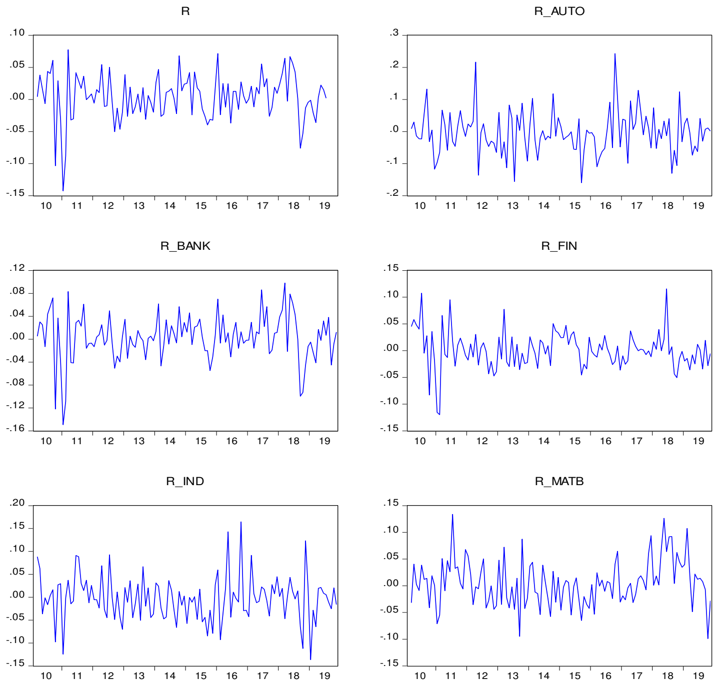

Figure 2 (a): Monthly stock retuns on Tunisian stock markets from 2010M02 to 2019M07; Tunindex and 5 sectoriel stock returns.

Results of LM test for various ARCH order (q = 1 and 2) provide strong evidence of rejecting the null hypothesis of constant variance [of no ARCH(1) or no ARCH(2) effects] for R, R_BANK, and R_FIN (the LM test show a p-value below 0.05) ; see Table1.[8] This is in accordance with results of DBS test given also at Table1. The **squares** series, however exhibits serial dependence in the second moment of TUNindex returns series R_t, Bank and Finance Retuns series. Rejecting H_0 (of independance) indicates the presence of ARCH effect in the TUNindex returns R_t, Bank and Finance Retuns series, therefore we can conclude that the variance of these returns is no-constant for all periods specified. BDS test do not supports for nonlinearity in all of consedered series.[9]

Table B 1 (and Table B 3, see Appendice I) reports the pair wise correlation coefficient estimations of stock returns (and Squared returns) for period from 2010M02 to 2019M07. We applied the Tunindex index return R (Squared R) to analyze its correlation with sectorial stock Returns (Squared returns). It is clear to see from the Table B 1 that the coefficients of all sectorial stock return are significant related to TUNindex with much lower correlations with TUNindex

[8] We assume a constant mean model and the LM test is applied to compute the test statistic value TR^2, where T is the number of observations and R^2 is the coefficient of multiple correlation obtained from regressing the squared residuals on q own lagged values.

[9] Once the volatility is confirmed in data (for R, R_BANK, and R_FIN), we can proceed our analysis further to estimate the parameters of both conditional mean and conditional variance equations for univarite specifications for these series. This can be done in subsequent paper.

compared to R_AUTO, R_IND, and R_MATB return index. This indicates that the sectorial stock markets are relatively non separated market with overall markets in Tunisia economic during this period. Bank and Finance get positive correlation with TUNindex, although the correlation between R_bank and R is higher than the correlation between R_FIN and R. From Table B 3, It is clear that TUNindex Squared return is related to R_BANK, R_FIN, and R_IND Squared return. R_AUTO squared return is related to R_IND squared return. And R_BANK squared return is related to R_FIN, and R_IND squared return.

In order to identify if the index return is integrated or stationary, we conduct the unit root test. In accordance with Figure 2 (a), in Table B 3 (see Appendice I), traditional ADF and PP (and KPSS) tests results, show that for all return series, the null hypothesis of the presence of a unit root (stationarity) is rejected (not rejected) at the 5% level of significance.[10]

Table1 Basic statistics of Tunisian monthly stock returns from 2010M02 to 2019M07.

	R	R_AUTO	R_BANK	R_FIN	R_IND	R_MATB
Mean	**0.003793**	**-0.003588**	**0.003427**	**0.001009**	**-0.002636**	**0.005721**
Test of Hypothesis: Mean = 0.	(0.2586)	(0.5672)	(0.7581)	(0.3630)	(0.5663)	(0.1559)
Median	0.006093	-0.005278	0.004195	-0.000205	-0.001546	0.004245
Maximum	0.077494	0.242752	0.098027	0.115497	0.164319	0.133561
Minimum	-0.142611	-0.159740	-0.149215	-0.119671	-0.136387	-0.098913
Std. Dev.	0.035507	0.067629	0.040595	0.035375	0.049587	0.043320
Skewness	-0.802058	0.546170	-0.714985	0.054336	0.357373	0.412705
Kurtosis	5.179202	4.431049	4.953860	5.381032	4.323355	3.35290
Jarque-Bera	34.4749*	15.80040*	28.57912*	27.69548*	11.02788*	3.92847
P-value	0.000000	0.000371	0.000001	0.000001	0.004030	0.14026
Ljung-Box test (LB)						
LB(10)	14.487	6.2343	16.466	18.983	12.28	33.926
P-value	0.152	0.795	0.087	0.040	0.267	0.000
$LB^2(10)$	30.409	4.4216	36.942	27.239	7.485	11.255
P-value	0.001	0.926	0.000	0.002	0.679	0.338

ARCH-LM test						
q=1	9.588454	0.879670	15.58506	6.244880	0.018135	0.586420
p-value	0.0020	0.3483	0.0001	0.0125	0.8929	0.4438

[10] In all cases : when only a constant term was included in the model and when neither constant nor trend is included in the regression.

q=2	9.841302	1.174099	15.47718	6.449704	0.091164	0.718403
p-value	0.0073	0.5560	0.0004	0.0398	0.9554	0.6982

BDS	**R^2**	**R^2_AUTO**	**R^2_BANK**	**R^2_FIN**	**R^2_IND**	**R^2_MATB**
Statistic	0.066290	0.001329	0.056433	0.114171	0.025751	0.016328
z-Statistic	2.652494	0.056034	2.034179	4.432052	0.956560	0.657919
p-value	**0.0080**	0.9553	**0.0419**	**0.0000**	0.3388	0.5106

Note : p-value is reported for Test of Hypothesis: Mean = 0.

However, the existence of a possible structural break in the series, a common feature among the selected sectors during the period of analysis, may compromise the power of these tests or result in spurious stationarity. Therefore, to avoid these potential problems, we use the unit root test with endogenous structural breaks as proposed by (Zivot & Andrews, 1992).[11] Results give the previous conclusion. To test the null hypothesis of independance against the alternative of linear or non-linearity dependance, we employ the BDS independent test as described in (Brock, Dechert, Scheinkman, & LeBaron, 1996).[12] Results are given at Table1. Null hypothesis of independance is rejected for R, R_BANK, and R_FIN. This mean that significant dependance present in these series may be linear dependence or non-linear dependence. Hence, we may conclude that R_AUTO, R_IND, and R_MATB follow a white noise processes. To see if multivariate specification can be proposed for sectorial returns, we applied granger causality test to squared values of return series. From Table 2, we conclude that VECH models are to be determed for (R_BANK, R_FIN, R_MATB, R_IND). Figure 2 (b) shows some dependence in the individual asset returns with high peaks of volatility. This is further confirmed by the Ljung-Box (LB) test reported in summary (Table1). The jumps are particularly associated with Yesamin 2011 revolution periods for all of the series as the jumps are around 2010-2011 and later between 2016 and 2017. The spikes and the LB-Q statistics on the squared series suggests that the

[11] Results are illustrated at this Table. Structural breaks unit root test (One break) :

Variable	R	R_BANK	R_FIN
Date of breack	2011M01	2011M01	2010M07
Minimize Dickey-Fuller t-statistic	-10.29009*	-9.822557*	-10.22082*
Conclusion	SL2	SL2	SL2

Note : Citical values at 5% level is -4.443649.

[12] The BDS test is a portmanteau test for time based dependence in a series. It can be used for testing against a variety of possible deviations from independence including linear dependence, non-linear dependence, or chaos.

percentage changes of the series have some ARCH effects. All these test results reveal that we jointly model the observed facts of the first and second moments of the data generating process to investigate dependence structure of the variables within the multivariate framework, which will be discussed below.

Macro economic instability as volatility of consumer price index (Vol_LCPI), volatility of exchange rate (Vol_exrate) and volatility of Oil Price (Vol_LOP) can have effect on stock returns.[13] Table B 4 illustrates OLS results of effect of these factor on each of considered stock return mean. It is clear that Vol_LCPI has significant effect on R_FIN, R_BANK, and on R_MATB. Vol_lop has significant effect only on R_FIN, while Vol_exrate has significant effect on R_IND.

Table 2: Significant Granger Causality test results for **squared sectorial returns.**

Null Hypothesis:	Obs	F-Statistic	p-value
R^2_BANK does not Granger Cause R^2_FIN		8.80967	0.0003
R^2_MATB does not Granger Cause R^2_FIN	115	2.85537	0.0618
R^2_IND does not Granger Cause R^2_MATB		2.63690	0.0761

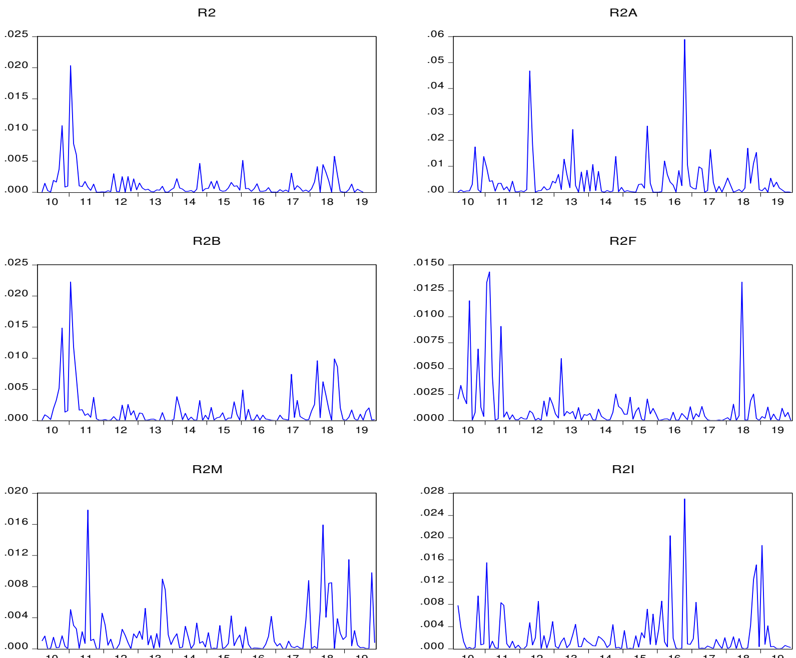

Figure 2 (b) : **Squared returns** for the Monthly time series (A : Automobile, Bank, F : finance, M :Matb, and I : Industry)

[13] The monthly Macro economic variable CPI, is collected from INS for the period from 2010M02 to 2019M07 (T = 114 observations). Source for variable OP : oil price [Europe Brent Spot Price FOB (Dollars per Barrel)] is EIA.

III. Volatility Modeling

Consider a multivariate return series R_t= $(R_{1t}, R_{2t}, \ldots, R_{kt})$ be a vector of returns of k number of assets at time index t (t = 1, 2, 3, …, T). The set of information available at time t is denoted by Ψ_{t-1}. We adopt the same approach as the univariate case by rewriting the series as :[14]

$$R_t = \mu_t + \varepsilon_t \text{ (1)}$$

where $\mu_t = E[R_t/\Psi_{t-1}]$ is the conditional expectation of R_t given the past information Ψ_{t-1}, and $\varepsilon_t = [\varepsilon_{1t}, \ \varepsilon_{2t}, \ \varepsilon_{3t}, \ \ldots, \varepsilon_{kt}]$ is the shock, or innovation, of the series at time t. The ε_t process is assumed to follow the conditional expectation of a multivariate time series model

$$\varepsilon_t \,/\, \Psi_{t-1} = H_t^{0.5} v_t, \text{ (2)}$$

where $v_t = [v_{1t}, \ v_{2t}, \ v_{3t}, \ \ldots, v_{kt}]$ is the independent and identically distributed (i.i.d.) random vectors of order k ×1 with E$[v_t]$ = 0 and E$[v_t v_t']$ = I , where I is the Identity matrix. The conditional covariance matrix of ε_t given Ψ_{t-1} is a $k \times k$ positive-definite matrix H_t defined by H_t = Cov(ε_t |Ψ_{t-1}).[15]

Model (1) with (2) can be written more compactly as $R_t \,/\, \Psi_{t-1}$~ D(μ_t, H_t), where D(., .) is some specified probability distribution.

Multivariate GARCH models are in spirit very similar to their univariate counterparts, except that the former also specify equations for how the covariances (or correlations) move over time. For general introductions to MGARCH models, reader can see (Bollerslev, Engle, & Wooldridge, 1988), (Bollerslev, Engle, & Nelson, 1994), (Bauwens, Laurent, & Rombouts, 2006), (Silvennoinen & Terasvirta), and (Engle R. F., 2009).

Several different multivariate GARCH (MGARCH) formulations have been proposed in the literature, including the basic VECH, the diagonal VECH, the BEKK models, the CCC, and the DCC specifications. Each of these is discussed in turn below.[16] These models are classified on 2 groups : one for conditional covariance presentation and the other for conditional correlation presentation.

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[14] The mean equations for returns i is the following:

$$R_{it} = \mu_{it} + \varepsilon_{it}, \text{ i = 1} \equiv \text{BANK, 2} \equiv \text{FIN, ...}$$

[15] Multivariate volatility modeling is concerned with the time evolution of H_t . We refer to a model for the $\{H_t\}$ process as a volatility model for the return series R_t .

[16] For an excellent survey of MGARCH models, see (Bauwens, Laurent, & Rombouts, 2004).

A. Conditional covariance presentation

Allowing the news and GARCH effects on volatility, various parameterizations for H_t have been proposed in the literature specifying (2) as given in the following subsections.

1. VECH model

A common specification of the VECH model is :[17]

$$\mathrm{VECH}(\mathrm{H_t}) = \Omega + \mathrm{A}\ \mathrm{VECH}(\varepsilon_{t-1}\varepsilon'_{t-1}) + \mathrm{B}\ \mathrm{VECH}(\mathrm{H_{t-1}}),\ (3)$$

$$\varepsilon_{t-1}|\Psi_{t-1} \sim D(0,\ H_t\ ),$$

where the coefficient matrices are $\Omega(\alpha_{ij,0})$, $A\ (\alpha_{ij})$, and $B\ (\beta_{ij})$. For the **bivariate** case (k = 2):[18]

$$\varepsilon_t = \begin{pmatrix}\varepsilon_{1t}\\ \varepsilon_{2t}\end{pmatrix},\ \mathrm{VECH}(H_t) = \begin{bmatrix}\sigma_{11,t}\\ \sigma_{22,t}\\ \sigma_{12,t}\end{bmatrix} \equiv \begin{bmatrix}\sigma^2{}_{1,t}\\ \sigma^2{}_{2,t}\\ \sigma_{12,t}\end{bmatrix},\ \text{and } \mathrm{VECH}(\varepsilon_t\varepsilon'_t) = \begin{bmatrix}\varepsilon_{1t}{}^2\\ \varepsilon_{2t}{}^2\\ \varepsilon_{1t}\varepsilon_{2t}\end{bmatrix}.$$

The VECH(1, 1) model in full is then given by the following **system**:

$$\begin{cases}\sigma^2{}_{1,t} = \alpha_{11,0} + \alpha_{11}\varepsilon^2{}_{1,t-1} + \alpha_{12}\varepsilon^2{}_{2,t-1} + \alpha_{13}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + \beta_{11}\sigma^2{}_{1,t-1} + \beta_{12}\sigma^2{}_{2,t-1} + \beta_{13}\sigma_{12,t-1}\\ \sigma^2{}_{2,t} = \alpha_{21,0} + \alpha_{21}\varepsilon^2{}_{1,t-1} + \alpha_{22}\varepsilon^2{}_{2,t-1} + \alpha_{23}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + \beta_{21}\sigma^2{}_{1,t-1} + \beta_{22}\sigma^2{}_{2,t-1} + \beta_{23}\sigma_{12,t-1}\\ \sigma_{12,t} = \alpha_{31,0} + \alpha_{31}\varepsilon^2{}_{1,t-1} + \alpha_{32}\varepsilon^2{}_{2,t-1} + \alpha_{33}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + \beta_{31}\sigma^2{}_{1,t-1} + \beta_{32}\sigma^2{}_{2,t-1} + \beta_{33}\sigma_{12,t-1.}\end{cases}$$

As the number of assets employed in the model increases, the estimation of the VECH model can quickly become infeasible.

2. Diagonal VECH model

(Bollerslev, Engle, & Wooldridge, 1988) introduce a restricted version of the general MVECH model of the conditional covariance. If A and B are assumed to be **diagonal**, the model will be known as a diagonal VECH as follow :[19]

$$\sigma_{ij,t} = \alpha_{ij,0} + \alpha_{ij}\varepsilon_{i,t-1}\varepsilon_{j,t-1} + \beta_{ij}\sigma_{ij,t-1}, for\ i,j = 1,2.\ (4)$$

Specifically, the **bivariate** DVECH(1, 1) satisfy the following system :

[17] The VECH operator takes the 'upper triangular' portion of a matrix, and stacks each element into a vector with a single column.

[18] $\mathrm{VECH}(\varepsilon_t\varepsilon'_t) = \mathrm{VECH}\begin{pmatrix}\varepsilon_{1t}\\ \varepsilon_{2t}\end{pmatrix}(\varepsilon_{1t}\quad \varepsilon_{2t}) = \mathrm{VECH}\begin{bmatrix}\varepsilon_{1t}{}^2 & \varepsilon_{1t}\varepsilon_{2t}\\ \varepsilon_{1t}\varepsilon_{2t} & \varepsilon_{2t}{}^2\end{bmatrix} = \begin{bmatrix}\varepsilon_{1t}{}^2\\ \varepsilon_{2t}{}^2\\ \varepsilon_{1t}\varepsilon_{2t}\end{bmatrix}.$

[19] For an algebric form see Annexe F.

$$\begin{cases} \sigma^2{}_{1,t} = \alpha_{11,0} + \alpha_{11,1}\varepsilon^2{}_{1,t-1} + \beta_{11,1}\sigma^2{}_{1,t-1} \\ \sigma^2{}_{2,t} = \alpha_{22,0} + \alpha_{22,1}\varepsilon^2{}_{2,t-1} + \beta_{22,1}\sigma^2{}_{2,t-1} \\ \sigma_{21,t} = \alpha_{21,0} + \alpha_{21,1}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + \beta_{21,1}\sigma_{21,t-1,} \end{cases}$$

where each element of H_t depends only on its own past value and the corresponding product term in $\varepsilon_{t-1}\varepsilon'_{t-1}$. That is, each element of a DVEC model follows a GARCH(1, 1) type model. Furthermore, this simple model **does not** allow for **dynamic** dependence between volatility series.[20] However, the model **may not** produce a **positive semi**-definite (PSD) **covariance matrix**. [21]

3. BEKK model

(Engle & Kroner, 1995) proposed a BEKK model that can be viewed also as a restricted version of the VECH model. The BEKK (q, p) model has the following form :[22]

$$H_t = \text{AA}' + \sum_{h=1}^{q} A_h\,(\varepsilon_{t-h}\varepsilon'_{t-h})A'_h + \sum_{h=1}^{p} B_h\,H_{t-h}B'_h$$

Where AA' is k× k, **lower triangular** matrix of constants (containing the intercept parameters of the volatility model), A_h and B_h are $k \times k$. The **diagonal** parameters in matrices A_h and B_h measure the effect of own past shocks and past volatility of market i on its conditional volatility. The off-diagonal elements in matrix A_h ($\alpha_{ij,h}$) and B_h ($\beta_{ij,h}$) i, j = 1, 2, 3, …, k measure respectively the cross-market effects **of shock spillover** and the cross effect of **volatility spillover**.

Based on the **symmetric** parameterization of the model, H_t is almost surely positive definite provided that AA' is **positive** definite. This model also allows for **dynamic** dependence between the volatility series.[23]

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[20] As summarized in (Ding & Engle, 2001), there are several approaches for specifying coefficient matrices that restrict H_t to be PSD, possibly by **reducing the number of parameters** (for more details see Annexe G).

[21] To guarantee the positive-definite constraint, (Engle & Kroner, 1995) propose the BEKK model.

[22] This model allows for dynamic dependence between the volatility series. This model is statistically sound but the interpretation of the model parameters is not straight forward.

[23] **EViews** does not estimate the general form of BEKK in which A and B are unrestricted. However, a common and popular form, diagonal BEKK, may be specified that restricts A and B to be **diagonals**. This Diagonal BEKK model is identical to the **Diagonal** VECH model where the coefficient matrices are **rank one matrices**.

On the other hand, the model has several **disadvantages**. First, the parameters in A_h and B_h **do** not have direct **interpretations** concerning lagged values of volatilities or shocks. Second, the **number** of parameters employed is $k^2(p + q) + k(k + 1)/2$, which **increases** rapidly with p and q.[24]

B. Conditional Correlation MGARCH Models

Since H_t is a symmetric matrix, it can be **take the** following form

$$H_t = (\sigma_{ij,t}) = \Gamma_t \Lambda_t \Gamma_t \qquad (5)$$

where Λ_t is the **conditional correlation** matrix of ε_t and Γ_t is $k\times k$ diagonal matrix consisting of the conditional **standard deviations** of elements of ε_t;

$$\Gamma_t = diag(\sqrt{\sigma^2}_{1,t}, \sqrt{\sigma^2}_{2,t}, \dots, \sqrt{\sigma^2}_{kk,t}).$$

In CC models, H_t is decomposed into a matrix of conditional correlations Λ_t and a diagonal matrix of conditional variances Γ_t where each conditional variance follows a univariate GARCH process and the **parameterizations** of Λ_t vary across models.

Because Λ_t is symmetric with unit diagonal elements, the time evolution of H_t is governed by that of the conditional variances $\sigma^2_{ii,t}$ and the elements $\rho_{ij,t}$ of Λ_t, where $j < i$ and $1 \le i \le k$. Therefore, to model the volatility of ε_t, it suffices to consider the **conditional variances** and **correlation** coefficients of ε_{it} .
Bollerslev (1990) specifies the elements of the conditional covariance matrix as given by the following system :

$$\sigma_{ij,t} = \boldsymbol{\rho_{ij,t}} \ \sigma_{i,t}\sigma_{j,t} \quad (6)$$

Where $\sigma^2_{i,t}$ is modeled by the following univariate GARCH process

$$\sigma^2_{i,t} = \alpha_{ii,0} + \alpha_{ii,1}\varepsilon^2_{i,t-1} + \beta_{ii,1}\sigma^2_{i,t-1}.$$

Equation (6) indicates that CC models use nonlinear combinations of univariate GARCH models to represent the conditional covariances and that the parameters in the model for $\boldsymbol{\rho_{ij,t}}$ describe the extent to which the errors from equations i and j move together. We can define then the $k(k + 1)/2$-dimensional vector

$$\Xi_t = (\sigma^2_{1,t}, , \dots, \sigma^2_{k,t}, \rho_t')',$$

to be estimated, where ρ_t is $k(k-1)/2$-dimentional vector obtained by stacking column of the correlation matrix Λ_t, where,[25]

[24] Some applications show that many of the estimated parameters are statistically insignificant, introducing additional complications in modeling.

[25] Restrictions may be imposed on the constant term using **variance targeting** so that :

$$\alpha_{ii,0} = \sigma^2(1 - \alpha_{ii,1} - \beta_{ii,1})$$

Where σ^2 is the unconditional variance. Exogenous variables can be included in the mean/variance specification, with individual coefficients or common coefficients. For common

$$\rho_{ij,t} = \frac{\sigma_{ij,t}}{\sqrt{\sigma^2{}_{i,t}\sigma^2{}_{j,t}}} \quad (7)$$

The following three CC models differ in how they **parameterize** Λ_t.

1. Constant Conditional Correlation (CCC)

To keep the number of volatility equations low, (Bollerslev, 1990) considers the special case in which the correlation coefficient

$$\rho_{ij,t} = \rho_{ij}$$

is time-invariant, with $|\rho_{ij}| < 1$.[26] Under such an assumption, ρ_{ij} is a **constant** parameter and the volatility model consists of k equations for

$$\Xi_t^* = (\sigma^2{}_{1,t}, \dots, \sigma^2{}_{k,t}).$$

A GARCH(1, 1) model for Ξ_t^* becomes

$$\Xi_t^* = \alpha_0 + \alpha_1 \varepsilon^2{}_{t-1} + \beta_1 \Xi_{t-1}^*,$$

where $\varepsilon^2{}_{t-1} = (\varepsilon^2{}_{1,t-1}, \varepsilon^2{}_{2,t-1}, \dots, \varepsilon^2{}_{k,t-1})'$, α_0 is k-dimensional positive vector, and α_1 and β_1 are k × k non-negative definite matrices.

Volatility **forecasts** of the model can be obtained by using forecasting methods similar to those of a vector ARMA(1, 1) model. The 1-step ahead volatility forecast at the forecast origin *h* is

$$\Xi_h^*(1) = \alpha_0 + \alpha_1 \varepsilon^2{}_h + \beta_1 \Xi_h^*.$$

For the *l*-step ahead forecast for the marginal volatilities of ε_{it}, we have

$$\Xi_h^*(l) = \alpha_0 + (\alpha_1 + \beta_1)\Xi_h^*(l-1),\ l > 1.$$

The *l*-step ahead forecast of the covariance between ε_{it} and ε_{jt} is

$$\hat{\rho}_{ij}\sqrt{\sigma^2{}_{i,h}(l)\sigma^2{}_{j,h}(l)}$$

where $\hat{\rho}_{ij}$ is the estimate of ρ_{ij} and $\sigma^2{}_{i,h}(l)$ are the elements of $\Xi_h^*(l)$.

2. Non constant conditional correlation

Using Equation (5), several authors have proposed parsimonious models for Λ_t to describe the time-varying correlations. We discuss **two** such developments for *k*-dimensional returns.

---

coefficients, exogenous variables are assumed to have the same slope, λ, for every equation. Individual coefficients allow each exogenous variable effect δ_i to differ across equations ;

$$\mu_{it} = \delta_i X_{1i,t} + \lambda X_{2i,t}$$

$$\sigma^2{}_{i,t} = \alpha_{ii,0} + \alpha_{ii,1}\varepsilon^2{}_{i,t-1} + \beta_{ii,1}\sigma^2{}_{i,t-1} + \delta_i Z_{1i,t} + \lambda Z_{2i,t} \ . \ (8)$$

[26] It is for this reason that the model is known as a constant conditional correlation (CCC) MGARCH model. Restricting Λ_t to a constant matrix reduces the number of parameters and simplifies the estimation but may be too strict in many empirical applications.

a) Dynamic conditional correlation (DCC)

(Engle R. F., 2002) introduced a dynamic conditional correlation (DCC) MGARCH model in which the conditional quasicorrelations Λ_t follow a GARCH(1, 1)-like process. (Engle R. F., 2002) propose the model

$$\Lambda_t = \Upsilon_t \mathrm{Q}_t \Upsilon_t,$$

where Q_t=($\mathrm{q}_{ij,t}$) is a k$\times k$ positive-definite matrix,

$$\Upsilon_t = diag(q_{11,t}^{-1/2}, \dots, q_{,t}^{-1/2})$$

and

$$\mathrm{Q}_t = (1 - \theta_1 - \theta_2)\mathrm{Q} + \theta_1 a_{t-1} a'_{t-1} + \theta_2 \mathrm{Q}_{t-1} \quad (9)$$

where a_t is the standardized innovation vector with elements $a_{it} = \varepsilon_{it}/\sqrt{\sigma^2{}_{i,t}}$, Q is the unconditional covariance matrix of a_t, θ_1 and θ_2 are non-negative scalar parameters satisfying $0 < \theta_1 + \theta_2 < 1$, and Υ_t is a normalization matrix to guarantee that Λ_t is a correlation matrix.

b) Varying conditional correlation (VCC)

(Tse & Tsui, 2002) derived the varying conditional correlation (VCC) MGARCH model in which the conditional correlations at each period are a weighted sum of a time-invariant component, a measure of recent correlations among the residuals, and last period's conditional correlations.

(Tse & Tsui, 2002) assume that the conditional correlation matrix Λ_t follows the model :

$$\Lambda_t = (1 - \theta_1 - \theta_2)\Lambda + \theta_1 \Lambda_{t-1} + \theta_2 \Psi_{t-1} \quad (10)$$

where θ_1 and θ_2 are scalar parameters ($0 \leq \theta_1 + \theta_2 < 1$), Λ is k$\times k$ positive-definite matrix with unit diagonal elements, and Ψ_{t-1} is the k$\times k$ **sample correlation** matrix using shocks from t −m, . . . , t − 1 for a prespecified *m*.

3. Test for volatility spillovers Effects

Refer to the multivariate volatility model of Section III.A.2 (DVECH), the following hypotheses are of interest to test for volatility spillovers effects across assets. Considering **six stock market returns**, covariances take the following system :

σ1,2 = α+ α (1,2)*ε1(-1)* ε2(-1) + β (1,2)* σ1,2(-1)

σ1,3 = α + α (1,3)* ε1(-1)* ε3(-1) + β (1,3)* σ1,3(-1)

σ1,4 = α + α (1,4)* ε1(-1)* ε4(-1) + β (1,4)* σ1,4(-1)

σ1,5 = α + α (1,5)* ε1(-1)* ε5(-1) + β (1,5)* σ1,5(-1)

$\sigma_{1,6} = \alpha + \alpha\ (1,6)^* \ \varepsilon_1(-1)^* \ \varepsilon_6(-1) + \beta\ (1,6)^* \ \sigma_{1,6}(-1)$

$\sigma_{2,3} = \alpha + \alpha\ (2,3)^* \ \varepsilon_2(-1)^* \ \varepsilon_3(-1) + \beta\ (2,3)^* \ \sigma_{2,3}(-1)$

$\sigma_{2,4} = \alpha + \alpha\ (2,4)^* \ \varepsilon_2(-1)^* \ \varepsilon_4(-1) + \beta\ (2,4)^* \ \sigma_{2,4}(-1)$

$\sigma_{2,5} = \alpha + \alpha\ (2,5)^* \ \varepsilon_2(-1)^* \ \varepsilon_5(-1) + \beta\ (2,5)^* \ \sigma_{2,5}(-1)$

$\sigma_{2,6} = \alpha + \alpha\ (2,6)^* \ \varepsilon_2(-1)^* \ \varepsilon_6(-1) + \beta\ (2,6)^* \ \sigma_{2,6}(-1)$

$\sigma_{3,4} = \alpha + \alpha\ (3,4)^* \ \varepsilon_3(-1)^* \ \varepsilon_4(-1) + \beta\ (3,4)^* \ \sigma_{3,4}(-1)$

$\sigma_{3,5} = \alpha + \alpha\ (3,5)^* \ \varepsilon_3(-1)^* \ \varepsilon_5(-1) + \beta\ (3,5)^* \ \sigma_{3,5}(-1)$

$\sigma_{3,6} = \alpha + \alpha\ (3,6)^* \ \varepsilon_3(-1)^* \ \varepsilon_6(-1) + \beta\ (3,6)^* \ \sigma_{3,6}(-1)$

$\sigma_{4,5} = \alpha + \alpha\ (4,5)^* \ \varepsilon_4(-1)^* \ \varepsilon_5(-1) + \beta\ (4,5)^* \ \sigma_{4,5}(-1)$

$\sigma_{4,6} = \alpha + \alpha\ (4,6)^* \ \varepsilon_4(-1)^* \ \varepsilon_6(-1) + \beta\ (4,6)^* \ \sigma_{4,6}(-1)$

$\sigma_{5,6} = \alpha + \alpha\ (5,6)^* \ \varepsilon_5(-1)^* \ \varepsilon_6(-1) + \beta\ (5,6)^* \ \sigma_{5,6}(-1)$

Volatility **Spillovers** from stock j to each stock i ≠ j can then be tested in the following null hypotheses:

$$H_{0,1}:\ \alpha_{1j} = \ \beta_{1j} = 0,\ \text{for } j = 2, 3, 4, 5, 6,$$

against

$$H_{1,1}: \alpha_{1j} \neq 0,\ \ \beta_{1j} \neq 0,$$

$$H_{0,2}:\ \alpha_{2j} = \ \beta_{2j} = 0,\ \text{for } j = 3, 4, 5,\ 6$$

against

$$H_{1,2}: \alpha_{2j} \neq 0,\ \ \beta_{2j} \neq 0,$$

$$H_{0,3}:\ \alpha_{3j} = \ \beta_{3j} = 0,\ \text{for } j = 4, 5,\ 6$$

against

$$H_{1,3}: \alpha_{3j} \neq 0,\ \ \beta_{3j} \neq 0,$$

$$H_{0,4}:\ \alpha_{4j} = \ \beta_{4j} = 0,\ \text{for } j = 5,\ 6$$

against

$$H_{1,4}: \alpha_{4j} \neq 0,\ \ \beta_{4j} \neq 0,$$

$$H_{0,5}:\ \alpha_{5j} = \ \beta_{5j} = 0,\ \text{for } j = 6$$

against

$$H_{1,5}: \alpha_{5j} \neq 0,\ \ \beta_{5j} \neq 0,$$

by applying **Chi-square** tests.

IV.Results and discussion

We have Monthly data on the stock returns of five sectors—Bank, Finance, Industry, Material, and Automobil, from 2010M02 to 2019M07—in the variables

R_Bank, R_Fin, R_Ind, R_Matb, and R_Auto respectively. We model the conditional means of the returns as a constant or a function of some explicative variables and the conditional covariances as a MGARCH- DVECH, -DBEKK, -DCC, and -CCC process in which the variance of each disturbance term follows a GARCH(1, 1) process. ARCH estimation uses maximum likelihood to jointly estimate the parameters of the mean and the variance equations. Given a specification for the mean equation and a distributional assumption, all that we require is a specification for the conditional covariance (correlation) matrix.
In first stage, we consider the first system

$$\begin{cases} R_t = \mu + \varepsilon_t \\ H_t = \mathrm{Cov}(\varepsilon_t \,|\Psi_{t-1}) \end{cases} \quad (I)$$

with only μ as a constant conditional mean. Then, we consider the second system

$$\begin{cases} R_t = \lambda_1 X_{1,t} + \lambda_2 X_{2,t} + \lambda_3 X_{3,t} + \varepsilon_t \\ H_t = \mathrm{Cov}(\varepsilon_t \,|\Psi_{t-1}) \end{cases} \quad (II)$$

where expected mean depends on several macroeconomic variables reflecting macro economic instability effect ; $X_{1,t}, X_{2,t},$ and $X_{3,t}$ are respectively volatility of oil price in log, volatility of consumer price index in log, (LCPI), and volatility of excghange rate in log.

We can estimate the parameters of MGARCH models by maximum likelihood (ML), assuming that the errors come from a multivariate normal distribution or **Student's** t-distribution.[27] Both the ML estimator and the quasi–maximum likelihood (QML) estimator, which drops the normality assumption, are assumed to be consistent and normally distributed in large samples; see (Jeantheau, 1998), (Berkes & Horvath, 2003), (Comte & Lieberman, 2003), (Ling & McAleer, 2003), and (Fiorentini & Sentana, 2007). The QML parameter estimates are the same as the ML estimates, but the VCEs are different.[28]

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[27] Assuming multivariate **normality**, the log likelihood contributions for GARCH models are given by

$$l_t = -\frac{1}{2}k.log(2\pi) - \frac{1}{2}\log(|H_t|) - \frac{1}{2}\varepsilon'_t H_t \varepsilon_t$$

where k is the number of mean equations, and ε_t the k vector of mean equation residuals.
For **Student's** *t*-distribution, the contributions are of the form

$$l_t \propto C - \frac{1}{2}\log(|H_t|) - \frac{1}{2}(\eta + k)log\left(1 + \frac{\varepsilon'_t {H_t}^{-1}\varepsilon_t}{\eta - 2}\right)$$

where η is the estimated degree of freedom.

[28] The choice between the multivariate normal and the multivariate t distributions is one between robustness and efficiency. If the disturbances come from a multivariate Student t, then the ML estimates will be **consistent and efficient**, while the QML estimates based on the multivariate normal assumption will be **consistent but not efficient**. In contrast, if the disturbances come from a well-behaved distribution that is neither multivariate Student t nor multivariate normal, then the ML estimates based on the multivariate Student t assumption will not be consistent, while the QML estimates based on the multivariate normal assumption will be consistent but not efficient (see STATA documentation ts.pdf pp 353-4).

For both system, QML estimator is obtained under **Student** distribution. This result is based on the assumption that the multivariate t-distribution has common but unknown degrees of freedom. The shapeparameter η is estimated to 10 (approximately). Matrix of the ARCH and GARCH parameters are A (α_{ij}), and B (β_{ij}).

C. System (I)

In This section, we consider the first system

$$\begin{cases} R_t = \mu + \varepsilon_t \\ H_t = \mathrm{Cov}(\varepsilon_t \,|\Psi_{t-1}) \end{cases}$$

with μ is a constant as conditional mean. System (I) is estimated with different conditional covariance specifications given by DVECH, DBEKK, and CCC. Results are illustrated at Table B 5.

From Figure A 5 and Figure A 6 (see Appendice II), the **correlation** looks to be time varying, which is a general characteristic of DVECH and DBEKK models. For instance, the time-varying correlations of the DBEKK(1, 1) model appear to be less volatile. That from CCC model (see Table B 5, Appendice II), all the estimated constant conditional **correlation** ρ_{ij} parameters are **positive** and significant indicates that the returns on these stocks **rise or fall together**. Theses correlations reflect the agent's behavior in the sector depending on the state of the economy. ρ_{ij} parameters have **decreasing** evolution for full period or for recent year for almost all i and j. From Figure A 6 (for DBEKK model), only correlation between TUNindex return and R_Fin (or R_BANK) and correlation between R_FIN and R_IND (or R_MATB) which have recent **encreasing** evolution after a **decreasing** pattern till 2016 or 2018.

While the log likelihood value is lower for DVECH and DBEKK, we may compare the three models by looking at model selection criterion. The Akaike, (Schwarz) and Hannan-Quinn all show lower (higher) information criterion values for the CCC specification than VECH and DBEKK model, suggesting that the time-varying Diagonal specifications are not preferred**. CCC** model is then the **best**. From Table B 5 (see Appendice II), not all parameter estimates are significant at the 5% level, and the fitted conditional mean and volatility model are

$$R_{1t} = -0.000359 + \varepsilon_{1t}$$

$$\sigma^2_{1,t} = 0.000515 + 0.063777\varepsilon^2_{1,t-1} + 0.477957\sigma^2_{1,t-1},\ 1 \equiv \text{Return}$$

$$R_{2t} = -0.004210 + \varepsilon_{2t}$$

$$\sigma^2_{2,t} = 26.38E-07-0.090209\,\varepsilon^2_{2,t-1} + \mathbf{1.084453}\sigma^2_{2,t-1},\ 2 \equiv \textbf{Fin}$$

$$R_{3t} = 0.000590 + \varepsilon_{3t}$$
$$\sigma^2_{3,t} = 0.000942 + 0.117784\,\varepsilon^2_{3,t-1} + 0.239348\sigma^2_{3,t-1},\ 3 \equiv \text{Bank}$$

$$R_{4t} = -0.009345 + \varepsilon_{4t}$$
$$\sigma^2_{4,t} = 0.000296 + 0.155613\varepsilon^2_{4,t-1} + \mathbf{0.719893}\sigma^2_{4,t-1},\ 4 \equiv \textbf{Ind}$$

$$R_{5t} = -0.002443 + \varepsilon_{5t}$$
$$\sigma^2_{5,t} = 0.000189 + 0.158480\varepsilon^2_{5,t-1} + \mathbf{0.760376}\sigma^2_{5,t-1},\ 5 \equiv \textbf{Matb}$$

and

$$R_{6t} = -0.014398 + \varepsilon_{6t}$$
$$\sigma^2_{6,t} = 0.000323\mathbf{-0.158443}\varepsilon^2_{6,t-1} + \mathbf{1.11614}\sigma^2_{6,t-1},\ 6 \equiv \textbf{Auto.}$$

Only coefficients in **bold** are significant (in 5% or 10% level). System (I) in CCC specification implies no dynamic volatility dependence between different markets. As expected all Constant Conditional Correlations (CCC) are positive (Table B 5). This reflects a simultaneous growth between different sector. The estimated correlation ρ_{ij} are between 0.295445 and 0.933575, reflecting some higher integration within Tunisian stock markets.

The Ljung–Box statistics as model checking approach is to apply the multivariate *Q*-statistics to the *sixth standardized residual series*. For this particular CCC(1, 1) model, we have $Q_2(10)$ = 361.4224 (0.4690), where the number in parentheses denotes *p*-value. Based on this statistic, the mean equation is then adequate at the 5% significance level.

Figure A 7, Figure A 8, and Figure A 9 (see Appendice II) show repectively the fitted **volatilities** of the CCC, DVECH, and DBEKK(1, 1) models. Comparing these Figures, there are some differences between the three fitted volatility models. However, it is clear that for **Bank, Financial** sectors, and TUNindex return volatility are ***decreasing*** while for **Auto** sector and **MATB** sector, volatility is ***increasing***.

Since Schwarz critiria is minimum for DBEKK specification, this model may be the best but we can not analyse Volatility **Spillover** for each return with this diagonal model. So, with DVECH model, the existence of any causal relation among variance and covariance included in H_t (implying that the off-diagonal coefficients of $A(\alpha_{ij})$ and $B(\beta_{ij})$ are statistically significant) will be investigated.[29]

[29] In fact, α_{ij} and β_{ij} respectively measure the effect of the own and cross past shock and past conditional volatility of the other markets.

Looking to DVECH model results (Table B 5, Appendice II), almost all $\alpha_{1(i,j)}$ are not significant except $\alpha_{1(1,2)} \approx 0.23$ and $\alpha_{1(1,3)} \approx 0.3$. Hence that, we can conclude the presence of a significance and **positive effect** only of **cross shock** (news) of Finance and Bank stock returns on Tunindex return. And there is unidirectional effect between **Tunindex** and **financial** and **Bank** stock market.

The fitted conditional mean, volatility, and covariance of **DVECH model** is summed as follow : [30]

R = 0.0014923+ ε1	$\sigma^2$1 = **0.000309 + 0.310962** $\varepsilon^2$1 (-1) **+ 0.525732** $\sigma^2$1(-1)
R_FIN = -0.00353+ ε2	$\sigma^2$2 = **0.0003099 +** 0.27796 $\varepsilon^2$2(-1) **+ 0.7021** $\sigma^2$2(-1)
R_BANK = 0.0031458+ ε3	$\sigma^2$3 = **0.0003099 + 0.29646** $\varepsilon^2$3(-1) **+ 0.5353** $\sigma^2$3(-1)
R_IND = **-0.010267+** ε4	$\sigma^2$4 = **0.0003099+ 0.151388** $\varepsilon^2$4(-1) + **0.743438** $\sigma^2$4(-1)
R_MATB = -0.0031543+ ε5	$\sigma^2$5 = **0.0003099** + 0.16156 $\varepsilon^2$5(-1) + **0.784216** $\sigma^2$5(-1)
R_AUTO = **-0.015434+** ε6	$\sigma^2$6 = **0.0003099 -0.07314984** $\varepsilon^2$6(-1) + **1.02114** $\sigma^2$6(-1)

With

σ1,2 = 0.0003099 + **0.2284** ε1(-1) ε2(-1) + **0.5959** σ 1,2(-1)

σ 1,3 = 0.0003099+ **0.29271** ε1(-1) ε3(-1) + **0.517866** σ 1,3(-1)

σ 1,4 = 0.000309 + 0.12962 ε1(-1) ε4(-1) + **0.62094** σ 1,4(-1)

σ 1,5 = 0.000309 + 0.125595 ε1(-1) ε5(-1) + **0.64067** σ 1,5(-1)

σ 1,6 = 0.0003099 -0.027536 ε1(-1) ε6(-1) + **0.7508** σ 1,6(-1)

σ 2,3 = 0.00030991 + 0.208801 ε2(-1) ε3(-1) **+ 0.59174** σ 2,3(-1)

σ 2,4 = 0.0003099 + 0.0538 ε2(-1) RESID4(-1) + **0.70460** σ 2,4(-1)

σ 2,5 = 0.0003099 + 0.02198 ε2(-1) ε5(-1) + **0.725672** σ 2,5(-1)

σ 2,6 = 0.0003099 + 0.006479 ε2(-1) ε6(-1) + **0.75882** σ 2,6(-1)

σ 3,4 = 0.000309 + 0.07271 ε3(-1) ε4(-1) + **0.6152** σ 3,4(-1)

σ 3,5 = 0.0003099 + 0.07441 ε3(-1) ε5(-1) + **0.630839** σ 3,5(-1)

σ 3,6 = 0.0003099 -0.00923 ε3(-1) ε6(-1) + **0.660136** σ 3,6(-1)

σ 4,5 = 0.0003099 + 0.03062 ε4(-1) ε5(-1) + **0.75712** σ 4,5(-1)

σ 4,6 = 0.000309 -0.00152 ε4(-1) ε6(-1) + **0.78651** σ 4,6(-1)

σ 5,6 = 0.0003099 -0.12678 ε5(-1) ε6(-1) + **0.83521** σ 5,6(-1)

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[30] Only coefficient in **bold** are significant (in 1%, 5% or 10% level).

The results also demonstrate **spillover effects** in the volatility models since all $\beta_{1(i,j)}$ are significant at the 0.01 level. The test results at **Table 3** suggest significant volatility spillovers from BANK, FIN, IND, MATB, and AUTO sectors to TUNindex stock, from BANK, IND, MATB, and AUTO sectors to FIN sector, from IND, MATB, and AUTO sectors to BANK sector. The test result also suggests significant **volatility spillovers** from **MATB** sector, and **AUTO** sector to **IND** sector and from **AUTO** sector to **MATB** sector. These observations suggest that these Tunisian's sectorial asset markets are interlinked and transmit volatility spillovers across sectorial asset markets. This information is useful in planning for future investment decisions both by individuals and financial institutions to minimize risk.

Table 4 reveals that ML estimation based on Student t distribution is adequate with data analysis results since all residuals (except from TUNindex equation) are not Normally distributed.

Table 3 : **Volatility spillover** effect from 2010M02 to 2019M07.

Hypothesis spillover	**From j =**	**To i =**	**Chi-square**	**p-Value**
$H_{0,1}$	BANK, FIN, IND, MATB, and AUTO	R≡1	219.1257	0.0000
$H_{0,2}$	BANK, IND, MATB, and AUTO	FIN≡2	165.9716	0.0000
$H_{0,3}$	IND, MATB, and AUTO	BANK≡3	106.1206	0.0000
$H_{0,4}$	**MATB, and AUTO**	**IND**≡4	183.0273	0.0000
$H_{0,5}$	**AUTO**	**MATB**≡5	163.6256	0.0000

Table 4 : Normality hypothesis results for residuals of **DVECH** Model .

Component	**Jarque-Bera**	**df**	**p-Value**
TUNindex≡1	1.423358	2	0.4908
BANK≡2	249.9431	2	0.0000
FIN ≡3	15.06615	2	0.0005
IND≡ 4	17.21621	2	0.0002
MATB ≡5	2168.750	2	0.0000
AUTO ≡6	107.7597	2	0.0000
Joint	2560.159	12	0.0000

D. System (II)

Then, we consider the second system

$$\begin{cases} R_t = \lambda_1 X_{1,t} + \lambda_2 X_{2,t} + \lambda_3 X_{3,t} + \varepsilon_t \\ H_t = \mathrm{Cov}(\varepsilon_t \,|\Psi_{t-1}), \end{cases}$$

where expected mean depend on several macroeconomic variables reflecting instability effect ; $X_{1,t}, X_{2,t},$ and $X_{3,t}$ are respectively volatility of oil price in log, volatility of consumer price index in log (LCPI), and volatolity of excghange rate in log. System (II) is estimated with only DCC and CCC conditional correlation. Results for the second system are illustrated at Table B 6.

The MGARCH- DCC model reduces to the MGARCH - CCC model when θ_1 = θ_2= 0. The output shows that a Wald test rejects the null hypothesis that θ_1 = θ_2= 0 at all conventional levels since chi2(2) = 86.69 with p-value = 0.0000. These results indicate that the assumption of *time-invariant conditional correlations* maintained in the MGARCH-CCC model **is too restrictive** for these data. Moreover the log likelihood value is lower for CCC specification, and AIC show lower information criteria values for DCC model (see Table B 6 Appendice II), suggesting that the dynamic MGARCH-DCC is the preferred model. Only results of **DCC specification** will be then discussed.

Table B 6 (see Appendice II) first presents results for the mean or variance parameters used to model each dependent variable. Subsequently, the output table presents results for the conditional correlation parameters. For example, the conditional correlation between the standardized residuals for Bank and Finance is estimated to be 0.57. The higher conditional correlation, 0.92, is between the standardized residuals for TUNindex return and Bank, and is followed by conditional correlation of 0.66 between the standardized residuals for TUNindex return and Finance market.

Again, all the estimated conditional **correlation** parameters are positive and significant indicates that the returns on these stocks rise or fall together.

Table B 6 (see Appendice II) reports **Wald test** against the null hypothesis that all the coefficients on the independent variables in the mean equations are zero [Wald = 55.82 (p-value = **0.0000**)]. Here the null hypothesis is rejected at the 5% level. This mean that macro economic instability factors (as oil price volatility, consumer price index volatility, and exchange rate volatility) have significant effect on the mean of returns evolutions (at 5% or 10% level). Moreover, **VOL_exrate** has significant **positive** effect on R, R_FIN, and on R_MATB, while **VOL_LCPI** has significant **negative** effect on R_FIN and **VOL_LOP** has significant **negative** effect on R_AUTO.

Table B 6 indicates also that each of the univariate **ARCH** and univariate **GARCH** are statistically significant for **both TUNindex** return and **AUTO** sectorial returns. While for **Bank** sector only univariate **ARCH** is statistically significant. And for FINance, INDustry and MATB sectorial returns, only univariate **GARCH** are statistically significant. The **shot-run** volatility parameters $\alpha_{i,1}$ are significant between 0.01 and 0.1 level while the **long-run** parameters $\beta_{i,1}$ are significant at the 0.01 level. The value of R^2 is not reported in the table because the model is highly non-linear therefore R^2 is not a meaningful measure of goodness of fit. Further, the multivariate model is tested for model adequacy using the Ljung-Box (**LB**) statistics [Portmanteau test for white noise (test for model adequacy)] on the residuals and squared residuals of the model.

Table 5 provides the Ljung-Box test results. The **LB** test results fail to suggest any model inadequacy of serial dependence of the model errors.

We plot the **dynamic correlations** across sectorial markets (Figure 3). Theses correlations reflect the agent's behavior in the sector depending on the state of the economy. In fact, there is a dynamic correlation over the period and across sectors. First, we find the θ_2 coefficient is positively significant (see Table B 6 in Appendice II). Most series show an effect of the Yesameen 2011 Revolution in Tunisia 2011. Second, in the following, we look at Figure 3 to analyse evolution of these correlations.

Table 5 : Univariate and multivariate Ljung-Box test for model adequacy.

	Return	R_FIN	R_BANK	R_IND	R_MATB	R_AUTO	Multivar
Portmanteau (LB-Q) statistic	26.694	26.6762	30.9286	44.1172	47.2282	32.0311	26.6943
Prob > chi2(40)	0.9471	0.9474	0.8477	0.3017	0.2011	0.8112	0.9471
Portmanteau (LB-Q²) statistic	35.974	26.8140	40.9574	26.3955	34.6251	23.5288	35.9747
Prob > chi2(40)	0.6520	0.9452	0.4283	0.9517	0.7104	0.9823	0.6520

The correlation of the **TUNindex return** and all considered sector is dynamic and positive throughout the sample period. The positive relationship became strong during the Yesameen 2011 Revolution with **BANK and FINance** sectors. The correlation reached the peak level about **0.97**.

The correlation of the **FINance** sector and **all** other considered sector is also dynamic and positive throughout the sample period. The positive relationship vary between 0.17 and 0.82, and became strong during the Yesameen 2011 Revolution with **BANK** sector. The correlation reached the peak level of **0.82** *between Financial sector and bank sector*.

However, the correlation between sector **MATB** market and sector **AUTO** market wasn't affected at the beginning by the revolution. In fact, the correlation is still *low* and approximately equal to 0.21. This correlation has a stationnary evolution around 0.2.

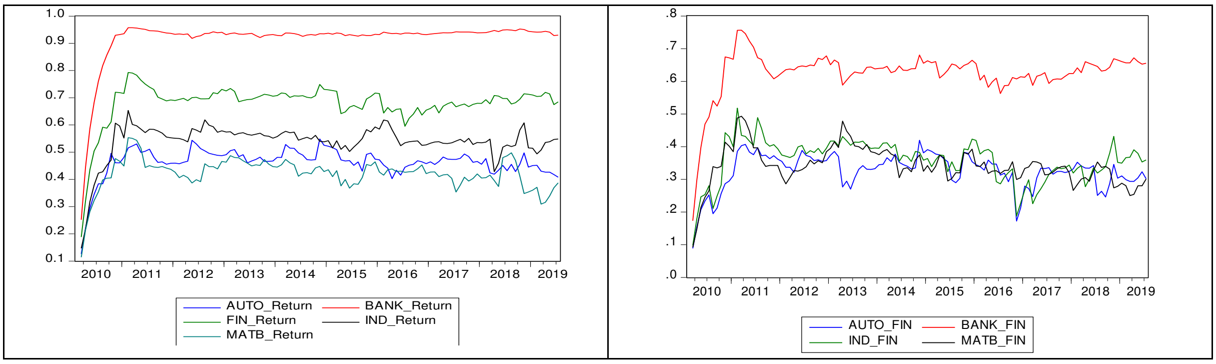
At the beginning of 2010, we note *independence* between **Bank** and **Matb** markets, as the correlation is near zero (equal 0.057). Followed by an *increase*, the correlation reached the peak level of 0.74 at the end of 2017 and is *decreased* later to reach level less than 0.2.

For **all the others** (except correlation between IND and Bank and correlation between IND and AUTO), the correlation, during revolution have a peak around 0.5 - 0.6, showing that **integration** has been *increased* by the revolution to get *decreasing* and very low value throughout the rest of sample period.

Correlation between **IND** and **Bank** and between **IND** and **AUTO** have rather a *stationary* puttern with some outliers for the latter at the end of 2016 (low value near .02) and the begening of 2017 and 2019 (high values less than 0.6).

From Figure 4, the graph show that the in-sample predictions are quite similar for the conditional variances except for AUTO sector and that the dynamic forecasts converge to similar levels. It also shows that the ARCH and GARCH parameters cause substantial time-varying volatility for AUTO sector.

In addition, the results indicate that increases in the futures oil prices lead to lower returns on the AUTO stock, and increased input prices (via exchange rate volatility) lead to lower returns on the MATB stock. Increases in LCPI volatility (and in exchange rate volatility) lead to lower (higher) return on the Finance stock. In addition, increases in exchange rate volatility (**a depreciation of local money**) lead to higher return on the TUNindex stock. Two sectors, Bank and INDustry, have no significant effect of considered volatility of macro economic variables : oil price, CPI (consumer price index) and exchange rate in log.



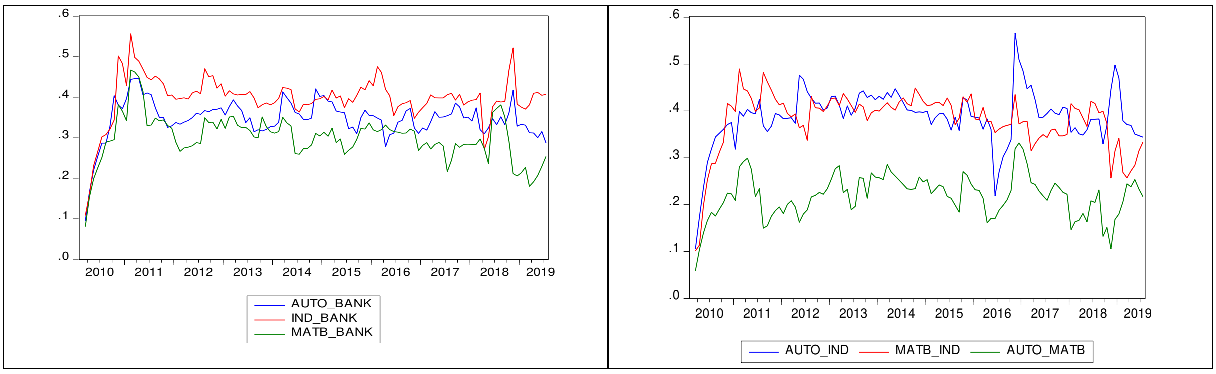


Figure 3: Evolution of D C **correlation** from MGARCH(1, 1)-**DCC** model for System (II).

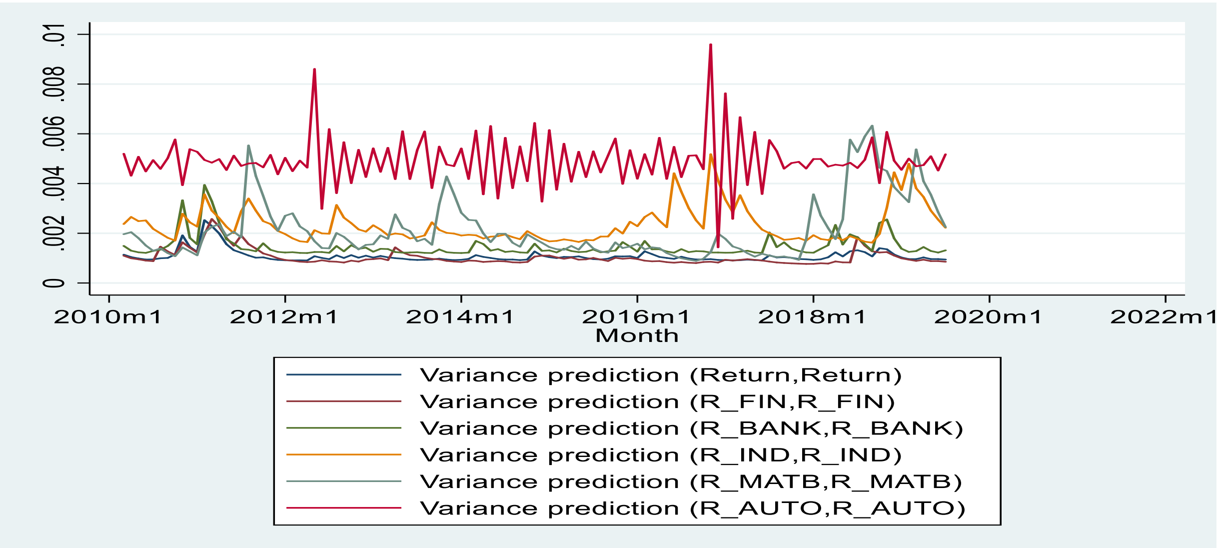


Figure 4: C **Variance** prediction from MGARCH(1, 1)-**DCC** model for System (II)

V. Conclusion

This paper aims at examining the volatility experiences in 5 tunisian sectorial stock index series and TUNindex series. The Monthly returns of stock indices of 5 sectors (namely : BANK, FINancial service, AUTOmobile, INDdustry, and MATerials (MATB)) have been considered from 2010M02 to 2019M07. The objectif in this paper is to proposes multivariate GARCH volatility models to assess the dynamic interdependence among volatility of returns. Two system are considered.

The first System, with conditional (C) constant (C) mean, allows for market interaction. Results from the DVECH model reveals that some sectorial stock markets are **interdependent,** the presence of a significance and **positive effect** of **cross shock** of TUNindex return on FINance and BANK stock returns, and volatility is **predictable**. C corroletation (C), ρ_{ij} , have **decreasing** evolution for full period or for recent years for almost all i and j except CC between Tunindex return and R_FIN (or R_BANK) and CC between R_FIN and R_IND (or R_MATB). The asymptotic chi-square tests for volatility spillovers effects suggests significant **volatility spillovers** From MATB sector, and AUTO sector to IND sector and from AUTO sector to MATB sector. And there is unidirectional effect between **TUNindex** and **financial** and **Bank** stock market.

The second system, with macroeconomic factor **instability effects** as conditional mean**,** examine the CCC and dynamic (D) CC between different sectors. Three macroeconomic factor are considered ; oil price volatility, consumer price index volatility, and exchange rate volatility.

The conditional correlations are time invariant is a restrictive assumption. The MGARCH-DCC model nest the MGARCH-CCC model. When we test the time-invariance assumption with **Wald tests** on the parameters of this more general model, we reject the null hypothesis that these conditional correlations are time invariant. We examine then the estimated result of time-varying variance-covariance by the **DCC (1, 1) model**. The main result supports the hypotheses of DCC. The DCC provides evidence of cross border relationship within sectors. We do find evidence of **integration** of some sectors through the volatility.

Moreover, Macro economic instability factors have significant effect on the mean of returns evolutions (at 5% or 10% level). Hence, **VOL_exrate** has significant **positive** effect on TUNindex return R, on R_FIN, and on R_MATB, while **VOL_LCPI** has significant **negative** effect on R_FIN, and **VOL_LOP** has significant **negative** effect on R_AUTO.

VI. Bibliographie

Aftab, H., Beg, R., Sun, S., & Zhou, Z. (2019). Testing and Predicting Volatility Spillover—A Multivariate GJR-GARCH Approach. *Theoretical Economics Letters, 9*, 83-99.

Baba, Y. (1992). The demand for M1 in USA, 1960-1988. *Review of Economic Statistics, 59*, 25-61.

Bauwens, L., Laurent, S., & Rombouts, J. V. (2004). Multivariate GARCH models: a survey. *Journal of Applied Econometrics (to appear)*.

Bauwens, L., Laurent, S., & Rombouts, J. V. (2006). Multivariate GARCH models: A survey. *Journal of Applied Econometrics, 21*, 79–109.

Becketti, S. (2013). *Introduction to Time Series Using Stata* . College Station, TX: Stata Press.

Berkes, I., & Horvath, L. (2003). The rate of consistency of the quasi-maximum likelihood estimator. *Statistics and Probability Letters, 61*, 133–143.

Bollerslev, T. (1990). Modelling the coherence in short-run nominal exchange rates: A multivariate generalized ARCH model. *Review of Economics and Statistics, 72*, 498–505.
Bollerslev, T., & Wooldridge, J. M. (1992). Quasi Maximum Likelihood Estimation and Inference in Dynamic Models with Time Varying Covariances. *Econometric Reviews, 11*(2), 143-72.
Bollerslev, T., Chou, R. Y., & Kroner, K. F. (1992). ARCH modeling in finance. *Journal of Econometrics, 52*, 5–59.
Bollerslev, T., Engle, R. F., & Wooldridge, J. M. (1988). A capital asset pricing model with time-varying covariances. *Journal of Political Economy , 96*, 116 –131.
Bollerslev, T., Engle, R., & Nelson, D. B. (1994). *ARCH Models.* Handbook of Econometrics, 4,: in R.F. Engle and D. McFadden eds.
Brock, W. A., Dechert, D., Scheinkman, H., & LeBaron, B. (1996). A Test for Independence Based on the Correlation Dimension. *Econometric Reviews, 15*, 197--235.
Christiansen, C. (2007). Volatility-Spillover Effects in European Bond Markets. *European Financial Management, 13*(5), 923-948.
Comte, F., & Lieberman, O. (2003). Asymptotic theory for multivariate GARCH processes. *Journal of Multivariate Analysis, 84*, 61–84.
Davidson, R., & MacKinnon, J. G. (1993). *Estimation and Inference in Econometrics.* New York: Oxford University Press.
Ding, Z., & Engle, R. F. (2001). Large Scale Conditional Covariance Matrix Modeling, Estimation and Testing. *Academia Economic Paper, 29*, 157–184.
Elliot, G., Rothenberg, T., & Stock, J. (1996). Efficient test for an autoregressive unit root. *Econometrica, 64*, 813–836.
Enders, W. (2004). *Applied Econometric Time Series.* 2nd ed. New York: Wiley.
Engle, R. (1982). Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation. *Econometrica, 50*, 987–1007.
Engle, R. F. (2009). *Anticipating Correlations: A New Paradigm for Risk Management.* Princeton, NJ: Princeton University Press.
Engle, R. F. (2002). Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of Business & Economic Statistics, 20*, 339–350.
Engle, R. F., & Kroner, K. F. (1995). Multivariate Simultaneous generalized ARCH. *Econometric Theory, 11*, 122-150.
Engle, R., & Granger, C. (1987). Cointegration and error correction representation : Estimation and testing. *Econometrica, 55*, 251-276.
Engle, R., & Kroner, K. (1995). Multivariate Simultaneous Generalized ARCH. *Econometric Theory, 11*, 122-150.
Fama, E. (1990). Stock returns, Expected Returns, and Real Activity. *Journal of Finance*, 1189-1108.
Fiorentini, G., & Sentana, E. (2007). On the efficiency and consistency of likelihood estimation in multivariate conditionally heteroskedastic dynamic regression models . *Working paper 0713, CEMFI, Madrid, Spain. ftp://ftp.cemfi.es/wp/07/0713.pdf.*
Greene, W. H. (2018). *Econometric Analysis.* 8th ed. New York: Pearson.
Hamilton, J. D. (1994). *Time Series Analysis.* Princeton, New Jersy: Princeton university press.
Hammoudeh, S., & Li, H. (2008). Sudden changes in volatility in emerging markets: The case of Gulf Arab stock markets. *International Review of Financial Analysis, 17*, 47–63.

Hammoudeh, S., Yuan, Y., & McAleer, M. M. (2009). Shock and volatility spillovers among equity sectors of the Gulf Arab stock markets. *The Quarterly Review of Economics and Finance, 49*(3), 829-842.

Harris, D., McCabe, B., Leybourne, & S.J. (2003). Same limit theory for autocovariances whose order depends on sample size. *Econ. Theory, 10*, 829–864.

Harris, R., & Pisedtasalasai, A. (2006). Return and Volatility Spillovers between Large and Small Stocks in the UK. . *Journal of Business Finance and Accounting, 33*(9-10), 1556-1571.

Harvey, A. C. (1989). *Forecasting, Structural Time Series Models and the Kalman Filter.* Cambridge: Cambridge University Press.

Harvey, D., Leybourne, S., & Xiao, B. (2008.). A powerful test for linearity when the order of integration is unknown. *Stud. Nonlinear Dyn. Econ.12, Article 2.*

Hassan, S., & Malik, F. (2007). Multivariate GARCH modeling of sector volatility Transmission. *Quarterly Review of Economics and Finance, 47*, 470-480.

Jeantheau, T. (1998). Strong consistency of estimators for multivariate ARCH models. *Economic Theory, 14*, 70–86.

Kapetanios, G., Shin, Y., & Snell, A. (2003). Testing for a unit root in the nonlinear star framework. *Journal of Econometrics, 12*, 359–379.

Kouki, I., Harrathi, N., & Haque, . (2011). A Volatility Spillover among Sector Index of International Stock Markets. *Journal of Money, Investment and Banking, 22*.

Koulakiotis, A., Dasilas, A., & Papasyriopoulos, N. (2009). Volatility and error transmission Spillover effects: Evidence from three European financial regions. *The Quarterly Review of Economics and Finance, 49*(3), 858-869.

Lee, J., & Strazicich, M. (2003). . Minimum LM unit root test with two structural breaks. *Rev. Econ. Stat., 85* (4), 1082–1089.

Lee, J., & Strazicich, M. C. (2004). Minimum LM Unit Root Test with One Structural Break. *Working Paper 17. Department of Economics, Appalachian State*.

Li, H., & Majerowska, E. (2008). Testing stock market linkages for Poland and Hungary: A multivariate GARCH approach. *Research in International Business and Finance, 22*, 247-266.

Ling, S., & McAleer, M. (2003). Asymptotic theory for a vector ARM–GARCH model. *Economic Theory, 19*, 280–310.

Luukkonen, R., Saikkonen, P., & Teräsvirta, T. (1988). Testing linearity against smooth transition autoregressive models. *Biometrica, 75*, 491–499.

Maddala, G., & Kim, I.-M. (1998). *Unit roots, Cointegration, and Structural Change.* Cambridge university press.

Malik F. and Hammoudeh, S. (2007). Shock and volatility transmission in the oil, US and Gulf equity markets. . *International Review of Economics and Finance, 16*, 357-368.

MALLIKARJUNA, M., & RAO, R. P. (2019). Volatility experience of major world stock markets. *Theoretical and Applied Economics, 4*(621), 35-52.

Ng, S., & Perron, P. (2001). Lag length selection and the construction of unit root tests with good size and power. *Econometrica, 69*, 1519–1554.

Silvennoinen, A., & Terasvirta, T. (s.d.). *Multivariate GARCH models.* 2009: In Handbook of Financial Time Series, ed.

Stock, J. H., & Watson, M. W. (2015). *Introduction to Econometrics.* Updated 3rd ed. Hoboken, NJ: Pearson.

Teräsvirta, T. (1994). Specification, estimation and evaluation of smooth transition autoregressive models. *J. Am. Stat. Assoc., 89*, 208–218.

Tse, Y. K., & Tsui, A. K. (2002). A multivariate generalized autoregressive conditional heteroscedasticity model with time-varying correlations. *Journal of Business & Economic Statistics, 20*, 351–362.

Wongswan, J. (2006). Transmission of information across international equity markets. *Review of Financial Studies, volume, 19*(4), 1157–1189.

Wooldridge, J. M. (2016). *Introductory Econometrics: A Modern Approach.* 6th ed. Boston: Cengage.

Zivot, E., & Andrews, D. (1992). Further evidence on the great crash, the oil price shock and the unit root Hypothesis. *Journal of Business and Economic Statistics, 10*.

VII. Appendice I : Selected review and data Analysis

A. Table of Selected Review

Table 1 (a): Selected review

Author	Problem/technic	Data	Results
(Harris & Pisedtasalasai, 2006)	Return and spillover effects	FTSE100, FTSE250 and FTSE Small Cap equity	Volatility **transmission** mechanism between **large** and **small** stocks in the UK is asymmetric
(Wongswan, 2006)	Information transmission	U.S. Japan Korean and Thai equity markets	There is a large and significant **association** between developed market and emerging market equity volatility at **short time** horizons
(Christiansen, 2007)	Mean and volatility spillover	US and Europ	Negligible mean-**spillover** but volatility spillover effects was substantial.
(Hassan & Malik, 2007)	Mean and conditional variance	US **sector** indexes	**Transmission of shocks** and volatility among different sectors
(Malik F. and Hammoudeh, 2007)	Volatility and shock transmission ; **MGARCH**	US equity, Gulf equity and global crude oil	Significant **transmission** among second moments Gulf equity markets are the recipients of volatility from the oil market.
Zarour and Siriopoulos (2008)	**Univariate** CGARCH		Existence of **volatility** composition into **short run** and **long run** components
(Hammoudeh & Li, 2008)	**Univariate** GARCH, persistence of volatility, (ICSS) algorithm[31]	For 5 Gulf Cooperation Council (GCC) stock markets	These stock markets are more **sensitive** to major global events than to local and regional factors

[31] Iterated cumulative sums of squares algorithm

(Li & Majerowska, 2008)		Emerging stock markets and the developed markets	Evidence of returns and volatility **spillovers** from the developed to the emerging markets implying that foreign investors may benefit from risk reduction by adding emerging markets' stocks to their portfolio
(Hammoudeh, Yuan, & McAleer, 2009)	VAR(1)–GARCH(1,1) model	3 major sectors (Service, Banking and Industrial/or Insurance) in 4 GCC's economies (Kuwait, Qatar, Saudi Arabia and UAE)	Past own volatilities matter more than past shocks and there are moderate volatility **Spillovers** between the **sectors** within the individual countries, with the exception of Qatar
(Koulakiotis, Dasilas, & Papasyriopoulos, 2009)	**Multivariate** GARCH-BEKK, examine the transmission of news (both volatility and error) between portfolios of cross-listed equities	Three European financial regions.	Finnish and Danish portfolios of cross-listed equities are the main **transmitters** of volatility relative to the Swedish and Norwegian portfolios of cross-listed equities.
(Kouki, Harrathi, & Haque, 2011)	Volatility **Spillove**r among Sector, VAR-BEKK model, dynamic conditional correlation (DCC)	5 sectors (banking, financial service, industrial, real estate and oil) in International Stock Markets (two zones)	Evidence of cross border relationship within sectors. Evidence of **integration** of some **sectors** through the volatility.
(Aftab, Beg, Sun, & Zhou, 2019)	**Multivariate** VAR-BEKK-GJR-GARCH	Australia's domestic stock, bond, and money markets	Domestic financial markets are **interdependent** and volatility is **predictable**. Volatility **spillovers** from stock market to bond and to money markets due to common news.
(MALLIKARJUNA & RAO, 2019)	*GARCH* *EGARCH* *TGARCH*	Australia (ASX 200), Canada (TSX), France (CAC 40), Germany (DAX), Japan (NIKKEI 225) South Korea (KOSPI), Switzerland (SMI), United Kingdom (FTSE 100), and the United States of America (S&P 500). The	The volatility is highly **persistent** in all the markets, informational asymmetries and leverage effects exist in the developed and emerging markets, whereas the **frontier** markets do not exhibit any tendencies of informational **asymmetries and leverage** effects except the stock market of Argentina.

		markets in the emerging group are Brazil (BOVESPA), China (SSEC), Egypt (EGX 30), India (SENSEX), Indonesia (IDX), Mexico (BMV IPC), Russia (MOEX), South Africa (JSE 40), Thailand (SET), and Turkey (BIST 100). The markets in the frontier category are Argentina (S&P MERVAL), Estonia (TSEG), Kenya (NSE 20), Sri Lanka (CSE AS), and **Tunisia (TUNINDEX).**	

B. Figures

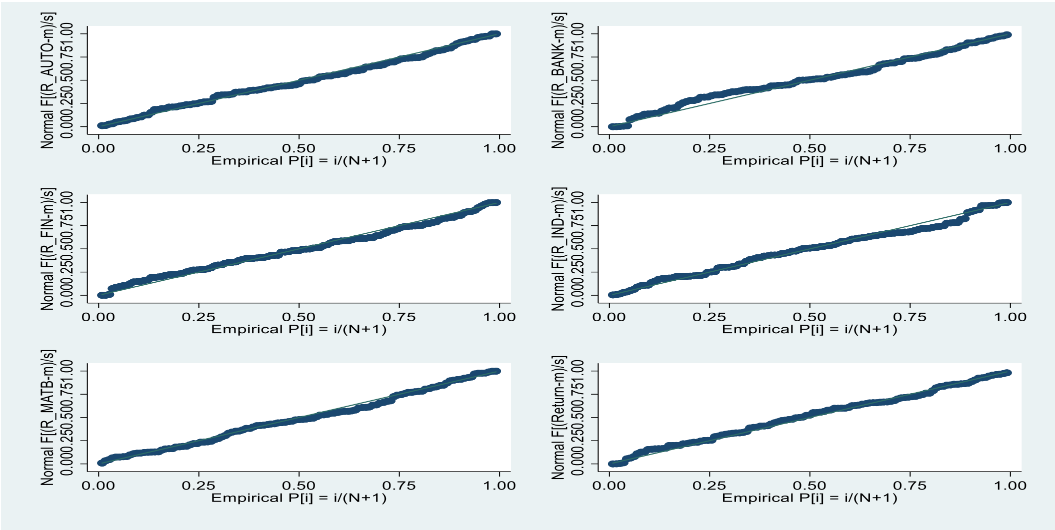


Figure A 1: The quantiles of normal distribution plot.

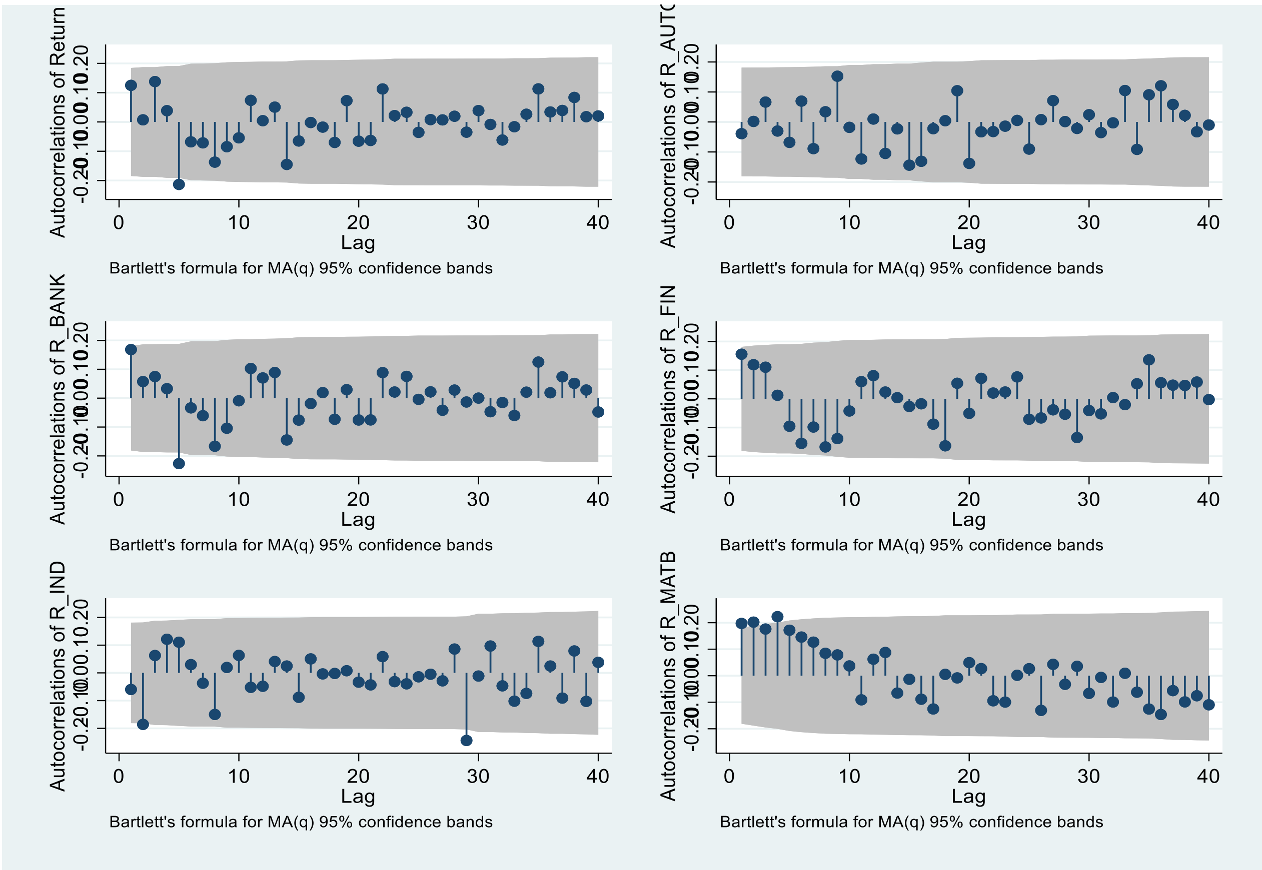


Figure A 2: Correlograms of autocorrelation function for **Monthly Return** data.

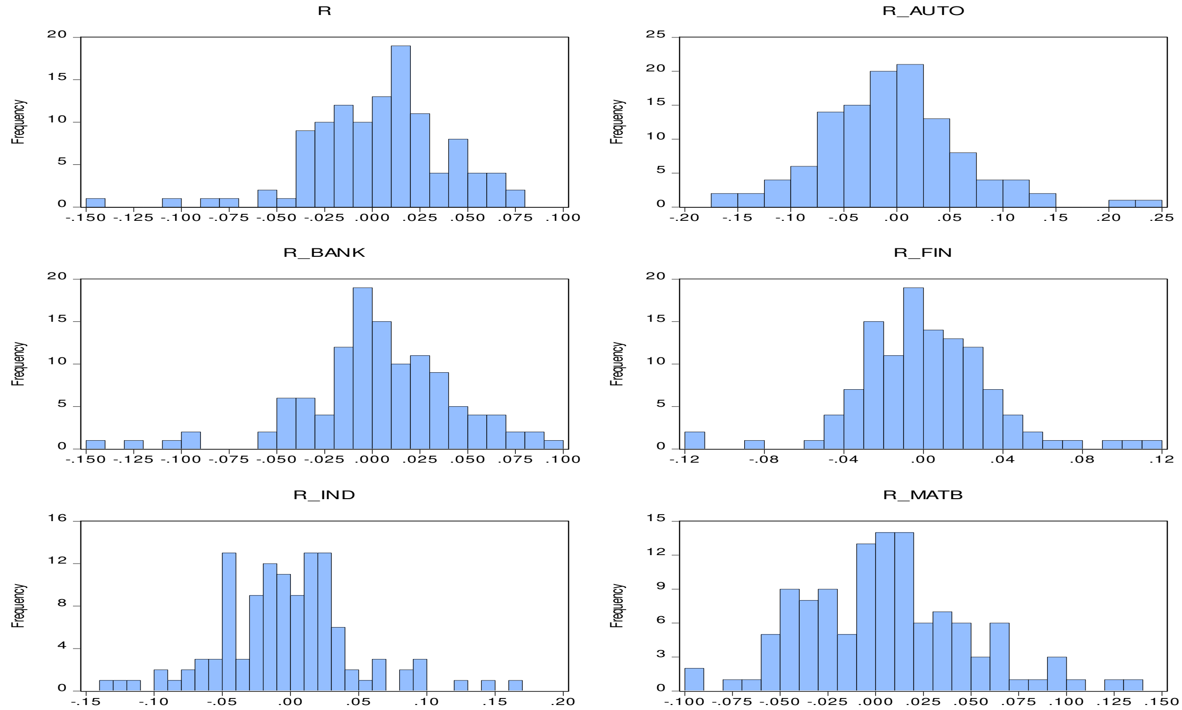


Figure A 3 : Histogram of Monthly Stock return by sector.

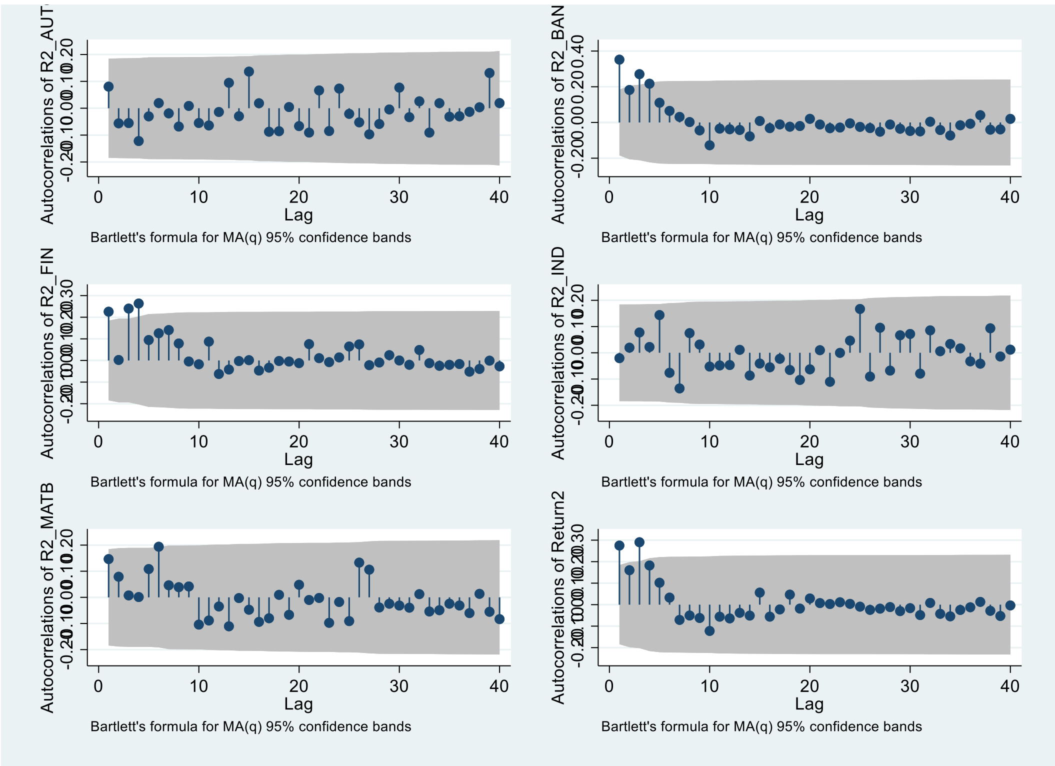


Figure A 4: Correlograms of autocorrelation function **for Squared Monthly Return** data.

C. Tables

Table B 1: Correlations between Different stock Markets **returns** in level for period 2010M02-2019M07.

Correlation **Probability**	**R**	**R_AUTO**	**R_BANK**	**R_FIN**	**R_IND**	**R_MATB**
R	1.000000					

R_AUTO	0.468642	1.000000				
	0.0000	-----				
R_BANK	0.941535	0.352801	1.000000			
	0.0000	0.0001	-----			
R_FIN	0.716654	0.323154	0.661598	1.000000		
	0.0000	0.0005	0.0000	-----		
R_IND	0.554835	0.409363	0.413083	0.384554	1.000000	
	0.0000	0.0000	0.0000	0.0000	-----	
R_MATB	0.425288	0.234617	0.306709	0.342560	0.394976	1.000000
	0.0000	0.0124	0.0010	0.0002	0.0000	-----

Table B 2: Correlations between Different stock Markets returns in **squared level** for period 2010M02-2019M07.

Correlation Probability	R^2	R^2_AUTO	R^2_BANK	R^2_FIN	R^2_IND	R^2_MATB
R^2	1.000000					
R^2_AUTO	0.070701	1.000000				
	0.4568	-----				
R^2_BANK	0.935731	-0.000404	1.000000			
	0.0000	0.9966	-----			
R^2_FIN	0.634896	-0.026941	0.575420	1.000000		
	0.0000	0.7770	0.0000	-----		
R^2_IND	0.268240	0.391530	0.228190	0.144112	1.000000	
	0.0041	0.0000	0.0151	0.1278	-----	
R^2_MATB	0.118294	-0.032910	0.102424	0.079996	0.085265	1.000000
	0.2121	0.7293	0.2804	0.3996	0.3692	-----

Table B 3: Unit root test results for Monthly data **stock returns** (PP, ADF, and KPSS).

				PP test			
		R	R_AUTO	R_BANK	R_FIN	R_IND	R_MATB
With Constant	t-Statistic	-9.3188	-11.1033	-9.0822	-9.2381	-11.6389	-9.4411
	Prob.	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		***	***	***	***	***	***
Without Constant & Trend	t-Statistic	-9.2551	-11.1175	-9.0730	-9.2753	-11.6094	-9.3915
	Prob.	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		***	***	***	***	***	***
				ADF test			
		R	R,AUTO	R,BANK	R,FIN	R,IND	R,MATB
With Constant	t-Statistic	-9.2491	-11.1032	-9.0052	-9.1885	-11.5025	-5.4089
	Prob.	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		***	***	***	***	***	***
Without Constant & Trend	t-Statistic	-9.1985	-11.1174	-8.9900	-9.2278	-11.4972	-5.3539
	Prob.	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		***	***	***	***	***	***
				KPSS test			
		R	R,AUTO	R,BANK	R,FIN	R,IND	R,MATB
With Constant	t-Statistic	0.0888	0.0560	0.1007	0.0846	0.1481	0.2044
	Prob.	n0	n0	n0	n0	n0	n0

Table B 4: Macroeconomic instability effect on Mean returns.

Dependent Variable: R_FIN				R_BANK			R_IND			R_MATB			R_AUTO		
Variable	Coefficient	Std. Error	P-value	Coefficient	Std. Error	P-value	Coefficient	Std. Error	P-value.	Coefficient	Std. Error	P-value	Coefficient	Std. Error	P-value
VOL_LCPI	-4054.315	1192.628	**0.0009**	-2475.148	1439.858	**0.0884**	-2743.753	1753.538	0.1206	-6054.396	1408.371	**0.0000**	-8.594226	2451.429	0.9972
VOL_LEXRATE	32.63307	15.35449	**0.0358**	25.48829	18.53745	0.1720	38.59145	22.57592	**0.0902**	-22.92594	18.13207	0.2088	34.28737	31.56092	0.2797
VOL_LOP	0.733387	0.434920	**0.0946**	0.438497	0.525079	0.4055	-0.774829	0.639470	0.2283	-0.063327	0.513596	0.9021	-1.106129	0.893972	0.2186
C	0.025378	0.013898	0.0706	0.015960	0.016779	0.3436	0.015591	0.020435	0.4471	0.079724	0.016412	0.0000	-0.010025	0.028567	0.7263
Adjusted R-squared			0.117819			0.021000			0.041766			0.144814			-0.004286
Log likelihood			225.3908			204.1033			181.8320			206.6018			143.9730
F-statistic			5.986040			1.800815			2.627242			7.321904			0.840670
Prob(F-statistic)			**0.000815**			0.151309			**0.053963**			**0.000162**			0.474452
Mean dependent var			0.001474			0.003570			-0.002448			0.007040			-0.003625
S.D. dependent var			0.035691			0.040904			0.050352			0.042808			0.068759
Akaike info criterion			-3.918422			-3.541652			-3.147470			-3.585873			-2.477398
Schwarz criterion			-3.821877			-3.445107			-3.050925			-3.489328			-2.380853
Hannan-Quinn criter.			-3.879245			-3.502475			-3.108293			-3.546696			-2.438221
Durbin-Watson stat			1.858202			1.675200			2.192646			1.956068			2.089861

VIII. Appendice II : Econometric results

D. Figures

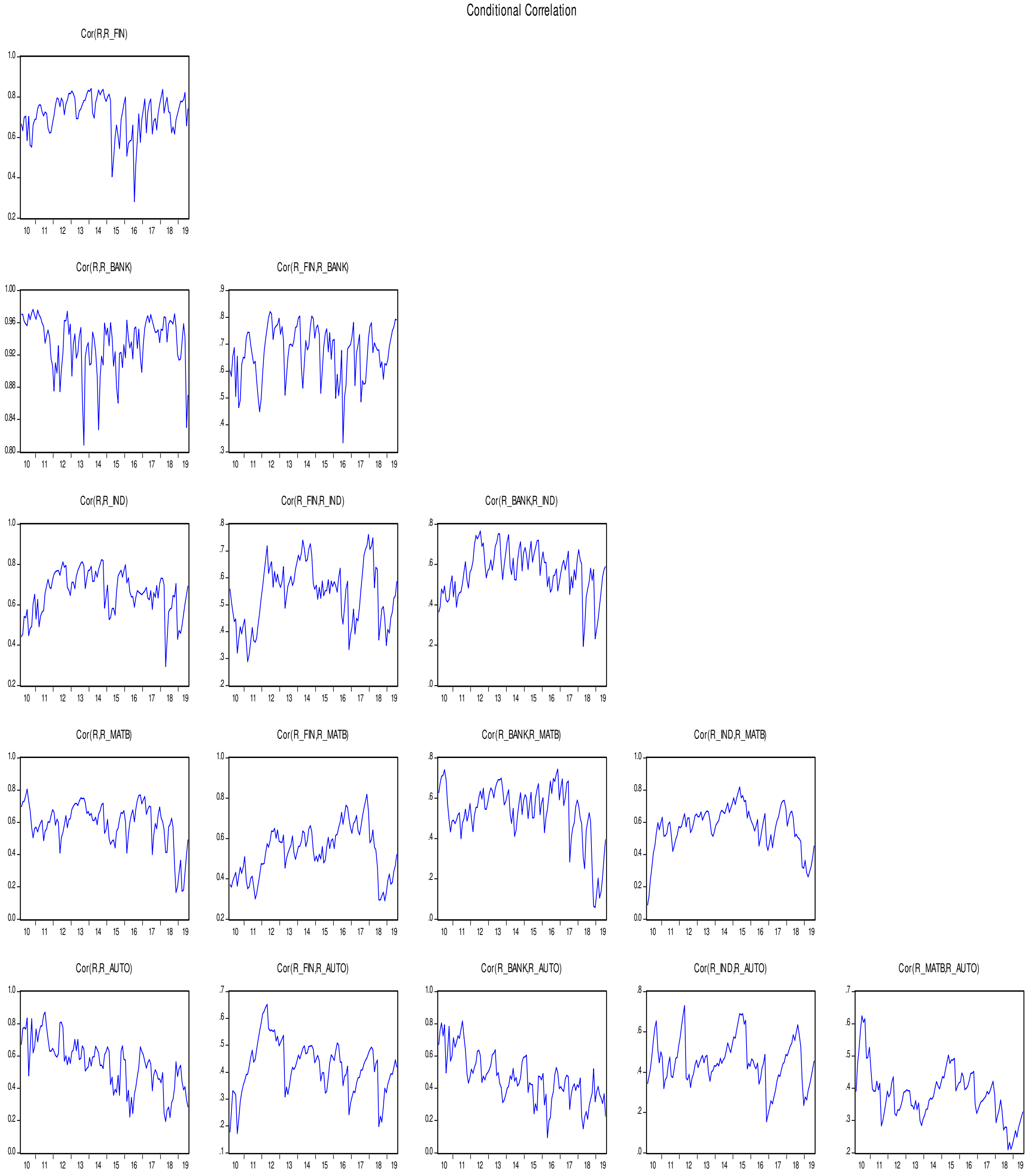


Figure A 5： Time-varying **correlations** from **DVECH** model (system I).

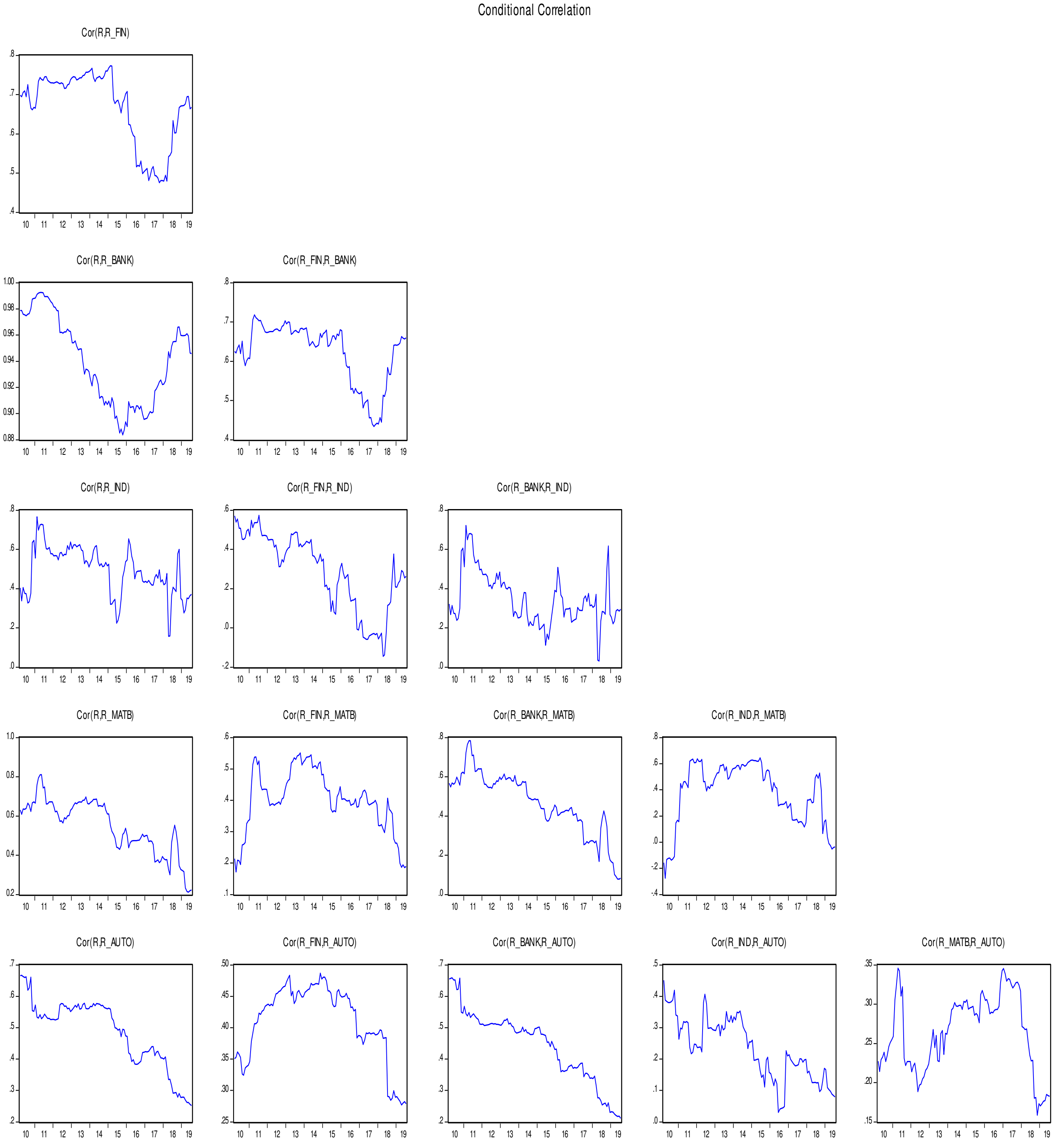


Figure A 6：Time-varying **correlations** from **DBEKK** model (system I).

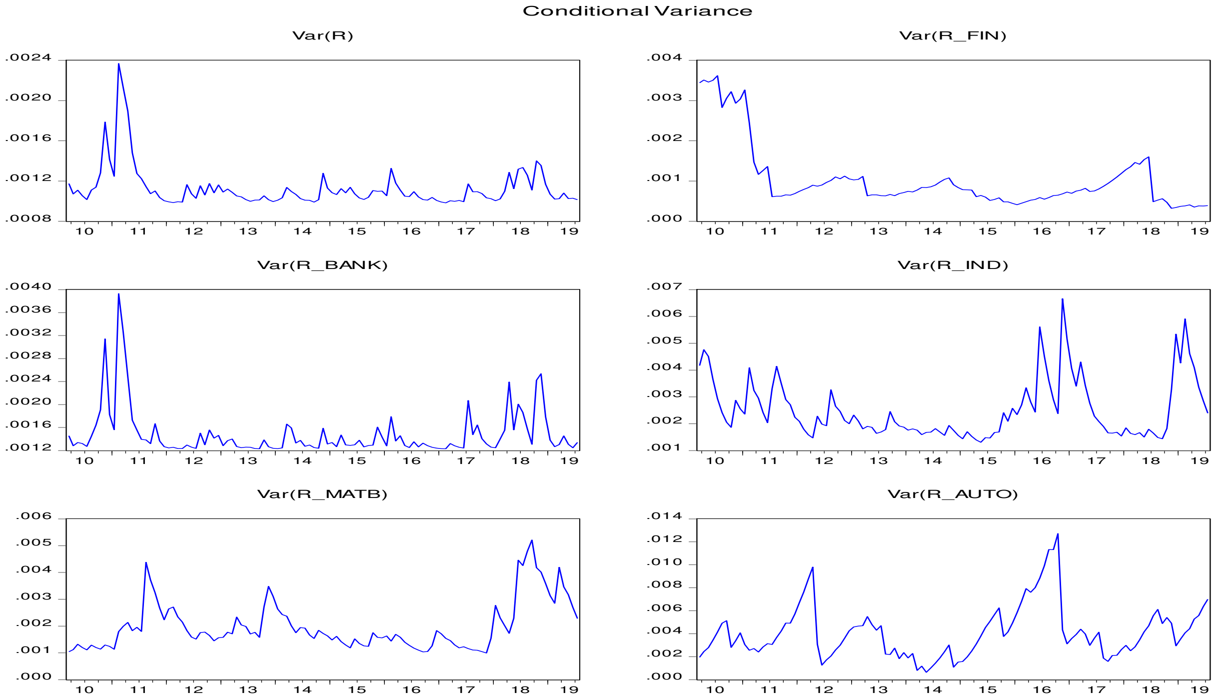


Figure A 7：Conditional **variances** from **CCC** model (system I).

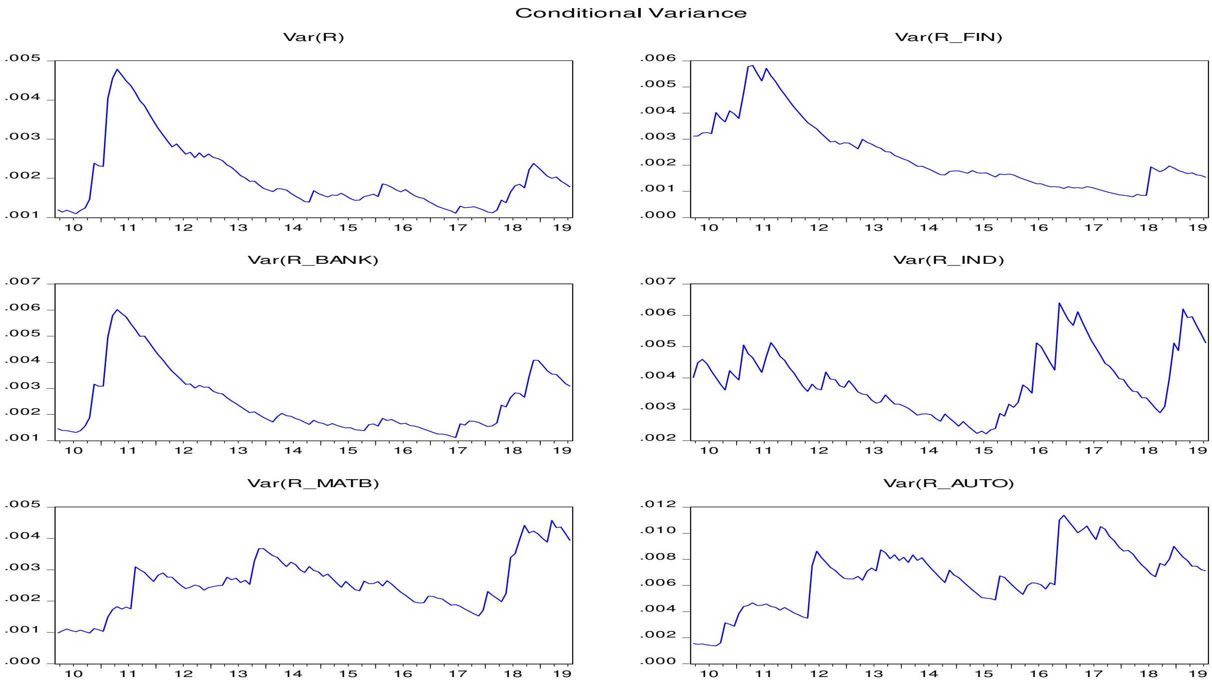


Figure A 8：Conditional **variances** from **DVECH** model (system I).

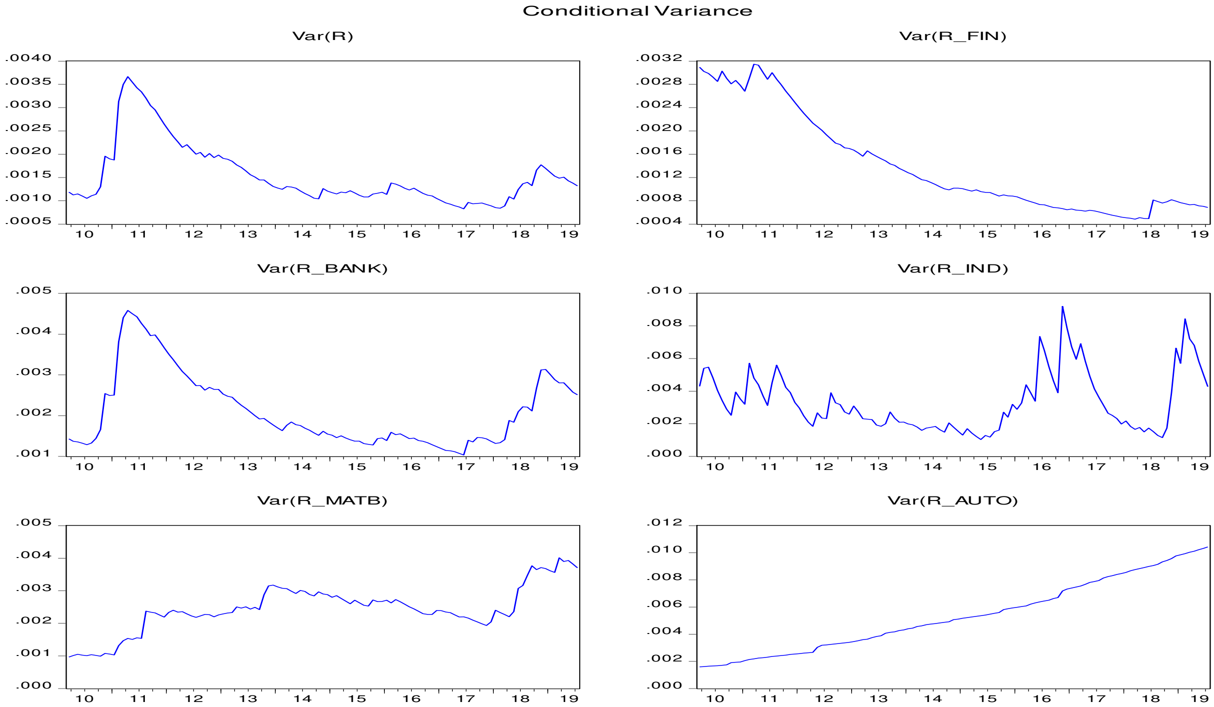
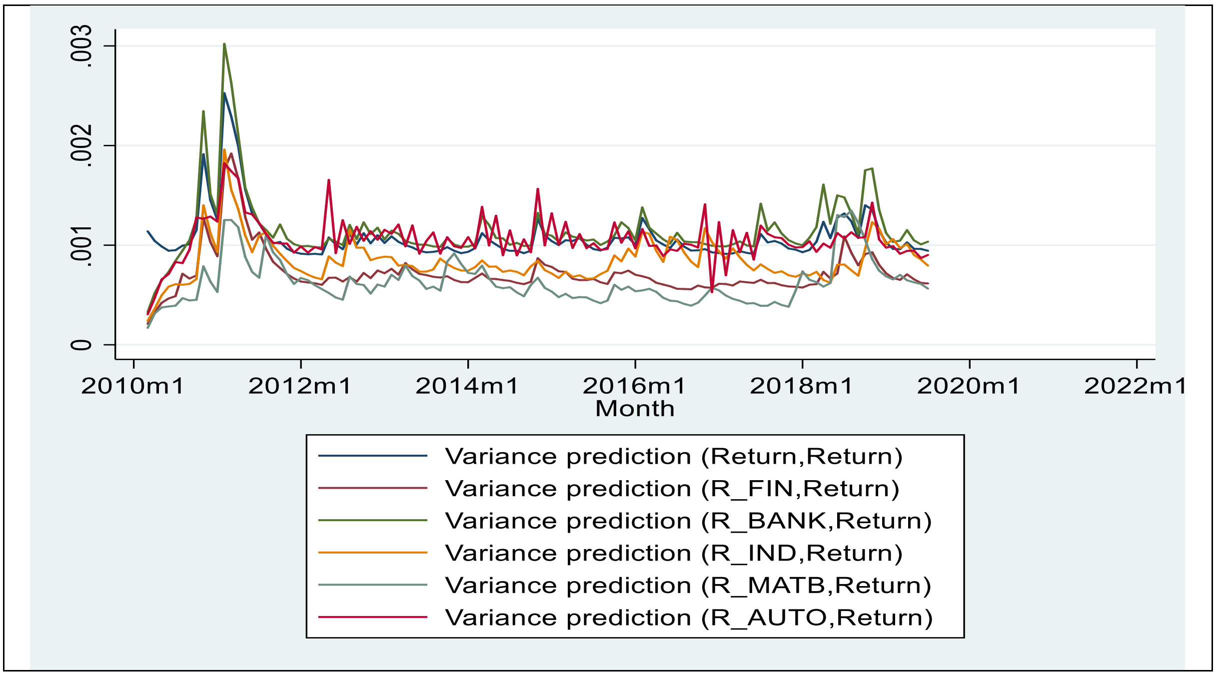


Figure A 9: Conditional **variances** from **DBEKK** model (system I).



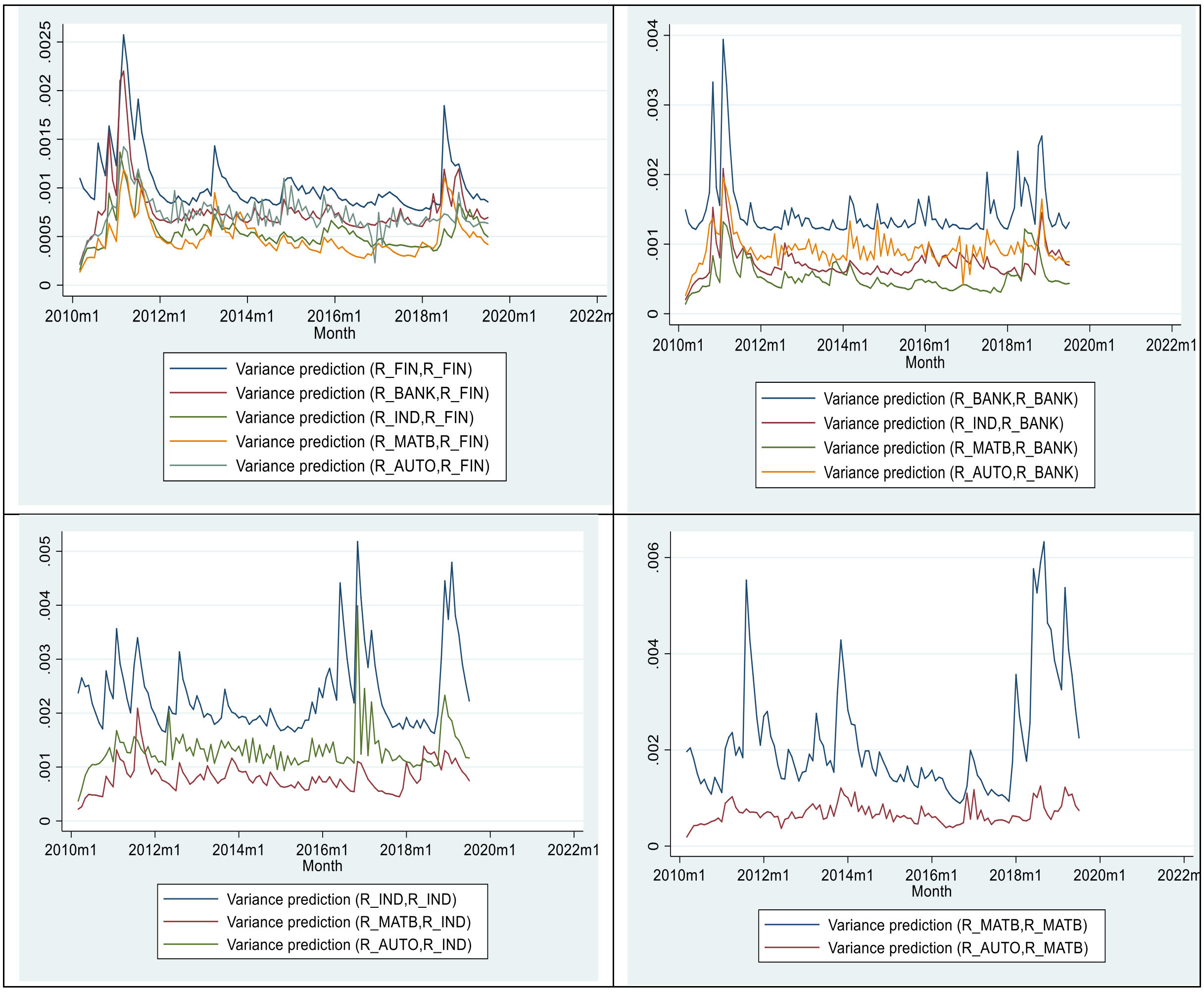


Figure A 10: C Variance prediction by sector from MGARCH(1, 1)-DCC model for System (II).

E. Tables

Table B 5: DVECH, DBEKK, and CCC results of system (I) for sectorial index returns and Tunindex return.[32]

	Diagonal VECH			**Diagonal BEKK**				**CCC**		
	Coefficient	SdError	P-value	Coefficient	Sd. Error	P-value		Coefficient	Sd Error	P-value
α_0	0.000303	9.38E-05	**0.0012**	6.18E-06	6.74E-06	0.3593	$\alpha_{Return,1}$	0.063777	0.081389	0.4333
$\alpha_{1(1,1)}$	0.311209	0.12449	**0.0124**	0.253148	0.029773	0.0000	$\beta_{Return,1}$	0.477957	0.401852	0.2343
$\alpha_{1(1,2)}$	0.229111	0.12968	**0.0773**				$\alpha_{Return,0}$	0.000515	0.000435	0.2368
$\alpha_{1(1,3)}$	0.292378	0.11475	**0.0108**				$\alpha_{FIN,1}$	-0.090209	0.031026	0.0036
$\alpha_{1(1,4)}$	0.127848	0.08957	0.1535				$\beta_{FIN,1}$	1.084453	0.043225	0.0000
$\alpha_{1(1,5)}$	0.129830	0.09940	0.1915				$\alpha_{FIN,0}$	6.38E-07	2.50E-05	0.9796
$\alpha_{1(1,6)}$	0.026771	0.03988	0.5021				$\alpha_{BANK,1}$	0.117784	0.096963	0.2245
$\alpha_{1(2,2)}$	0.339504	0.23187	0.1431	0.158303	0.04217	0.0002	$\beta_{BANK,1}$	0.239348	0.384733	0.5339
$\alpha_{1(2,3)}$	0.215427	0.13267	0.1044				$\alpha_{BANK,0}$	0.000942	0.000577	0.1025
$\alpha_{1(2,4)}$	0.026376	0.11757	0.8225				$\alpha_{IND,1}$	0.155613	0.090986	0.0872
$\alpha_{1(2,5)}$	0.000155	0.13713	0.9991				$\beta_{IND,1}$	0.719893	0.127193	0.0000
$\alpha_{1(2,6)}$	0.036459	0.06080	0.5488				$\alpha_{IND,0}$	0.000296	0.000281	0.2927
$\alpha_{1(3,3)}$	0.294752	0.11519	**0.0105**	0.245108	0.03193	0.0000	$\alpha_{MATB,1}$	0.158480	0.114324	0.1657
$\alpha_{1(3,4)}$	0.072864	0.08942	0.4152				$\beta_{MATB,1}$	0.760376	0.170529	0.0000
$\alpha_{1(3,5)}$	0.096394	0.10286	0.3487				$\alpha_{MATB,0}$	0.000189	0.000198	0.3379
$\alpha_{1(3,6)}$	0.036947	0.05027	0.4624				$\alpha_{AUTO,1}$	-0.158443	0.052037	0.0023
$\alpha_{1(4,4)}$	0.147592	0.07042	**0.0361**	0.456899	0.04915	0.0000	$\beta_{AUTO,1}$	1.116145	0.058491	0.0000
$\alpha_{1(4,5)}$	0.022063	0.09918	0.8240				$\alpha_{AUTO,0}$	0.000323	0.000212	0.1283
$\alpha_{1(4,6)}$	0.001727	0.05454	0.9747				$\rho_{Return,R,FIN}$	0.652363	0.078558	**0.0000**
$\alpha_{1(5,5)}$	0.176369	0.11623	0.1292	0.231210	0.04541	0.0000	$\rho_{Return,R,BANK}$	0.933575	0.016861	**0.0000**
$\alpha_{1(5,6)}$	-0.088360	0.06232	0.1562				$\rho_{Return,R,IND}$	0.589572	0.080966	**0.0000**
$\alpha_{1(6,6)}$	-0.063860	0.02430	**0.0086**	0.083730	0.02749	0.0023	$\rho_{Return,R,MATB}$	0.472450	0.110125	**0.0000**

[32] This is done by Eviews 10.

$\beta_{1(1,1)}$	0.524912	0.07886	**0.0000**	0.971839	0.00506	0.0000	$\rho_{Return,R,AUTO}$	0.494502	0.104651	**0.0000**
$\beta_{1(1,2)}$	0.592141	0.08480	**0.0000**				$\rho_{FIN,BANK}$	**0.585304**	0.096327	**0.0000**
$\beta_{1(1,3)}$	0.518898	0.07546	**0.0000**				$\rho_{FIN,IND}$	0.429432	0.099233	**0.0000**
$\beta_{1(1,4)}$	0.619171	0.09084	**0.0000**				$\rho_{FIN,MATB}$	0.349484	0.118202	**0.0031**
$\beta_{1(1,5)}$	0.634235	0.09620	**0.0000**				$\rho_{FIN,AUTO}$	0.398441	0.120265	**0.0009**
$\beta_{1(1,6)}$	0.718966	0.06607	**0.0000**				$\rho_{BANK,IND}$	0.428228	0.102199	**0.0000**
$\beta_{1(2,2)}$	0.698777	0.08905	**0.0000**	0.978662	0.00354	0.0000	$\rho_{BANK,MATB}$	0.342241	0.119776	**0.0043**
$\beta_{1(2,3)}$	0.592962	0.08962	**0.0000**				$\rho_{BANK,AUTO}$	0.380455	0.120631	**0.0016**
$\beta_{1(2,4)}$	0.702059	0.09835	**0.0000**				$\rho_{IND,MATB}$	0.489588	0.093238	**0.0000**
$\beta_{1(2,5)}$	0.717366	0.09548	**0.0000**				$\rho_{IND,AUTO}$	0.388151	0.113787	**0.0006**
$\beta_{1(2,6)}$	0.753731	0.08101	**0.0000**				$\rho_{MATB,AUTO}$	0.295445	0.122144	**0.0156**
$\beta_{1(3,3)}$	0.532757	0.07382	**0.0000**	0.976728	0.00486	0.0000				
$\beta_{1(3,4)}$	0.618670	0.09801	**0.0000**							
$\beta_{1(3,5)}$	0.628313	0.10168	**0.0000**							
$\beta_{1(3,6)}$	0.645872	0.09882	**0.0000**							
$\beta_{1(4,4)}$	0.742161	0.06503	**0.0000**	0.915972	0.01691	0.0000				
$\beta_{1(4,5)}$	0.749839	0.09405	**0.0000**							
$\beta_{1(4,6)}$	0.781695	0.08089	**0.0000**							
$\beta_{1(5,5)}$	0.771773	0.08905	**0.0000**	0.983463	0.00729	0.0000				
$\beta_{1(5,6)}$	0.828525	0.06667	**0.0000**							
$\beta_{1(6,6)}$	1.011419	0.01823	**0.0000**	1.003728	0.00301	0.0000				
Log likelihood			1385.65			1383.284				1424.161
Akaike info criterion			-23.6398			-24.12891				-24.49843
Schwarz criterion			-22.4330			-23.64619				-23.53298
Hannan-Quinn criter.			-23.1501			-23.93303				-24.10666

Note : Return = (1), FIN = (2), BANK = (3), IND = (4), MATB = (5), AUTO = (6)

Table B 6: Macro economic effects in DCC and CCC models for 5 sectorial index returns and TUNindex return.[33]

Method	DCC				CCC			
Variable	Coef.	Robust Std. Err.	z	P-value	Coef.	Robust Std. Err.	z	P-value
Return								
VOL_lcpi	-678.79	510.97	-1.33	0.18	-741.11	492.26	-1.51	0.13
VOL_lop	0.14	0.52	0.28	0.78	0.09	0.58	0.15	0.88
VOL_lexrate	**21.36**	**10.34**	**2.07**	**0.04**	22.49	9.98	2.25	**0.02**
$\alpha_{\text{Return},1}$	0.07	0.04	1.93	**0.05**	0.06	0.04	1.60	0.11
$\beta_{\text{Return},1}$	0.51	0.10	5.06	**0.00**	0.50	0.11	4.64	0.00
$\alpha_{\text{Return},0}$	0.00	0.00	3.53	**0.00**	0.00	0.00	3.56	0.00
R_FIN								
VOL_lcpi	**-1560.04**	**579.4**	**-2.69**	**0.01**	-1614.85	590.5	-2.73	**0.01**
VOL_lop	0.29	0.39	0.74	0.46	0.24	0.39	0.60	0.55
VOL_lexrate	**36.46**	**14.37**	**2.54**	**0.01**	36.94	14.11	2.62	**0.01**
$\alpha_{\text{FIN},1}$	0.08	0.08	0.94	0.35	0.06	0.08	0.85	0.40
$\beta_{\text{FIN},1}$	0.69	0.16	4.40	**0.00**	0.70	0.16	4.36	0.00
$\alpha_{\text{FIN},0}$	0.00	0.00	1.39	0.16	0.00	0.00	1.32	0.19
R_BANK								
VOL_lcpi	-510.78	574.98	-0.89	0.37	-585.79	557.08	-1.05	0.29
VOL_lop	0.19	0.47	0.39	0.69	0.12	0.56	0.22	0.83
VOL_lexrate	19.29	12.00	1.61	0.11	20.81	11.55	1.80	**0.07**
$\alpha_{\text{BANK},1}$	0.12	0.06	1.95	**0.05**	0.11	0.06	1.68	0.09
$\beta_{\text{BANK},1}$	0.24	0.16	1.46	0.14	0.21	0.22	0.97	0.33
$\alpha_{\text{BANK},0}$	0.00	0.00	2.73	**0.01**	0.00	0.00	2.37	0.02
R_IND								
VOL_lcpi	-1061.66	804.09	-1.32	0.19	-1181.45	802.29	-1.47	0.14
VOL_lop	-0.70	0.61	-1.14	0.25	-0.75	0.64	-1.16	0.24
VOL_lexrate	23.17	21.47	1.08	0.28	25.59	21.65	1.18	0.24
$\alpha_{\text{IND},1}$	0.12	0.12	0.99	0.32	0.12	0.11	1.06	0.29
$\beta_{\text{IND},1}$	0.70	0.15	4.55	**0.00**	0.70	0.14	5.04	0.00
$\alpha_{\text{IND},0}$	0.00	0.00	1.37	0.17	0.00	0.00	1.41	0.16
R_MATB								
VOL_lcpi	-475.61	675.79	-0.70	0.48	-581.07	674.33	-0.86	0.39
VOL_lop	-0.68	0.45	-1.51	0.13	-0.68	0.42	-1.64	0.10
VOL_lexrate	**15.75**	**8.61**	**1.83**	**0.07**	16.73	8.46	1.98	**0.05**
$\alpha_{\text{MATB},1}$	0.23	0.16	1.44	0.15	0.23	0.16	1.46	0.14
$\beta_{\text{MATB},1}$	0.70	0.13	5.40	**0.00**	0.70	0.12	5.81	0.00
$\alpha_{\text{MATB},0}$	0.00	0.00	1.22	0.22	0.00	0.00	1.21	0.22
R_AUTO								

[33] This is done by Stata 15.

	DCC				CCC			
VOL_lcpi	-480.01	1189.26	-0.40	0.69	-439.60	1187.24	-0.37	0.71
VOL_lop	**-1.26**	**0.65**	**-1.92**	**0.05**	-1.25	0.68	-1.85	**0.06**
VOL_lexrate	18.94	21.90	0.86	0.39	16.17	22.22	0.73	0.47
$\alpha_{AUTO,1}$	0.08	0.02	4.04	**0.00**	0.08	0.02	4.37	0.00
$\beta_{AUTO,1}$	-0.85	0.07	-12.89	**0.00**	-0.84	0.07	-12.82	0.00
$\alpha_{AUTO,0}$	0.01	0.00	5.38	**0.00**	0.01	0.00	5.36	0.00

Table B 6 (suite): DCC and CCC results for 5 sectorial index return and TUNindex return.

	DCC				**CCC**			
	Coef.	Robust Std. Err.	z	P-value	Coef.	Robust Std. Err.	z	P-value
$\rho_{Return,R,FIN}$	0.66	0.08	8.04	0.00	0.67	0.06	10.67	0.00
$\rho_{Return,R,BANK}$	0.92	0.02	44.88	0.00	0.93	0.02	59.93	0.00
$\rho_{Return,R,IND}$	0.58	0.09	6.68	0.00	0.58	0.07	7.92	0.00
$\rho_{Return,R,MATB}$	0.45	0.09	4.95	0.00	0.47	0.08	6.07	0.00
$\rho_{Return,R,AUTO}$	0.47	0.07	6.33	0.00	0.49	0.06	8.00	0.00
$\rho_{FIN,BANK}$	0.57	0.09	6.66	0.00	0.59	0.07	8.48	0.00
$\rho_{FIN,IND}$	0.41	0.11	3.68	0.00	0.41	0.10	4.33	0.00
$\rho_{FIN,MATB}$	0.35	0.11	3.20	0.00	0.37	0.09	4.26	0.00
$\rho_{FIN,AUTO}$	0.41	0.11	3.93	0.00	0.39	0.08	4.58	0.00
$\rho_{BANK,IND}$	0.39	0.11	3.45	0.00	0.42	0.09	4.47	0.00
$\rho_{BANK,MATB}$	0.31	0.11	2.85	0.00	0.34	0.09	3.67	0.00
$\rho_{BANK,AUTO}$	0.34	0.09	3.89	0.00	0.37	0.07	5.01	0.00
$\rho_{IND,MATB}$	0.49	0.09	5.43	0.00	0.47	0.09	5.32	0.00
$\rho_{IND,AUTO}$	0.35	0.11	3.23	0.00	0.37	0.09	4.22	0.00
$\rho_{MATB,AUTO}$	0.26	0.09	2.74	0.01	0.26	0.09	3.06	0.00
Adjustment								
θ_1	0.04	0.03	1.36	0.17				
θ_2	0.71	0.14	5.16	**0.00**				
df								
η	9.57	2.51	3.81	**0.00**	10.10	2.84	3.56	**0.00**
Wald	**55.82**				46.27			
(p-value)	**(0.0000)**				(0.0003)			
AIC	**-2738.522**				-2737.802			
BIC	-2604.88				**-2609.615**			
LL	**1418.261**				1415.901			

IX. Annexe : Some algebra

F. DVECH model in Algebric form

(Bollerslev, Engle, & Wooldridge, 1988) introduce a restricted version of the general MVECH model of the conditional covariance with the following formulation:

$$H_t = \Omega + \sum_{h=1}^{q} A_h \bullet \varepsilon_{t-1}\varepsilon'_{t-1} + \sum_{h=1}^{p} B_h \bullet H_{t-1},$$

where the coefficient matrices $\Omega(\alpha_{ij,0})$, A_h $(\alpha_{ij,h})$, and B_h $(\beta_{ij,h})$ are **symmetric matrices**, and the operator "•" is the element by element (Hadamard) product. "Ω" is the constant matrix coefficient, A_h is the coefficient matrix for the ARCH term and B_h is the coefficient matrix for the GARCH term. Each matrix contains k(k+1)/2 parameters.[34]

For q = p =1, this is referred to as the diagonal VECH(1, 1) model or DVECH(1, 1) model :

$$H_t = \Omega + A \bullet \varepsilon_{t-1}\varepsilon'_{t-1} + B \bullet H_{t-1},$$

where Ω , A, and B are k × k **symmetric matrices** of parameters (k=2 for the bivariate case). The coefficient matrices may be parametrized in several ways. The most general way is to allow the parameters in the matrices to vary without any restriction.[35] In that case the model may be written **in single equation** format as:

$$\sigma_{ij,t} = \alpha_{ij,0} + \alpha_{ij}\varepsilon_{i,t-1}\varepsilon_{j,t-1} + \beta_{ij}\sigma_{ij,t-1}, for\ i,j = 1,2,\ \ldots$$

G. : Several approaches for reducing parameters number

For example, one may use the rank **Cholesky factorized** matrix of the coefficient matrix. This method is labeled the **Full Rank Matrix** in the coefficient **Restriction** selection option of the system ARCH dialog in Eviews.[36]

A **second** method, which is term **Rank One** (in Eviews), reduces the number of parameter estimated to k and guarantees that the conditional covariance is PSD. In this case, the estimated **raw** matrix is restricted, with all but the first column of coefficients equal to zero. There are **two other** covariance specifications that we may employ. **First**, the values in the k×k matrix may be a constant, so that:

$$B = \beta\,\iota\,\iota'$$

---

[34] This method is labeled the **Full Rank Matrix** in the coefficient **Restriction** selection option with Eviews.

[35] *i.e.* parameterize them as **indefinite matrices**.

[36] While this method contains the same number of parameters as the indefinite version, it does ensure that the conditional covariance is PSD.

Where β is a scalar ι and is a vector of ones. This **Scalar** specification implies that for a particular term, the parameters of the variance and covariance equations are restricted to be the same. **Alternately**, the matrix coefficients may be parameterized as **Diagonal** so that all **off** diagonal elements are restricted to be **zero**. In both of these parameterizations, the coefficients are not restricted to be positive, so that is not guaranteed to be PSD. **Lastly**, for the constant matrix Ω, we may also impose a **Variance Target** on the coefficients which restricts the values of the coefficient matrix so that:

$$\Omega = \Omega_0 \bullet (\iota\,\iota' - A - B)$$

Where Ω_0 is the **unconditional sample** variance of the residuals. When using this option, the **constant** matrix is **not estimated**, reducing the number of estimated parameters.