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Profit-enhancing entries in mixed oligopolies*

Junichi Haraguchi† and Toshihiro Matsumura‡

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Abstract

Mixed oligopolies are characterized by private and public enterprises. Entry into these markets was restrictive, but has now been relaxed by deregulations; as a result, private firms have entered mixed oligopolies. An increase in the number of private firms increases competition among private firms and reduces the profit of incumbent private firms, given the privatization policy remains unchanged. However, an increase in the number of private firms may in turn affect privatization policy, and thus, indirectly affect private firms' profits. Therefore, the overall effect on private firms' profit is ambiguous. In this study, we thus investigate how the number of private firms affects the profit of each private firm in mixed oligopolies. For this end, we use a linear-quadratic production cost function, which covers two popular model formulations in the mixed oligopoly literature. We show that, if the degree of privatization is exogenous, the profit of each private firm is decreasing in the number of private firms. However, if the degree of privatization is endogenous, the relationship between the number of private firms and profit takes an inverted-U shape under a plausible range of cost parameters. Our results imply that there can exist multiple equilibria in free-entry markets with different degrees of privatization.

JEL classification numbers: D43, H44, L33

Key words: optimal degree of privatization, profit-enhancing entry, multiple long-run stable equilibria

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†Corresponding author: Faculty of Economics, Kanagawa University, 3-27-1, Rokkakubashi, Kanagawa-ku, Yokohama, Kanagawa, 221-8686, Japan. Phone:(81)-45-481-5661. E-mail:jyunichi.haraguchi@gmail.com

‡Institute of Social Science, The University of Tokyo, 7-3-1, Hongo, Bunkyo-ku, Tokyo 113-0033, Japan. Phone:(81)-3-5841-4932. Fax:(81)-3-5841-4905. E-mail:matsumur@iss.u-tokyo.ac.jp

1 Introduction

Despite the global trend of state-owned public enterprises towards privatization, public enterprises with substantial ownership shares from public sectors remain active in various industries and manage a significant ratio of the world's resources (Megginson and Netter, 2001; La Porta et al., 2002). According to an OECD report, over 10% of the 2,000 largest companies are public enterprises, with sales equivalent to approximately 6% of global GDP (Kowalski et al., 2013). Chinese, Russian, and Indian public enterprises, in particular, occupy significant positions in many industries (Gupta, 2005; Chen, 2017; Fridman, 2018).

Public enterprises managed to prevent private monopolies in natural monopoly markets because of significant economies of scale in these markets. However, due to recent technological progress and expansion of market size, many markets that contain public enterprises are no longer characterized by significant economies of scale. As a result, a number of public enterprises coexist with private enterprises in a wide range of industries.[1] We call the markets in which both private and public firms coexist mixed oligopolies.

Moreover, recent deregulation has significantly loosened the entry and exit restrictions in mixed oligopolies, and private enterprises have therefore entered many mixed oligopolies in industries such as energy, telecommunications, transportation, banking, and insurance. The optimal privatization policies in these mixed oligopolies and their economic consequences have attracted extensive attention from economic, legal, and political science researchers, as well as policymakers.[2]

The literature showed that an increase in the number of private firms raises the optimal degree of privatization (Lin and Matsumura, 2012; Matsumura and Okamura, 2015), which in turn reduces the output of the public firm; thus, it raises the profit of each private firm.[3] However, given the

[1] Examples include the United States Postal Service, Amtrak, TVA, La Poste, KfW, Volkswagen, Renault, Areva, Japan Post, Development Bank of Japan, Tokyo Electric Power Corporation, Korea Electric Power Corporation, Indian Telecommunications Service, National Aviation Company of India Limited, Gazprom, PJSC Aeroflot Russian Airlines, PetroChina Company, and China Petroleum and Chemical Corporation.

[2] For examples of mixed oligopolies and recent developments in this field, see Escrihuela-Villar and Gutiérrez-Hita (2018), Kim et al. (2019), Mahanta (2019), and the works cited therein.

[3] For example, in the Japanese banking industry, the number of firms declined continuously, and the privatization programs of public banks were postponed and downscaled. By contrast, in the Japanese electric power industry, the number of firms continuously increased, and J-Power was fully privatized. These facts are consistent with the above

privatization policy, an increase in the number of private firms strengthens the competition among private firms, reducing the profit of each private firm. Therefore, it is ambiguous whether an increase in the number of private firms leads more or less competition in mixed oligopolies under endogenous privatization policies.[4]

The above concern regarding an increase in number of firms versus profit is significant for two reasons. First, if an increase in the number of private firms increases the profit of private firms, the number of firms many be an inappropriate measure of the competitiveness in the market. In other words, a larger number of firms may indicate less acerbic competition in mixed oligopolies. Thus, this result has implications for the context of anti-trust policies in mixed oligopolies.

Second, if an increase in the number of private firms increases the profit of private firms, then a new entry in the market triggers the entry of other private firms in free-entry markets, making the long-run equilibrium unstable. Several studies on free-entry mixed oligopolies assumed that the profit of each private firm decreases as the number of private firms increases (Matsumura and Kanda, 2005; Lee et al., 2018); however, this assumption may be problematic. Therefore, the relationship between the profit of each firm and the number of private firms should be investigated to clarify the long-run equilibrium stability.

Haraguchi and Matsumura (2020a) investigated a mixed oligopoly model with constant marginal costs, and showed that each private firm's profit is increasing in the number of private firms unless the optimal degree of privatization is one (i.e., unless the optimal public ownership share is zero). Although models with constant marginal costs are popular in the literature on mixed oligopolies (Pal, 1998), models with increasing marginal costs are further popular in mixed oligopolies (De Fraja and Delbono, 1989, Matsumura and Shimizu, 2010). Moreover, in the mixed oligopoly literature on free-entry markets, increasing marginal costs are standard and most studies adopt them. Therefore, it is essential to investigate this problem under increasing marginal costs.

In addition, even in a non-free-entry market, constant marginal cost and increasing marginal

theoretical results.

[4]Incumbent firms' profits can increase with a new entry even in private oligopolies. See Coughlan and Soberman (2005), Chen and Riordan (2007), Ishibashi and Matsushima (2009), Mukherjee and Zhao (2009), and Ishida et al. (2011). However, our approach is different from theirs because they do not discuss policy changes.

cost models could have opposite policy implications on optimal privatization policies (Matsumura and Okamura, 2015). Therefore, the implications of the results based on the constant marginal cost assumption can be restrictive.[5]

We use a linear-quadratic production cost function that covers two popular model formulations in the literature on mixed oligopolies— constant marginal cost and quadratic cost functions. We find that, regardless whether the private firm's marginal cost is constant or increasing, if the public firm's marginal cost is constant, each private firm's profit is increasing in the number of private firms, unless the public firm is fully privatized in equilibrium (i.e., the equilibrium degree of privatization is one). This result suggests that, as long as public firm's marginal cost is constant, either a public monopoly or a pure private oligopoly, in which the public firm is fully privatized, emerges in equilibrium in free-entry markets.

However, if the public firm's marginal cost is increasing, each private firm's profit is either decreasing in the number of private firms or has an inverted-U shaped relationship with the number of private firms. An inverted-U shaped relationship emerges when the coefficient of the quadratic terms is small. When each private firm's profit is decreasing in the number of private firms, there is a unique long-run equilibrium (equilibrium under free entry), and it is stable. When an inverted-U shaped relationship emerges between profit and the number of private firms, there can be three long-run equilibria, and two are stable. Our results suggest that even under increasing marginal costs, uniqueness of the long-run equilibria is not always guaranteed.

We also compare a mixed duopoly with a mixed triopoly and investigate whether a private firm's entry raises the incumbent private firm's profit. We find that a new entry may raise the incumbent private firm's profit even when marginal costs are increasing. This result implies that our main results hold even if we consider an integer constraint for the number of firms and cost heterogeneity among private firms.

[5] Wang and Mukherjee (2012) investigated a Stackelberg mixed oligopoly with public leadership and Matsumura and Sunada (2013) introduced advertising competition in mixed oligopolies. They showed that private firms' profits can be increasing in the number of private firms. However, the driving force of this study is different from theirs because we consider neither sequential-move nor the advertising stage. Moreover, they assume constant marginal costs, while we allow increasing marginal costs as well.

The paper is divided into 6 sections. In section 2, we present the model. Section 3 presents the equilibrium analysis and main results. Section 4 discusses the properties of free entry markets. Section 5 compares a mixed duopoly with a mixed triopoly, and discusses whether a new entry of private firm increases the profit of incumbent private firm. Section 6 concludes the paper.

2 The model

We consider a mixed oligopoly in which one public firm (firm 0) initially owned by the government competes with n private firms (firms 1, 2,...,n). Let q_i be firm i's output. These firms produce homogeneous products for which the inverse demand function is $p(Q) = a - Q$, where p denotes price, a is a positive constant, and $Q := \sum_{i=0}^{n} q_i$. We assume that all private firms have an identical cost function, although we allow public and private firms to have different cost functions.[6] We assume linear-quadratic cost functions. The public firm's cost function is $c_0(q_0) = \gamma_0 q_0 + (\kappa_0/2)q_0^2$ and each private firm i's $c_i(q_i) = \gamma q_i + (\kappa/2)q_i^2$, where $\gamma_0, \kappa_0, \gamma$, and κ are non-negative constants. This model formulation of demand and cost covers several popular settings in the literature on mixed oligopolies. For example, if $\gamma_0 = \gamma = 0$ and $\kappa_0 = \kappa > 0$, this covers De Fraja and Delbono's (1989) model, which is popular in the literature on mixed oligopolies.[7] If $\gamma_0 > \gamma$ and $\kappa_0 = \kappa = 0$, this covers Pal's (1998) model, which is another popular model in mixed oligopolies.[8] Our model covers these two popular models in the literature on mixed oligopolies. As Matsumura and Okamura (2015) showed, two models can yield the opposite policy implications in mixed oligopolies, and we thus believe that the model formulation covering these two models is important.

Following the standard assumption in this field, we define social welfare as the sum of consumer surplus and profits of firms. Then, social welfare W is:

$$W = \int_0^Q p(q)dq - pQ + \sum_{i=0}^{n} \pi_i = \int_0^Q p(q)dq - \sum_{i=0}^{n} c_i(q_i).$$

[6] We relax this assumption in Section 5.

[7] De Fraja and Delbono (1989) assumed that public and private firms have identical cost functions; however, the model allowing cost difference between public and private firms are also widely used (Matsumura and Shimizu, 2010; Kawasaki et al., 2020).

[8] See also Mujumdar and Pal (1998) and Haraguchi and Matsumura (2020a,b).

Following Matsumura (1998), the public firm's objective function is $\alpha\pi_0 + (1-\alpha)W$.[9] $\alpha \in [0,1]$ represents the degree of privatization. Each private firm i's objective function is its profit, π_i.

The game runs as follows. In the first stage, the government chooses α to maximize social welfare. In the second stage, each firm simultaneously chooses its output to maximize its objective function. We solve this game by backward induction and the equilibrium concept is the subgame perfect Nash equilibrium.

Throughout this study, we assume interior solutions in the second stage. In other words, all firms produce positive outputs.

3 Equilibrium and main results

First, we solve the second stage game given α. The first order-condition for each private firm is:

$$p + p'q - \gamma - \kappa q. \tag{1}$$

The second-order condition is satisfied. The first-order condition of the public firm is:

$$p + \alpha p' q_0 - \gamma_0 - \kappa_0 q_0. \tag{2}$$

The second-order condition is satisfied.

These first-order conditions yield the following equilibrium quantities for the public and private firms in the second stage subgames:

$$q_0^S(\alpha, n) = \frac{(a-\gamma_0)(1+\kappa) - n(\gamma_0 - \gamma)}{(1+\kappa)(1+\kappa_0+\alpha) + n(\kappa_0+\alpha)}, \tag{3}$$

$$q^S(\alpha, n) = \frac{(a-\gamma)(\alpha+\kappa_0) + \gamma_0 - \gamma}{(1+\kappa)(1+\kappa_0+\alpha) + n(\kappa_0+\alpha)}. \tag{4}$$

Superscript S indicates the equilibrium outcomes in the second-stage subgame.

We respectively obtain the following equilibrium total output, price, each private firms' profit,

[9] For empirical evidence supporting this formulation, see Ogura (2018), as well as Seim and Waldfogel (2013) for empirical evidence for $\alpha = 0$.

and welfare:

$$Q^S(\alpha,n) = \frac{n(a-\gamma)(\alpha+\kappa_0)+(1+\kappa)(a-\gamma_0)}{(1+\kappa)(1+\kappa_0+\alpha)+n(\kappa_0+\alpha)}, \quad (5)$$

$$p^S(\alpha,n) = \frac{(1+\kappa)(a(\alpha+\kappa_0)+\gamma_0)+n\gamma(\alpha+\kappa_0)}{(1+\kappa)(1+\kappa_0+\alpha)+n(\kappa_0+\alpha)}, \quad (6)$$

$$\pi^S(\alpha,n) = \frac{2+\kappa}{2}\left(\frac{(\alpha+\kappa_0)(a-\gamma)+\gamma_0-\gamma}{(1+\kappa)(1+\kappa_0+\alpha)+n(\kappa_0+\alpha)}\right)^2, \quad (7)$$

$$W^S(\alpha,n) = \frac{X_1}{2((1+\kappa)(1+\kappa_0+\alpha)+n(\kappa_0+\alpha))^2}. \quad (8)$$

X_1 and other coefficients X_i that repeatedly appear throughout the paper are reported in Appendix A.

From (6) and (7), we obtain the following result.

Proposition 1 *(i) p^S is decreasing in n. (ii) π^S is decreasing in n.*

Proof See Appendix B.

Proposition 1 states that the both price and each private firm's profit are decreasing in n if α is given exogenously, which is intuitive. An increase in the number of private firms strengthens competition and reduces each firm's output and the price, which reduces the profit of each private firm.[10]

Next, we discuss the government's welfare maximization problem in the first stage. Let α^F be the equilibrium degree of privatization (superscript F indicates the first-stage). The first-order condition is:

$$\frac{\partial W^S(\alpha)}{\partial \alpha} = -\frac{(a-\gamma_0+\kappa(a-\gamma_0)-n(\gamma_0-\gamma))X_2}{((1+\alpha+\kappa_0)(1+\kappa)+n(\alpha+\kappa_0))^3} = 0. \quad (9)$$

The second-order condition is:

$$-\frac{((a-\gamma_0)(1+\kappa)^2-n(n+2)(\gamma_0-\gamma)+\kappa n(a-2\gamma_0-\gamma))^4}{((1+\kappa)(a-\gamma_0)-n(\gamma_0-\gamma))^2((1+\kappa)^2(1+\kappa_0)+n(\kappa+2\kappa_0(1+\kappa)+n\kappa_0))^3} < 0$$

and is satisfied under the assumption of the interior solution in the second stage (i.e., both q_0 and q are positive). The solution for (9), α^*, is:

$$\alpha^* = \frac{(\kappa_0(a-\gamma)+\gamma_0-\gamma)n}{((1+\kappa)^2+\kappa n)a-((1+\kappa)(1+\kappa+2n)+n^2)\gamma_0+n(2+\kappa+n)\gamma}. \quad (10)$$

[10]The public firm's profit is also decreasing in n.

Equilibrium α, α^F is

$$\alpha^F = \max\{0, \min\{\alpha^*, 1\}\}.$$

In other words, if the solution is interior (i.e., $\alpha^F \in (0,1)$), $\alpha^F = \alpha^*$.

From (10), we obtain the following result.

Lemma 1 *α^* is increasing in n.*

Proof See Appendix B.

The literature presented this result in various contexts (Lin and Matsumura, 2012, Matsumura and Okamura, 2015). An increase in α reduces the public firm's output while increasing that of private firms. The production substitution from the public firm to private firms improves welfare (Matsumura, 1998).[11] An increase in α reduces total output, thus reducing the consumer surplus. The equilibrium α is determined by the trade-off of these two effects. An increase in n strengthens the above welfare-improving effect because a larger number of private firms substitute the public firm's production. Moreover, an increase in n increases the total output given α, which weakens the above welfare-reducing effect of an increase in α. Therefore, the equilibrium degree of privatization is increasing in n unless it reaches the upper bound (i.e., one).

We then obtain the following result.

Proposition 2 *(i) $\alpha^F > 0$ (i.e., the optimal degree of privatization is strictly positive). (ii) If $\kappa_0 = \kappa > 0$ and $\gamma_0 = \gamma$, then $\alpha^F < 1$ (i.e., if both public and private firms have an identical cost function and the marginal cost is increasing, the optimal degree of privatization is less than one). (iii) Suppose that the solution in the first stage is interior (i.e., $\alpha^F < 1$). Then, $\pi^F(n)$ is increasing in n if and only if*

$$g(n) := (n^2 + 2(\kappa+1)n + (\kappa+1)^2)\kappa_0 - \kappa - 1 < 0. \tag{11}$$

Proof See Appendix B.

Proposition 2(i,ii) has already been demonstrated under general demand and cost conditions (Matsumura, 1998). From Proposition 2(i), the solution is interior if $\alpha^F < 1$.

[11] Lahiri and Ono (1988) comprehensively discussed the welfare-improving production substitution.

Proposition 2(iii) is our main result. It states that each private firm's profit can be increasing in the number of private firms. In other words, a new entry of a private firm may increase each incumbent private firm's profit. Given α, each private firm's profit decreases as n increases (Proposition 1). However, α increases as n increases, which increases each private firm's profit. Proposition 2(iii) states that the latter effect can dominate the former.

We now present the properties of π^F in detail. From Proposition 2, we obtain the following result that highlights the conditions under which each private firm's profit is increasing in n.

Proposition 3 *Suppose that the solution in the first stage is interior (i.e.,* $\alpha^F < 1$*). Then, (i)* $\pi^F(n)$ *is either (a) increasing in* n *for any* n*, (b) decreasing in* n *for any* n*, or (c) inverted-U shaped; (ii) (a) holds if and only if* $\kappa_0 = 0$*; and (iii) (b) holds if* $\kappa_0 \geq \kappa$ *and* $\kappa \geq \bar{\kappa} \simeq 0.618$.

Proof See Appendix B.

Proposition 3(i) states that various relationships between the number of private firms and the profit for each private firm can emerge in equilibrium. Haraguchi and Matsumura (2020a) showed that each private firm's profit is increasing in n when the marginal costs of all firms are constant (i.e., if $\kappa_0 = \kappa = 0$). Proposition 3(ii) states that this holds if the marginal cost of the public firm is constant, regardless whether the private firm's marginal cost is constant or increasing. In other words, only the assumption of constant marginal cost of the public firm is essential. Proposition 3 (iii) states that each private firm's profit is decreasing in n if c'' is large for both the public and private firms.

It also states that each private firm's profit can be increasing in n for small n, whereas it is always decreasing in n for large n as long as the public firm's marginal cost function is increasing. This result suggests that, when c'' is small, there can be multiple long-run equilibria (i.e., multiple equilibria in the free-entry market), whereas a unique equilibrium exists when c'' is large. We discuss this point in the next section. Moreover, our results suggest that the number of firms may be an inappropriate measure of the competitiveness of the market when κ is small because an increase in the number of firms may lead to a weaker competition in mixed oligopolies due to the change in privatization policy.

Our results have another implication for the literature on mixed oligopolies. In their pioneering work, De Fraja and Delbono (1989) assumed that $\gamma_0 = \gamma = 0$ and $\kappa_0 = \kappa$, allowing any positive κ. However, some subsequent papers on mixed oligopolies assumed $\kappa_0 = \kappa = 1$.[12] Our results clearly show that the assumption $\kappa_0 = \kappa = 1$ is not innocuous and can affect the properties of equilibria. In other words, setting $\kappa_0 = \kappa = 1$ is never normalized. Thus, we should carefully check the robustness of the results based on this assumption.

Propositions 2 and 3 suggest that an increase in n is more likely to increase each private firm's profit when κ_0 is smaller. We explain the intuition. When κ_0 is larger, a reduction in q_0 caused by a new entry (i.e., an increase in n) reduces firm 0's marginal cost more significantly, which in turn reduces the government's incentive to raise α. Therefore, a new entry is less likely to increase each private firm's profit when κ_0 is larger.

Moreover, Propositions 2 and 3 suggest that an increase in n is more likely to increase each private firm's profit when n is smaller. We explain the intuition below. Suppose that $\kappa_0 > 0$. When n is smaller, the difference between the marginal cost of a public firm and each private firm is larger. Thus, the production substitution from the public firm to the private firms improves welfare more significantly. Therefore, an increase in n more significantly reduces α when n is smaller, which is beneficial for private firms.

4 Free entry market

Here, we investigate a free-entry market in which n is endogenously determined by the zero profit condition.[13] When the private firm enters the market, it incurs an entry cost $F > 0$. In the first stage, each private firm chooses whether to enter the market. In the second stage, the government chooses α.[14] In the third stage, firms face quantity competition.

The second and third stages were analyzed in the previous section. We discuss the first stage.

[12] See, among others, Wang and Chen (2011), Lee and Xu (2018), and Cho et al. (2019).

[13] Needless to say, zero profit means zero excess profit, and each private firm obtains normal profit.

[14] This is an entry-then-privatization model (Xu et al., 2017; Lee et al., 2018, Haraguchi and Matsumura, 2020a). If the government chooses α before the entry of private firms, the equilibrium is unique, in contrast to the result in this section. For the properties of privatization-then-entry, see Matsumura and Kanda (2005) and Cato and Matsumura (2012).

As long as $n > 0$, the number of private firms is determined by the following zero profit condition.

$$\pi^F(n) = F. \tag{12}$$

If $\pi^F(0) < F$, the public monopoly (i.e., $n = 0$) constitutes an equilibrium, although there may be other equilibria in which (12) is satisfied for positive n.

We now present a result on the properties of long-run equilibria.

Proposition 4 *(i) If*

$$\pi^F(0) = \frac{(2+\kappa)(\kappa_0(a-\gamma)+\gamma_0-\gamma)^2}{2(1+\kappa_0)^2(1+\kappa)^2} > F, \tag{13}$$

there exists a unique and stable equilibrium. (ii) If (13) is not satisfied, there exists an equilibrium in which $n = 0$ *and* $\alpha = 0$. *(iii) If (13) is not satisfied and* $(\kappa+1)^2\kappa_0 - \kappa - 1 < 0$, *there exists* F *that yields three equilibria.*

Proof See Appendix B.

There are the following three patterns of long-run equilibria:

(1) Unique equilibrium is public monopoly equilibrium, in which $n^F = 0$ and $\alpha^F = 0$.

(2) Unique equilibrium is mixed oligopoly or private oligopoly equilibrium, in which $n^F > 0$ and $\alpha^F > 0$.[15]

(3) There are three equilibria. One is public monopoly equilibrium, another is mixed oligopoly equilibrium, and the last one is mixed or private oligopoly equilibrium.

Suppose case (a) in Proposition 3(i) emerges (i.e., π is increasing in n as long as $\alpha^F < 1$). An increase in n increases α^F and eventually $\alpha^F = 1$ holds. Then, a further increase in n decreases π because $\alpha^F = 1$ regardless of n, and thus, π is decreasing in n (Proposition 1(ii)). Therefore, the relationship between π and n is inverted-U shape, and all of (1)–(3) can emerge, depending on $\pi(0)$ (see Figure 1). The same is true when case (c) in Proposition 3 emerges.

Suppose case (b) in Proposition 3(i) emerges. Because π is decreasing in n, either (1) or (2) emerges, depending on $\pi(0)$. In other words, the equilibrium is always unique (see Figure 2).

[15] We call it a private oligopoly equilibrium if $\alpha^F = 1$ because all firms are owned by the private sector.

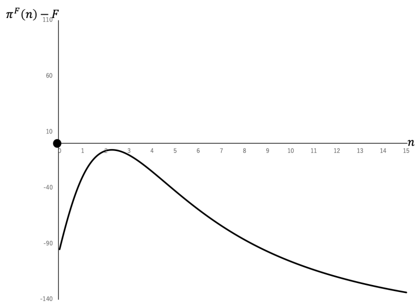

Figure 1-1

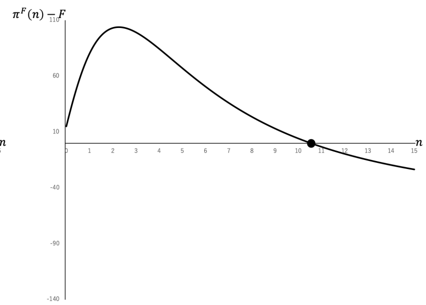

Figure 1-2

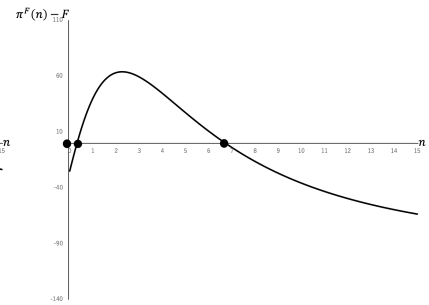

Figure 1-3

Figure 1: Numerical examples of $\pi^F(n) - F$ where $a = 100$, $\gamma_0 = 2$, $\gamma = 1$, $\kappa_0 = 0.1$, and $\kappa = 0.2$. $F = 170$, 60, and 100 in Figures 1-1, 1-2, and 1-3, respectively.

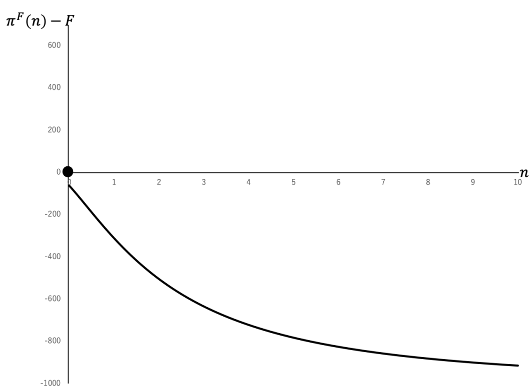

Figure 2-1

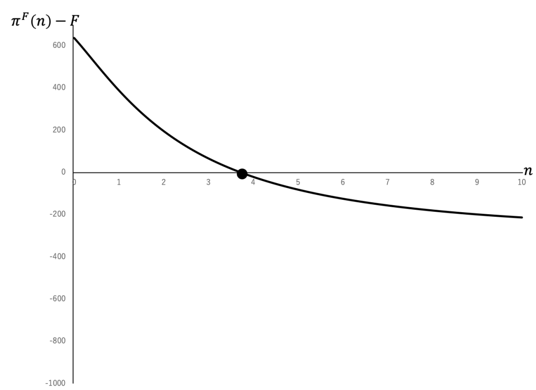

Figure 2-2

Figure 2: Numerical examples of $\pi^F(n) - F$ where $a = 100$, $\gamma_0 = 2$, $\gamma = 1$, and $\kappa_0 = \kappa = 1$. $F = 1000$ and 300 in Figures 2-1 and 2-2, respectively.

If κ_0 is small, multiple equilibria may exist and one of them is the public monopoly equilibrium. As we have shown in the previous section, if κ_0 is small, $\pi^F(n)$ is increasing in n for small n. This yields multiple equilibria, both public monopoly and mixed or private oligopoly equilibria. If private firms expect that other firms will enter the market, they expect that the equilibrium degree of privatization will be large, which increases the profit of each entrant. Therefore, each potential entrant has an incentive to enter the market, yielding a mixed or a private oligopoly equilibrium. If each private firm expects that other firms will not enter the market, the equilibrium degree of privatization will be small, which reduces the profit of each entrant. Therefore, each potential entrant does not enter the market. Under these conditions, multiple self-fulfilling long-run equilibria exist.

5 Integer constraint and heterogeneity among private firms

In the previous sections, we treated n as a continuous variable and ignored the integer constraint of the number of firms. Moreover, we did not allow cost heterogeneity among private firms.[16]

The general analysis of heterogeneous private firms with the integer constraint is difficult and intractable. Instead of this general analysis, we compare a mixed duopoly with a mixed triopoly and investigate the effect of the new entry of a private firm. Firm 0 is a public firm and firms 1 and 2 are private firms. Firm i's cost is $\gamma_i q_i + (\kappa_i/2)(q_i)^2$.

First, we discuss the mixed duopoly in which firms 0 and 1 compete. Let superscript DS be the equilibrium outcomes at the second stage (quantity competition stage) in the duopoly.

In the quantity competition stage, the equilibrium outputs are:

$$\begin{aligned} q_0^{DS}(\alpha) &= \frac{(1+\kappa_1)(a-\gamma_0)-(\gamma_0-\gamma_1)}{1+\kappa_1+(2+\kappa_1)(\alpha+\kappa_0)}, \\ q_1^{DS}(\alpha) &= \frac{(\alpha+\kappa_0)(a-\gamma_1)+(\gamma_0-\gamma_1)}{1+\kappa_1+(2+\kappa_1)(\alpha+\kappa_0)}. \end{aligned}$$

[16] Although asymmetry in private firms is rarely discussed in the literature on mixed oligopolies, it affects the equilibrium privatization policy. For asymmetric cost functions among private firms and their policy implications, see Haraguchi and Matsumura (2020b,c). For asymmetric objective functions among private firms and their policy implications, see Kim et al. (2019).

The resulting firm 1's profit and welfare are:

$$\pi_1^{DS}(\alpha) = \frac{(2+\kappa_1)((\alpha+\kappa_0)(a-\gamma_1)+\gamma_0-\gamma_1)^2}{2(1+\kappa_1+(2+\kappa_1)(\alpha+\kappa_0))^2},$$
$$W^{DS}(\alpha) = \frac{X_3}{2(1+\kappa_1+(2+\kappa_1)(\alpha+\kappa_0))^2}.$$

In the first stage, the government chooses α to maximize W^{DS}. The first-order condition for this maximization problem is:

$$\frac{\partial W^{DS}}{\partial \alpha} = -\frac{((1+\kappa_1)(a-\gamma_0)-(\gamma_0-\gamma_1))X_4}{(1+\kappa_1+(2+\kappa_1)(\alpha+\kappa_0))^3} = 0. \tag{14}$$

The second-order condition is:

$$-\frac{(a-4\gamma_0+3\gamma_1+\kappa_1(3a-4\gamma_0+\gamma_1)+\kappa_1^2(a-\gamma_0))^4}{(a-2\gamma_0+\gamma_1+\kappa_1(a-\gamma_0))^2(1+4\kappa_0(1+\kappa_1)+3\kappa_1+\kappa^2(1+\kappa_0))^3} < 0$$

and is satisfied. The solution for (14), α^{D*}, is:

$$\alpha^{D*} = \frac{\kappa_0(a-\gamma_1)+\gamma_0-\gamma_1}{a-4\gamma_0+3\gamma_1+\kappa_1(3a-4\gamma_0+\gamma_1)+\kappa_1^2(a-\gamma_0)}. \tag{15}$$

Let superscript DF denote the equilibrium outcomes at the first stage in the duopoly. Equilibrium α, α^{DF} is $\min\{\alpha^{D*}, 1\}$.[17]

Supposing that the solution is interior (i.e., $\alpha^{DF} < 1$), then:

$$\pi_1^{DF} = \frac{(2+\kappa_1)^3(\gamma_0-\gamma_1+\kappa_0(a-\gamma_1))^2}{2(\kappa_1+(1+\kappa_1)^2+\kappa_0(2+\kappa_1)^2)^2}. \tag{16}$$

Second, we discuss the mixed triopoly in which firms 0, 1, and 2 compete. Let superscript TS be the equilibrium outcomes at the second stage (quantity competition stage) in the triopoly.

In the quantity competition stage, the equilibrium outputs are:

$$q_0^{TS}(\alpha) = \frac{a-3\gamma_0+\gamma_1+\gamma_2+\kappa_1(a-2\gamma_0+\gamma_2)+\kappa_2(a-2\gamma_0+\gamma_1)+\kappa_1\kappa_2(a-\gamma_0)}{1+3(\alpha+\kappa_0)+(\kappa_1+\kappa_2)(1+2(\alpha+\kappa_0))+\kappa_1\kappa_2(1+\alpha+\kappa_0)},$$
$$q_1^{TS}(\alpha) = \frac{X_5}{1+3(\alpha+\kappa_0)+(\kappa_1+\kappa_2)(1+2(\alpha+\kappa_0))+\kappa_1\kappa_2(1+\alpha+\kappa_0)},$$
$$q_2^{TS}(\alpha) = \frac{X_6}{1+3(\alpha+\kappa_0)+(\kappa_1+\kappa_2)(1+2(\alpha+\kappa_0))+\kappa_1\kappa_2(1+\alpha+\kappa_0)}.$$

[17] We can show that $\alpha^{DF} > 0$ as long as all firms produce positive output in equilibrium.

The resulting firm 1's profit and welfare are:

$$\pi_1^{TS}(\alpha) = \frac{(2+\kappa_1)(X_5)^2}{2(1+3(\alpha+\kappa_0)+(\kappa_1+\kappa_2)(1+2(\alpha+\kappa_0))+\kappa_1\kappa_2(1+\alpha+\kappa_0))^2},$$
$$W^{TS}(\alpha) = \frac{X_7}{2(1+3(\alpha+\kappa_0)+(\kappa_1+\kappa_2)(1+2(\alpha+\kappa_0))+\kappa_1\kappa_2(1+\alpha+\kappa_0))^2}.$$

In the first stage, the government chooses α to maximize W^{TS}. The first-order condition for this maximization problem is:

$$\frac{\partial W^{TS}}{\partial \alpha} = -\frac{X_8(\alpha X_9 - X_{10})}{(1+3(\alpha+\kappa_0)+(\kappa_1+\kappa_2)(1+2(\alpha+\kappa_0))+\kappa_1\kappa_2(1+\alpha+\kappa_0))^3} = 0. \quad (17)$$

The second-order condition is:

$$-\frac{(X_{11})^4}{((1+\kappa_1)(1+\kappa_2)(a-\gamma_0)-(1+\kappa_1)(\gamma_0-\gamma_1)-(1+\kappa_2)(\gamma_0-\gamma_2))^2(X_{12})^3} < 0$$

and is satisfied. The solution for (17), α^{T*}, is $\alpha^{T*} = X_{10}/X_9$. Let superscript TF denote the equilibrium outcomes at the first stage in the triopoly. Equilibrium α, α^{TF} is $\min\{\alpha^{T*}, 1\}$.[18]

Suppose that the solution is interior (i.e., $\alpha^{TF} < 1$). Then

$$\pi_1^{TF} = \frac{(2+\kappa_1)(X_{13})^2}{2(X_{14})^2}. \quad (18)$$

Comparing (16) with (18), we obtain the following result.

Proposition 5 *Firm 2's entry increases firm 1's profit if and only if* $X_{15}\kappa_2 + X_{16} > 0$.

Proof See Appendix B.

Firm 2's entry may increase the degree of privatization, which may increase the profit of incumbent firm 1. The condition $X_{15}\kappa_2 + X_{16} > 0$ is messy and complicated. However, we obtain some results from this condition. First, suppose that $\kappa_0 = 0$. Then, if κ_2 is not too large or κ_1 is sufficiently large, firm 2's entry increases firm 1's profit. Second, if all firms have an identical cost function, firm 2's entry increases firm 1's profit if $\kappa \leq \hat{\kappa} \approx 0.17$. Again, this result suggests that assumption $\kappa_0 = \kappa = 1$, which is often adopted in the literature, is not innocuous and it can affect the properties of equilibria. The threshold value of 0.17 is smaller than that in the previous section,

[18]We can show that $\alpha^{TF} > 0$ as long as all firms produce positive output in equilibrium.

0.62. This implies that a new entry of the private firm is less likely to increase the incumbent's profit under the integer constraint of the number of firms, although this can happen even under symmetric increasing marginal costs.

We now present the numerical results. In Figure 3, we set $a = 10$, $\gamma_0 = 2$, and $\gamma_1 = \gamma_2 = 1$ and show the range of κ_1, κ_2 for $X_{15}\kappa_2 + X_{16} > 0$. Figure 3 suggests that firm 2's entry is more likely to increase firm 1's profit when κ_0 is smaller, κ_1 is larger, and κ_2 is smaller. A smaller κ_2 implies that the production substitution from firm 0 to firm 2 caused by an increase in α is more likely to improve welfare. Thus, firm 2's entry more significantly increases α when κ_2 is smaller, which increases firm 1's profit more significantly. A larger κ_1 implies that the production substitution from firm 0 to firm 1 caused by an increase in α less likely improves welfare. Thus, without firm 2's entry, α is small, which increases the difference in α with or without firm 2's entry. Therefore, firm 2's entry increases firm 1's profit more significantly when κ_1 is larger. As mentioned after Proposition 3, when κ_0 is large, a reduction in q_0 caused by firm 2's entry reduces firm 0's marginal cost, which reduces the government's incentive to increase α.[19] Therefore, firm 2's entry less is likely to increase firm 1's profit when κ_0 is larger.

6 Concluding remarks

In this study, we investigate whether the new entry of a private firm increases private incumbent firms' profits. To achieve that, we adopt a linear-quadratic cost function that covers two popular models in mixed oligopolies. We find that each private firm's profit is either (a) increasing in the number of private firms, (b) decreasing in the number of private firms, or (c) increasing (decreasing) for small (large) number of private firms. Case (a) emerges if and only if public firm's marginal cost is constant, and (b) when the coefficients in quadratic terms of both public and private firms is large. When (c) holds, free entry market may yield multiple equilibria. Moreover, our results suggest that the number of firms may not be an appropriate measure of competitiveness in mixed oligopolies under endogenous degree of privatization.

[19] Remember that firm 0's marginal cost is increasing in q_0.

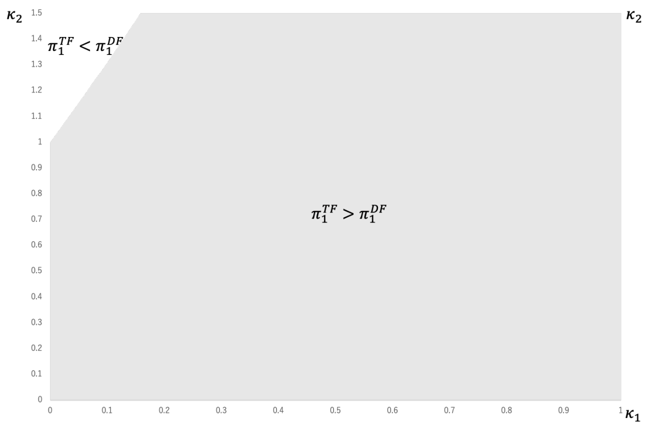

Figure 3-1

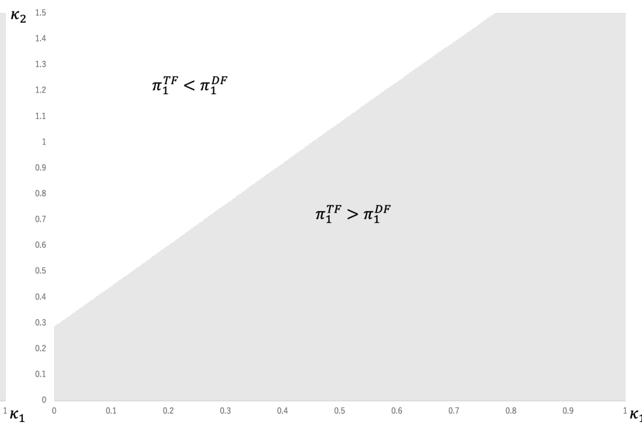

Figure 3-2

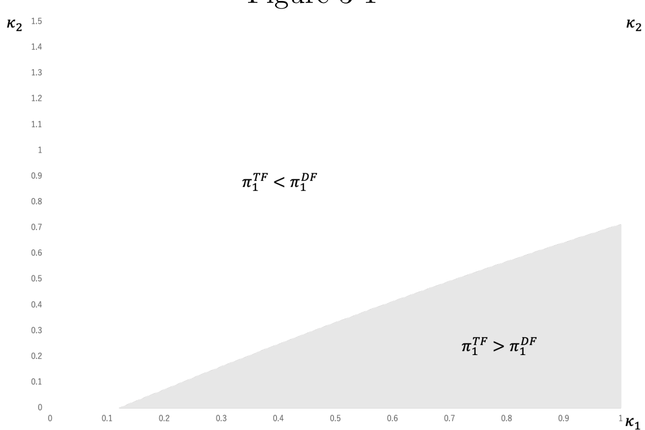

Figure 3-3

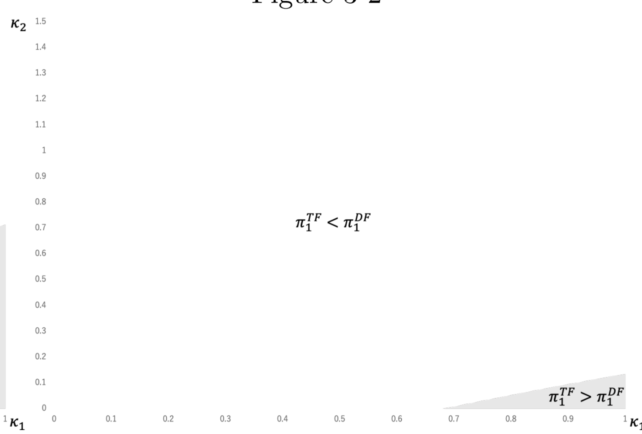

Figure 3-4

Figure 3: Numerical examples of firm 1's profit where $a = 10$, $\gamma_0 = 2$, and $\gamma_1 = \gamma_2 = 1$. $\kappa_0 = 0, 0.1, 0.2,$ and 0.3 in Figures 3-1, 3-2, 3-3, and 3-4, respectively.

We assume that all firms are domestic. In the literature on mixed oligopolies, the nationality of both public and private firms affects the equilibrium outcomes, especially the optimal privatization policy.[20] Moreover, the trade policy may affect the optimal privatization policy, and vice versa (Chang, 2005, 2007). We presume that the new entry of private firms is less likely to raise the incumbent private firms' profit when the new entrants are foreign firms. Future research should thus extend our analysis in this direction.

We focus on privatization policy and do not consider other policies, such as industrial policies or regulations. Moreover, we focus on entry decisions and the quantity choice of the firms, and do not consider other strategic actions such as R&D investments. For example, we assume that both production and entry costs are exogenous, but these may depend on both the policies and firms' actions. Future research should examine these factors.[21]

[20]Whether the private firm is domestic or foreign often yields contrasting results in the literature on mixed oligopolies. For the discussion on the nationality of private firms, see Corneo and Jeanne (1994), Fjell and Pal (1996), Pal and White (1998), Bárcena-Ruiz and Garzón (2005 a,b), and Chang and Ryu (2015). For the discussion on the nationality of public firms, see Lin and Matsumura (2012) and Sato and Matsumura (2019).

[21]For a discussion on entry taxes in mixed oligopolies, see Cato and Matsumura (2013). For the strategic manipulation of F by private firms or the government in free entry markets, see Matsumura and Yamagishi (2017a,b)

Appendix A

$$
\begin{aligned}
X_1 &:= ((1+\kappa)(a-\gamma_0)+n(a-\gamma)(\alpha+\kappa_0))^2+(2\alpha+\kappa_0)((1+\kappa)(a-\gamma_0)-n(\gamma_0-\gamma))^2 \\
&+ n(2+\kappa)((\alpha+\kappa_0)(a-\gamma)+\gamma_0-\gamma)^2, \\
X_2 &:= \alpha((1+\kappa)^2(a-\gamma_0)-n(n+2)(\gamma_0-\gamma)+\kappa n(a-2\gamma_0+\gamma))-n(\gamma_0-\gamma+\kappa_0(a-\gamma)), \\
X_3 &:= (2\alpha+\kappa_0)((1+\kappa_1)(a-\gamma_0)-(\gamma_0-\gamma_1))^2+(2+\kappa_1)((\alpha+\kappa_0)(a-\gamma_1) \\
&+ \gamma_0-\gamma_1)^2+((1+\kappa_1)(a-\gamma_0)+(\alpha+\kappa_0)(a-\gamma_1))^2, \\
X_4 &:= (\alpha(a-4\gamma_0+3\gamma_1+\kappa_1(3a-4\gamma_0+\gamma_1)+\kappa_1^2(a-\gamma_0))-(\kappa_0(a-\gamma_1)+\gamma_0-\gamma_1)), \\
X_5 &:= \alpha(a+\gamma_2)+\gamma_0-(1+2\alpha)\gamma_1+(a-2\gamma_1+\gamma_2)\kappa_0+(\alpha a+\gamma_0-(1+\alpha)\gamma_1)\kappa_2+(a-\gamma_1)\kappa_0\kappa_2, \\
X_6 &:= \alpha(a+\gamma_1)+\gamma_0-(1+2\alpha)\gamma_2+(a-2\gamma_2+\gamma_1)\kappa_0+(\alpha a+\gamma_0-(1+\alpha)\gamma_2)\kappa_1+(a-\gamma_2)\kappa_0\kappa_1, \\
X_7 &:= (2\alpha+\kappa_0)(a-3\gamma_0+\gamma_1+\gamma_2+\kappa_1(a-2\gamma_0+\gamma_2)+\kappa_2(a-2\gamma_0+\gamma_1)+\kappa_1\kappa_2(a-\gamma_0))^2 \\
&+ (2+\kappa_2)(\alpha(a+\gamma_2)+\gamma_0-(1+2\alpha)\gamma_1+(a-2\gamma_1+\gamma_2)\kappa_0+(\alpha a+\gamma_0-(1+\alpha)\gamma_1)\kappa_2 \\
&+ (a-\gamma_1)\kappa_0\kappa_2)^2+(2+\kappa_1)(\alpha(a+\gamma_1)+\gamma_0-(1+2\alpha)\gamma_2+(a-2\gamma_2+\gamma_1)\kappa_0 \\
&+ (\alpha a+\gamma_0-(1+\alpha)\gamma_2)\kappa_1+(a-\gamma_2)\kappa_0\kappa_1)^2+((a-\gamma_0)(1+\kappa_1+\kappa_2+\kappa_1\kappa_2) \\
&+ (\alpha+\kappa_0)(2a-\gamma_1-\gamma_2+(a-\gamma_1)\kappa_2+(a-\gamma_2)\kappa_1))^2, \\
X_8 &:= (1+\kappa_1\kappa_2)(a-\gamma_0)+\kappa_1(a-2\gamma_0+\gamma_2)+\kappa_2(a-2\gamma_0+\gamma_1)-2\gamma_0+\gamma_1+\gamma_2, \\
X_9 &:= a-9\gamma_0+4\gamma_1+4\gamma_2+\kappa_1(3a-12\gamma_0+\gamma_1+8\gamma_2)+\kappa_1^2(a-4\gamma_0+3\gamma_2)+\kappa_2(3a-12\gamma_0+8\gamma_1+\gamma_2) \\
&+ \kappa_2^2(a-4\gamma_0+3\gamma_1)+\kappa_1\kappa_2(8a-14\gamma_0+3\gamma_1+3\gamma_2)+\kappa_1^2\kappa_2(3a-4\gamma_0+\gamma_2)+\kappa_1\kappa_2^2(3a-4\gamma_0+\gamma_1) \\
&+ \kappa_1^2\kappa_2^2(a-\gamma_0), \\
X_{10} &:= (1+\kappa_2)^2(\gamma_0-\gamma_1)+(1+\kappa_1)^2(\gamma_0-\gamma_2) \\
&+ \kappa_0((1+\kappa_1)((a-\gamma_2)\kappa_1+a+\gamma_1-2\gamma_2)+(1+\kappa_2)((a-\gamma_2)\kappa_2+a-2\gamma_1+\gamma_2)), \\
X_{11} &:= a-9\gamma_0+4\gamma_1+4\gamma_2+\kappa_1(3a-12\gamma_0+\gamma_1+8\gamma_2)+\kappa_1^2(a-4\gamma_0+3\gamma_2)+\kappa_2(3a-12\gamma_0+8\gamma_1+\gamma_2) \\
&+ \kappa_1\kappa_2(8a-14\gamma_0+3\gamma_1+3\gamma_2)+\kappa_1^2\kappa_2(3a-4\gamma_0+\gamma_2)+\kappa_2^2(a-4\gamma_0+3\gamma_1)+\kappa_1\kappa_2^2(3a-4\gamma_0+\gamma_1) \\
&+ \kappa_1^2\kappa_2^2(a-\gamma_0),
\end{aligned}
$$

$$
\begin{aligned}
X_{12} &:= 1+3(\kappa_1+\kappa_2)+3\kappa_0(3+4(\kappa_1+\kappa_2))+2\kappa_1\kappa_2(4+7\kappa_0)+(1+4\kappa_0)(\kappa_1^2+\kappa_2^2) \\
&+ \kappa_1\kappa_2((3+4\kappa_0)(\kappa_1+\kappa_2)+\kappa_1\kappa_2(1+\kappa_0)), \\
X_{13} &:= 3\gamma_0-2\gamma_1-\gamma_2+(2\gamma_0-\gamma_1-\gamma_2)\kappa_1+((a-2\gamma_1+\gamma_2)(3+2\kappa_1)+(5a-7\gamma_1+2\gamma_2)\kappa_2 \\
&+ (3a-4\gamma_1+\gamma_2)\kappa_1\kappa_2)\kappa_0+(\gamma_0-\gamma_1)((2+\kappa_1)\kappa_2+3\kappa_1+5)\kappa_2+(a-\gamma_1)(2+\kappa_1)\kappa_0\kappa_2^2, \\
X_{14} &:= \kappa_1\kappa_2((1+\kappa_0)\kappa_1\kappa_2+(3+4\kappa_0)(\kappa_1+\kappa_2)+2(4+7\kappa_0)) \\
&+ (1+4\kappa_0)(\kappa_1^2+\kappa_2^2+3(\kappa_1+\kappa_2))+9\kappa_0+1, \\
X_{15} &:= -[((a-\gamma_2)\kappa_0^2+(\gamma_0-\gamma_2)\kappa_0)\kappa_1^3-((5a+\gamma_1-6\gamma_2)\kappa_0^2+5(\gamma_0-\gamma_2)\kappa_0)\kappa_1^2+(4(2a+\gamma_1-3\gamma_2)\kappa_0^2 \\
&+ (a+8\gamma_0-2\gamma_1-7\gamma_2)\kappa_0+\gamma_0-\gamma_1)\kappa_1+4(a+\gamma_1-2\gamma_2)\kappa_0^2+(a+4\gamma_0-3\gamma_1-2\gamma_2)\kappa_0+\gamma_0-\gamma_1], \\
X_{16} &:= \gamma_0-\gamma_2+(a-\gamma_2-6(\gamma_0-\gamma_1))\kappa_0-6(a+\gamma_1-2\gamma_2)\kappa_0^2+(4(\gamma_0-\gamma_2)+(4a-13\gamma_0+6\gamma_1+3\gamma_2)\kappa_0 \\
&- (13a+7\gamma_1-20\gamma_2)\kappa_0^2)\kappa_1+(4(\gamma_0-\gamma_2)+(4a-9\gamma_0+\gamma_1+4\gamma_2)\kappa_0-(9a+2\gamma_1-11\gamma_2)\kappa_0^2)\kappa_1^2 \\
&+ (\gamma_0-\gamma_2+(a-2\gamma_0+\gamma_2)\kappa_0-2(a-\gamma_2)\kappa_0^2)\kappa_1^3.
\end{aligned}
$$

Appendix B

Proof of Proposition 1

From (6), we obtain

$$\frac{\partial p^S(\alpha,n)}{\partial n} = -\frac{(1+\kappa)(\alpha+\kappa_0)((\alpha+\kappa_0)(a-\gamma)+\gamma_0-\gamma)}{((1+\kappa)(1+\kappa_0+\alpha)+n(\kappa_0+\alpha))^2} < 0.$$

From (7), we obtain

$$\frac{\partial \pi^S(\alpha,n)}{\partial n} = -\frac{(2+\kappa)(\alpha+\kappa_0)}{2}\left(\frac{(\alpha+\kappa_0)(a-\gamma)+\gamma_0-\gamma}{(1+\kappa)(1+\kappa_0+\alpha)+n(\kappa_0+\alpha)}\right)^2 < 0. \quad \blacksquare$$

Proof of Lemma 1

From (10), we obtain

$$\frac{\partial \alpha^*}{\partial n} = \frac{(\kappa_0(a-\gamma)+\gamma_0-\gamma)((a-\gamma_0)(1+\kappa)^2+n^2(\gamma_0-\gamma))}{(((1+\kappa)^2+\kappa n)a-((1+\kappa)(1+\kappa+2n)+n^2)\gamma_0+n(2+\kappa+n)\gamma)^2} > 0. \quad \blacksquare$$

Proof of Proposition 2

Because Proposition 2(i) and Proposition 2(ii) have already been shown in Matsumura (1998) under more general demand and cost conditions, we omit the proof. We then prove Proposition 2(iii). Substituting α^* into (7), we obtain

$$\pi^F(n) = \frac{(\kappa+2)(n+\kappa+1)^2(\kappa_0(a-\gamma)+\gamma_0-\gamma)^2}{2(\kappa_0 n^2+(2\kappa_0(1+\kappa)+\kappa)n+(1+\kappa_0)(1+\kappa)^2)^2}. \tag{19}$$

From (19), we obtain

$$\begin{aligned}\frac{\partial \pi^F(n)}{\partial n} &= -\frac{(\kappa+2)(n+\kappa+1)(\kappa_0(a-\gamma)+\gamma_0-\gamma)^2((n^2+2(\kappa+1)n+(\kappa+1)^2)\kappa_0-\kappa-1)}{(\kappa_0 n^2+(2\kappa_0(1+\kappa)+\kappa)n+(1+\kappa_0)(1+\kappa)^2)^3} \gtreqless 0\\ \iff &\quad (n^2+2(\kappa+1)n+(\kappa+1)^2)\kappa_0-\kappa-1 \lesseqgtr 0.\end{aligned}$$

This implies Proposition 2(iii). $\blacksquare$

Proof of Proposition 3

Substituting $\kappa_0 = 0$ into (11), we obtain $g(n) = -\kappa-1 < 0$. If $\kappa_0 > 0$, then $g(n) > 0$ for a sufficiently large n. These imply Proposition 3(ii).

If $\kappa_0 \geq \kappa > 0$, then $g(n)$ is increasing in n for $n > 0$. Then, $g(n)$ is positive for any $n > 0$ if $g(0) = (\kappa+1)(\kappa_0(\kappa+1)-1) > 0$. Since we suppose that $\kappa_0 \geq \kappa > 0$, we have $g(0) > 0$ if

$\kappa(\kappa+1)-1>0$. By solving $\kappa(\kappa+1)-1=0$, we have a unique positive solution $\bar{\kappa}=\frac{\sqrt{5}-1}{2}\simeq 0.618$. This implies Proposition 3(iii).

Since $g(n)$ is always negative if $\kappa_0=0$, then $\pi^F(n)$ is increasing in n for any $n>0$. This corresponds to case (a). Suppose that $\kappa_0>0$. Then, $g(n)$ is increasing in n for $n>0$. $g(n)=0$ has no positive solution if $g(0)\geq 0$. Then, $g(n)$ is always positive and $\pi^F(n)$ is decreasing in n for any $n>0$ if $g(0)\geq 0$. This corresponds to case (b). If $g(0)<0$, then $g(n)=0$ has one negative solution and one positive solution. Let $\breve{n}$ be the positive solution for $g(n)=0$. Then, $g(n)<(>)0$ if $n<(>)\breve{n}$. $\pi^F(n)$ increases in n for $n<\breve{n}$ and decreases in n for $n>\breve{n}$. This corresponds to case (c). No other case exists. These imply Proposition 3(i). ■

Proof of Proposition 4

If (13) holds (i.e., $\pi^F(0)>F$), then $n=0$ never constitutes an equilibrium. We now show that there exists unique n such that $\pi^F(n)-F=0$.

Proposition 3(i,ii) states that $\pi^F(n)$ is either (a) increasing in n for any n, (b) decreasing in n for any n, or (c) inverted U-shaped, as long as $\alpha^F=\alpha^*<1$. Proposition 1 states that when $\alpha^*\geq\alpha^F=1$, $\pi^F(n)$ is decreasing in n. Lemma 1 states that α^* is increasing in n. Moreover, $\lim_{n\to\infty}\pi^F=0$ because $\lim_{n\to\infty}q^S(1,n)=0$ and $q^S(1,n)\geq q^F(n)$. Therefore, from the continuity of π^F, there exists at least one positive solution for $\pi^F(n)-F=0$.

Suppose (b) holds. Then, $\pi^F(n)$ is decreasing in n; thus, the solution for $\pi^F(n)=F$ is unique. Suppose (c) holds. Let $\hat{n}$ be the solution for $\alpha^*=1$. π^F is increasing in n for $n<\min\{\hat{n},\breve{n}\}$, and decreasing in n for $n>\min\{\hat{n},\breve{n}\}$. Therefore, the unique equilibrium for $\pi^F(n)=0$ exists. Suppose (a) holds. π^F is increasing in n for $n<\hat{n}$, and decreasing in n for $n>\hat{n}$. Therefore, the solution for $\pi^F(n)-F=0$ is unique. These imply Proposition 4(i).

Henceforth, we consider the case in which (13) does not holds (i.e., $\pi^F(0)\leq F$). Then, $n=0$ constitutes an equilibrium. Given $n=0$, the optimal degree of privatization is zero. This implies Proposition 4(ii).

Suppose that $\pi^F(0)\leq F$. Then, $n=0$ constitutes an equilibrium. In addition, suppose $g(0)<0$. From Proposition 2(iii), we have that π^F is increasing in n for $n<\min\{\hat{n},\breve{n}\}$ and decreasing in n for $n > \min\{\hat{n}, \breve{n}\}$. Thus, if $\pi^F(0) - F$ is sufficiently close to zero, $\pi^F(n) = F$ has two positive solutions. These imply Proposition 4(iii). ■

Proof of Proposition 5

From (16) and (18), we obtain

$$\pi_1^{TS} - \pi_1^{DS} = \frac{(2+\kappa_1)[(A+B)(A-B)]}{2(\kappa_1 + (1+\kappa_1)^2 + \kappa_0(2+\kappa_1)^2)^2(X_{14})^2},$$

where $A := (\kappa_1 + (1+\kappa_1)^2 + \kappa_0(2+\kappa_1)^2)X_{13}$ and $B := (2+\kappa_1)(\gamma_0 - \gamma_1 + \kappa_0(a - \gamma_1))X_{14}$.

Since we suppose that equilibrium output is positive, then $q_1^{TS} = X_{13}/X_{14} > 0$. Thus, we have $X_{13} > 0$, because $X_{14} > 0$. As a result, $\pi_1^{TS} - \pi_1^{DS} > 0$ if and only if $A - B = X_{15}\kappa_2 + X_{16} > 0$. This verifies Proposition 5. ■

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