The Pareto distribution, asymmetric shape parameters, and equilibrium in an asymmetric Melitz model

Nakamoto, Yasuhiro

10 March 2020

The Pareto distribution, asymmetric shape parameters, and equilibrium in an asymmetric Melitz model

Yasuhiro Nakamoto*

March 10, 2020

Abstract

Incorporating the homogeneous good sector into the Melitz model, we re-consider the findings in Demidova (2008, International Economic Review) that two countries have different productivity distribution functions. Although the asymmetry of the productivity distribution function causes highly non-linear equations and incorporating the homogeneous good sector yields the difference in markup rates between sectors, we graphically conclude that there are no multiple equilibria and no pure exporters, and that the effects of trade liberalization on welfare are not qualitatively different from those in the one-sector Melitz model. Finally, supposing that trade specialization arises in the differentiated good sector, we confirm the welfare impacts of trade liberalization.

Keywords: Heterogeneous Firms, Country Asymmetries, Trade Liberalization, Welfare
JEL: F10, F12

*Faculty of Informatics, Kansai University, 2-1-1 Reizanji-cho,Takatsuki-shi, Osaka, 569-1095, Japan, e-mail: nakamoto@kansai-u.ac.jp

1 Introduction

In an economy that consists solely of the homogeneous good (numeraire) sector, the competitive market with free entry forces inefficient firms out of the market and secures the position of efficient firms; however, incorporating a differentiated good sector does not maintain the abovementioned efficiency but creates a distortion caused by the differences in the markups charged by firms in two sectors; namely, a zero markup in the numeraire sector and a positive constant markup in the differentiated good sector. The difference between single- and multiple-sector models matters and must be treated carefully. In this paper, incorporating the homogenous good sector into the one-sector Melitz model, we re-consider the findings in Demidova (2008).

In addition to different markup rates between two sectors, Demidova (2008) supposes that two countries have different productivity distribution functions, thereby showing that the shape heterogeneity in the productivity distribution functions creates a highly non-linear model structure.[1] As a result, one may expect a deviation of the findings obtained by the one-sector Melitz model: no multiple equilibria, no pure exporters, and the positive impacts of trade liberalization on welfare. Demidova (2008) also mentions that falling trade cost may lead to welfare loss in a country (See Proposition 1), and the appearance of pure exporters. However, the use of the general form of the productivity distribution function in Demidova (2008) may not lead to a fruitful finding in the analytic analysis. Specifically, Demidova (2008) provides Assumption 1 *to ensure that, in both countries, only firms producing in the domestic market can export, assuming that* $f_x/f > X$*, where* X *depends on the difference in productivity distributions*, which characterizes the productivity cutoff levels (See Lemma 3 in Demidova (2008)) and trade structure (See Lemma 4 in Demidova (2008)). However, we cannot see what X is in a concrete expression.

Following a large part of the literature such as Helpman et al. (2004), we assume a Pareto productivity distribution throughout. This assumption allows us to characterize our results in closed form despite country asymmetries by deriving a graphically novel analysis, and thus to inspect the underlying mechanism in the most transparent way. Solving our

[1] This asymmetry of productivity distribution is empirically supported in Balistreri et al. (2011), Hshieh and Ossa (2015), and Spearot (2016).

model analytically, we conclude that as long as two-way trade arises, there are no multiple equilibria and no pure exporter, and that the welfare impacts are qualitatively the same as in the one-sector Melitz model.

While the abovementioned findings are related to a re-consideration of Demidova's (2008) set-up, we also make an original analysis considering that trade specialization arises in the differentiated good sector. We then find that under this specialization, unilateral trade liberalization contributes welfare gain to the importing country, which is completely the opposite of the welfare impact of trade liberalization under two-way trade. The reason for this is that the trade liberalization that occurs under the specialization does not induce the entry of less productive firms, thereby implying that there is no resource allocation that deteriorates welfare in the importing country. As a result, the trade liberalization decreases the price of differentiated goods and hence yields the welfare gain in the importing country.

Our analysis is related to the important strands of literature regarding the asymmetric shape parameters of the Pareto distribution that governs productivity draws by country. Shape heterogeneity complicates the system of equilibrium equations so that little is known about the welfare effects of trade policy in such a setting except for Demidova (2008), Demidova and Rodríguez-Clare (2013), and Spearot (2016). When Demidova and Rodríguez-Clare (2013) built a small economy model of Melitz (2003), they showed that a decline in import costs always benefits the liberalizing country. Alternatively, we make use of an asymmetric two-country Melitz model, and then confirm the welfare effects of trade liberalization (i.e., a decrease in the iceberg cost). Spearot (2016) employs an amended version of Melitz and Ottaviano (2008) by removing the outside good, and allows for shape heterogeneity. He drives for the welfare effects of trade shocks theoretically, while his interest is in empirical analysis of the welfare effects of tariff shocks.

Our paper is organized as follows. Section 2 develops our model. Section 3 derives and discusses the free entry conditions. Section 4 analyzes the reversal of productivity cutoffs and the (non)existence and uniqueness of equilibrium. Section 5 examines the welfare effects of trade liberalization. Section 6 considers the case that trade in the differentiated good sector is specialized. Concluding remarks appear in Section 7.

2 Model

Except for the specification in Pareto productivity distribution, our model remains faithfully the same as that in Demidova (2008). Consider the world economy consisting of home and foreign countries (indexed by $i = H, F$) with two sectors: the homogenous and differentiated good sectors. As in Demidova (2008), we assume that the homogeneous good is produced under constant returns to scale, and that it is produced in each country and freely traded under incomplete specialization. As a result, we can choose the homonogeous good for our numéraire. This assumption also ensures factor price equalization across countries, and the prices and wages in both countries are then normalized, where labor is the sole factor of production in inelastic supply L and is immobile across countries.

2.1 Demand

The preference of a representative consumer in a country i is a Cobb-Douglas utility function over a continuum of goods indexed by ω and a homogeneous good N (our numéraire):

$$U^i = (C^i)^\beta (N^i)^{1-\beta}, \quad 0 < \beta < 1, \quad i = H, F. \tag{1}$$

where C^i is a consumption index over consumption of individual-domestic varieties $q_d^i(\omega)$ and -imported varieties $q_m^i(\omega)$: $C^i \equiv \left(\int_{\omega \in \Omega^i} q_d^i(\omega)^{\frac{\sigma-1}{\sigma}} d\omega + \int_{\omega \in \Omega^i} q_m^i(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}$ $(i = H, F)$. Ω^i is the set of available varieties in a country i, and $\sigma(> 1)$ is a constant elasticity of substitution between any two goods. The price index dual to C^i is $P^i = \left(\int_{\omega \in \Omega^i} p_d(\omega)^{1-\sigma} d\omega + \int_{\omega \in \Omega^i} p_x(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}}$, where $p_d(\omega)$ and $p_x(\omega)$ are the prices of domestic and exporting variety ω. As confirmed later, the price is symmetric between countries and thus, we omit the notation that specifies a country. We can then derive the optimal consumption for individual-domestic and -imported varieties ω: $q_d^i(\omega) = C^i \left(\frac{p_d(\omega)}{P^i} \right)^{-\sigma}$ and $q_m^i(\omega) = C^i \left(\frac{p_x(\omega)}{P^j} \right)^{-\sigma}$ $(i, j = H, F, \quad i \neq j)$.

2.2 Production in the differentiated-good sector

Denoting the iceberg cost by τ and the fixed production costs for the domestic and export goods by f and f_x, we can represent firm technology by cost functions $l_d^i(\varphi) = (f + q_d^i/\varphi)$ and $l_x^i(\varphi) = (f_x + \tau q_m^j/\varphi)$, where we assume that trade is costly such that $f_x > f$ and $\tau > 1$.

From the profit maximization in monopolistic competition, we obtain the pricing rules:

$$p_x(\varphi) = \tau p_d(\varphi), \quad p_d(\varphi) = \left(\frac{\sigma}{\sigma-1}\right)\left(\frac{1}{\varphi}\right), \tag{2a}$$

where $p_x(\varphi)$ and $p_d(\varphi)$ are symmetric between two countries. Defining $r_d^i(\varphi) = p_d(\varphi)q_d^i(\varphi)$ and $r_x^i(\varphi) = p_x(\varphi)q_m^j(\varphi)$ $(i = H, F,\ i \neq j)$ as the revenue earned by a country i's firm in domestic and foreign markets, we obtain:

$$r_d^i(\varphi) = R^i\left(\frac{p_d(\varphi)}{P^i}\right)^{1-\sigma}, \quad r_x^i(\varphi) = R^i\left(\frac{p_x(\varphi)}{P^j}\right)^{1-\sigma}, \quad i,j = H,F, \quad i \neq j. \tag{2b}$$

where R^i denotes aggregate revenue earned by country i's firms in the differentiated good sector. In what follows, we denote the profit functions of a country i's firm in domestic and export markets by $\pi_d^i(\varphi)$ and $\pi_x^i(\varphi)$ $(i = H, F)$, and therefore the total profit can be written as $\pi^i(\varphi) = \max\{0, \pi_d^i(\varphi)\} + \max\{0, \pi_x^i(\varphi)\}$. We can then show the zero-profit conditions as follows:[2]

$$\pi_d^i(\varphi^{i*}) = \frac{r_d^i(\varphi_d^{i*})}{\sigma} - f = 0, \quad \pi_x^i(\varphi^{i*}) = \frac{r_x^i(\varphi_x^{i*})}{\sigma} - f_x = 0, \quad i = H, F. \tag{2c}$$

where φ_d^{i*} and φ_x^{i*} show the productivity cutoff levels for serving the domestic and export markets. Using these expressions, we can show that the productivity cutoff levels for the domestic market (φ_d^{i*}) and export market (φ_x^{i*}) are linked as follows:[3]

$$\varphi_x^{F*} = A\varphi_d^{H*}, \quad \varphi_x^{H*} = A\varphi_d^{F*}, \quad A \equiv \tau\left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}} (> 1). \tag{3}$$

3 Industry equilibrium

The concrete entry-exit process follows the setting in Demidova (2008). There is a large pool of prospective entrants, M_e^i, into an industry in a country i, and firms are *ex-ante* identical. To enter, they must make an irreversible investment, modeled as a fixed entry cost f_e (measured in units of labor). In what follows, once the sunk entry cost is paid, firms draw

[2] Please observe that the revenue of a country i's firm in the export market is shown by $r_x^i(\varphi) = p_x(\varphi)q_m^j(\varphi)$, which is the same as $r^j(\tau^{-1}\varphi) = p(\tau^{-1}\varphi)q^j(\varphi)$(notation adapted) in Demidova (2008).

[3] From (2c), we obtain $r_d^H(\varphi_d^{H*}) = r_d^F(\varphi_d^{F*})$ and $r_x^H(\varphi_x^{H*}) = r_x^F(\varphi_x^{F*})$. Substituting (2a) and (2b) into these equations yields $\frac{\varphi_d^{H*}}{\varphi_d^{F*}} = \frac{P^F}{P^H}$ and $\frac{\varphi_x^{H*}}{\varphi_x^{F*}} = \frac{P^H}{P^F}$. Next, (2c) leads to $\frac{r_x^i(\varphi_x^{i*})}{r_d^i(\varphi_d^{i*})} = \frac{f_x}{f}$. Using this equation, from (2b) we can show that $\frac{\varphi_x^{H*}}{\varphi_d^{H*}} = A\frac{P^H}{P^F}$ and $\frac{\varphi_x^{F*}}{\varphi_d^{F*}} = A\frac{P^F}{P^H}$. Finally, using these equations and canceling out the relative price index $\frac{P^H}{P^F}$, we have (3).

their productivity φ from a common distribution $g^i(\varphi)$ that has a continuous cumulative distribution $G^i(\varphi)$, where we specify the function as follows:

$$G^i(\varphi) = 1 - \left(\frac{\hat{\varphi}^i}{\varphi}\right)^{k^i}, \quad i = H, F, \tag{4}$$

where $\hat{\varphi}^i > 0$ is the lower bound of the support of the productivity distribution. The shape parameter k^i indexes dispersion; lower values of k^i are associated with greater productivity dispersion. For average firm size to be finite, we require $k^i > \sigma - 1 (> 0)$, where $\sigma > 1$ is easily confirmed in (2a).[4] Therefore, the following inequality is always held:

$$\frac{A^{k^i} f}{f_x} = \tau^{k^i} \left(\frac{f_x}{f}\right)^{\frac{k^i - (\sigma-1)}{\sigma-1}} \quad (> 1). \tag{5}$$

After the entry, they face a constant exogenous probability of death δ. Therefore, in the stationary equilibrium, the mass of active firms that are alive in a country i, M^i is defined by $p^i_{in} M^i_e = \delta M^i$. We define the probability of successful entry by $p^i_{in} = \min\{D^i, X^i\}$, where $D^i \equiv (1 - G^i(\varphi^{i*}_d))$ and $X^i \equiv (1 - G^i(\varphi^{i*}_x))$ are the respective probabilities of successful entry into the domestic and export markets.

Unlike our paper, Demidova (2008) assumes a more general productivity distribution and provides the following assumption:

$$\frac{g^H(\varphi)}{1 - G^H(\varphi)} < \frac{g^F(\varphi)}{1 - G^F(\varphi)}. \tag{6}$$

According to Demidova (2008, p.1446), (6) means that (notation adapted)*for any given level φ, entrants in the Home with the productivity distribution $G^H(\cdot)$ have a better chance of obtaining a productivity draw than entrants in the Foreign with the productivity distribution* $G^F(\cdot)$. Moreover, assuming that $f_x/f > X$ but X is unclear, where she only mentions that X depends on the difference in productivity distributions (see Assumption 1 in Demidova (2008)), she finds an interesting order of productivity cutoffs: $\varphi^{F*}_d < \varphi^{H*}_d < \varphi^{H*}_x < \varphi^{F*}_x$.

With Pareto distributed productivity (4), the assumption (6) becomes $k^H < k^F$ (observe that the asymmetric lower bound $\hat{\varphi}^i$ is irrelevant here). Below, we will demonstrate that even if (6) is not satisfied (that is, $k^H = k^F$), the results of Demidova (2008) can still be obtained.

[4]This inequality is necessary given that the support for the Pareto distribution is unbounded from above and given the assumption of a continuum of firms: see Helpman et al. (2004), Redding (2011), and Melitz and Redding (2014) for details.

3.1 Free entry condition (FEC)

In an equilibrium with positive firm entry, we require that FEC holds; the expected value of a firm with productivity φ, $v^i(\varphi) = \max\{0, \bar{\pi}^i(\varphi)/\delta\}$, equals the sunk entry cost f_e^i, where the variable $\bar{\pi}^i$ is expected profits conditional on successful entry:[5]

$$\frac{\bar{\pi}^i}{\delta} = \frac{D^i \pi_d^i(\tilde{\varphi}_d^i(\varphi_d^{i*})) + X^i \pi_x^i(\tilde{\varphi}_x^i(\varphi_x^{i*}))}{\delta} = \frac{D^i \left[\left(\frac{\tilde{\varphi}_d^i(\varphi_d^{i*})}{\varphi_d^{i*}} \right)^{\sigma-1} - 1 \right] f + X^i \left[\left(\frac{\tilde{\varphi}_x^i(\varphi_x^{i*})}{\varphi_x^{i*}} \right)^{\sigma-1} - 1 \right] f_x}{\delta} = f_e, \tag{7}$$

Here, the weighted average productivities in the domestic and export markets are defined as: $\tilde{\varphi}_d^i(\varphi_d^{i*}) \equiv \left[\frac{1}{1-G^i(\varphi_d^{i*})} \int_{\varphi_d^{i*}}^{\infty} \varphi^{\sigma-1} g^i(\varphi) d\varphi \right]^{1/(\sigma-1)} = \left(\frac{k^i}{1+k^i-\sigma} \right)^{1/(\sigma-1)} \varphi_d^{i*}$ and $\tilde{\varphi}_x^i(\varphi_x^{i*}) \equiv \left[\frac{1}{1-G^i(\varphi_x^{i*})} \int_{\varphi_x^{i*}}^{\infty} \varphi^{\sigma-1} g^i(\varphi) d\varphi \right]^{1/(\sigma-1)} = \left(\frac{k^i}{1+k^i-\sigma} \right)^{1/(\sigma-1)} \varphi_x^{i*}$.

Substituting (4) into (7) and eliminating the export productivity cutoff level φ_x^{i*} by (3), we can derive FECs in each country:

$$\left(f(\varphi_d^{i*})^{-k^i} + f_x (A \varphi_d^{j*})^{-k^i} \right) = \Delta^i, \quad i, j = H, F, \quad i \neq j. \tag{8}$$

Notice that the right-hand side of (8) has a positive sign and consists of constant parameters alone $\Delta^i \equiv \frac{\delta f_e (k^i - \sigma + 1)}{(\sigma-1)(\hat{\varphi}^i)^{k^i}} (> 0)$.

It would be difficult to investigate the productivity cutoff levels that satisfy two FECs in (8) under two-way trade because of the highly nonlinear system in (8). Therefore, we try to characterize the productivity cutoff levels graphically. To do so, in our paper, FECs in home country (FEC-H) and foreign country (FEC-F), (8) are now rewritten as:

$$\text{FEC-H: } y = \frac{\Delta^H A^{k^H}}{f_x} - \frac{f A^{k^H}}{f_x} z, \quad \text{FEC-F: } y = f^{-k^H/k^F} \left(\Delta^F - f_x A^{-k^F} z^{k^F/k^H} \right)^{k^H/k^F}, \tag{9}$$

where we define $z \equiv \left(\varphi_d^{H*} \right)^{-k^H} (> 0)$ and $y \equiv \left(\varphi_d^{F*} \right)^{-k^H} (> 0)$. To depict FECs in (z, y) graph, we now confirm each slope of FEC-H and FEC-F, where we use (5):

$$\left. \frac{\partial y}{\partial z} \right|_{\text{FEC-H}} = -\frac{A^{k^H} f}{f_x} (< -1), \quad \left. \frac{\partial y}{\partial z} \right|_{\text{FEC-F}} = -\frac{A^{-k^F} f_x}{f} \left(\frac{z}{y} \right)^{\frac{k^F}{k^H} - 1} (< 0). \tag{10}$$

[5] To derive (7), we first explain that $\pi_d^i(\tilde{\varphi}_d^i) = ((\tilde{\varphi}_d^i/\varphi^{i*})^{\sigma-1} - 1) f$. From (2a), (2b)and (2c), we can derive $(\varphi_d^{i*})^{\sigma-1}/f = ((\sigma-1)/R^i)(\sigma/(\sigma-1))^{\sigma} (P^i)^{1-\sigma}$. Next, the profit of a firm with the weighted average productivity is $\pi_d^i(\tilde{\varphi}_d^i) = p(\tilde{\varphi}_d^i) q(\tilde{\varphi}_d^i) - (f + q(\tilde{\varphi}_d^i)/\tilde{\varphi}_d^i)$, which can be rewritten as $\pi_d^i(\tilde{\varphi}_d^i) = (R^i/(\sigma-1))((\sigma-1)/\sigma)^{\sigma} (1/P^i)^{1-\sigma} (1/\tilde{\varphi}_d^i)^{1-\sigma} - f$. Substituting the former equation into $\pi_d^i(\tilde{\varphi}_d^i)$, we obtain the expression for $\pi_d^i(\tilde{\varphi}_d^i) = ((\tilde{\varphi}_d^i/\varphi_d^{i*})^{\sigma-1} - 1) f$. By the same token, we can derive $\pi_x^i(\tilde{\varphi}_x^i) = ((\tilde{\varphi}_x^i/\varphi_x^{i*})^{\sigma-1} - 1) f_x$.

Figures 1(a) and 1(b) represent two FECs and the 45 degree line in (z,y) graph. Hereafter, FEC-H is depicted by a *solid* line with the negative slope as in Figure 1(a). On the contrary, FEC-F is expressed by a *chain* line ($k^H = k^F$) or a *chain* curve ($k^H \neq k^F$) as in Figure 1(b), which can be summarized in the following Lemma:

Lemma 1 *When $k^H = k^F$, FEC-F is a straight line. When $k^H > k^F$, FEC-F is convex to the origin, while when $k^H < k^F$, FEC-F is concave to the origin.*

Proof. The second derivatives of FEC-F in (9) are given by:

$$\left.\frac{\partial^2 y}{\partial z^2}\right|_{\text{FEC-F}} = \underbrace{\frac{f_x A^{-k^F} \Delta^F}{f} z^{\frac{k^F}{k^H}-2} \left(\Delta^F - f_x A^{-k^F} z^{\frac{k^F}{k^H}}\right)^{\frac{k^H}{k^F}-2}}_{(+)} \left(\frac{k^H - k^F}{k^H}\right).$$

This establishes Lemma 1. ■

We now mention two points that are important to characterize the existence and order of the productivity cutoff levels in the graphical analysis. First, we identify the coordinates on the z- and y-intercepts of FEC-H and FEC-F in Figures 1(a) and 1(b). Let z_H and z_F denote the z-coordinates on the z-intercept of FEC-H and FEC-F, and y_H and y_F the y-coordinates on the y-intercept of those as in Figures 1(a) and 1(b). After some algebra, we get:

$$z_H = \frac{\Delta^H}{f}, \quad z_F = A^{k^H} \left(\frac{\Delta^F}{f_x}\right)^{k^H/k^F}, \quad y_H = \frac{\Delta^H A^{k^H}}{f_x}, \quad y_F = \left(\frac{\Delta^F}{f}\right)^{k^H/k^F}. \tag{11}$$

Second, Figure 1(a) (Figure 1(b)) shows the point H^{45} (F^{45}), which is the intersection between FEC-H (FEC-F) and the 45-degree line. Hereafter, as in Figures 1(a) and 1(b), the intersection between FEC-H and a line is shown by a *round* point, while that between FEC-F and a line is shown by a *square* point. The z-coordinates of the points H^{45} and F^{45} are:

$$H^{45}: \quad z_{H^{45}} = \frac{\Delta^H}{f + f_x A^{-k^H}}, \quad F^{45}: \quad z_{F^{45}} = \left(\frac{\Delta^F}{f + f_x A^{-k^F}}\right)^{k^H/k^F}. \tag{12}$$

We must observe that Figure 1(b) omits the points F^{45} and the corresponding z-coordinates $z_{F^{45}}$ under $k^H > k^F$ and $k^H < k^F$, in order to avoid confusion due to too many points.

3.2 Aggregate expenditures, revenue, and the trade balance condition

In this subsection, we consider the trade balance conditions that guarantee two-way trade. First, we show the country-symmetry of the aggregate expenditures. Following the description in (1), we suppose that the superscript N shows the homogeneous good sector. Assuming that the country size L is fixed and symmetric between two countries, from (1) we can show that the aggregate expenditures on the homogenous good (E^N) and differentiated good (E) are symmetric between two countries: $E^N = N^i = (1-\beta)L$ and $E = C^i P^i = \beta L$ $(i = H, F)$.

Second, we consider the aggregate revenue earned by domestic firms in the differentiated good sector, defined by $R^i = \gamma^i L$, where γ^i is the fraction of labor employed in the differentiated good sector in a country i. Unlike closed economy, it does not always hold that $R^i = E$ in each country i under open economy. On the contrary, since the world expenditure on the differentiated goods equals the world revenues, it holds that $R^H + R^F = 2E$, which can be rewritten as:

$$\gamma^H + \gamma^F = 2\beta. \tag{13}$$

Since both countries have labor employed in the differentiated good sector under the two-way trade, the inequality $0 < \gamma^i < \min\{1, 2\beta\}$ must be satisfied from (13) and $0 < \gamma^i < 1$ $(i = H,\ F)$.

Next, we show the trade balance condition:[6]

$$M_x^H r_x^H(\tilde{\varphi}_x^H(\varphi_d^{H*})) + (1-\gamma^H)L - (1-\beta)L = M_x^F r_x^F(\tilde{\varphi}_x^F(\varphi_d^{F*})), \tag{14a}$$

where M_x^i is the mass of exporting firms in a country i. Denoting $b^i \equiv \frac{D^i r_d^i(\tilde{\varphi}_d^i(\varphi_d^{i*}))}{X^i r_x^i(\tilde{\varphi}_x^i(\varphi_x^{i*}))}$, we can show that the fraction γ^H is:[7]

$$\gamma^H = \left(1 + \frac{b^F - b^H}{b^F b^H - 1}\right)\beta, \tag{14b}$$

and (13) pins down the fraction γ^F.

To characterize the labor markets, from (14b) we depict three conditions by the $b^F = 1$, $b^H = 1$ and $b^F = b^H$ lines in (z, y) graph as in Figures 2(a) and 2(b). What is important is that only when $b^H > 1$ and $b^F > 1$, does two-way trade arise (See Appendix B). Then, each line is given by:

$$b^H = 1:\ y = \frac{A^{k^H} f}{f_x} z, \quad b^F = 1:\ y = A^{-k^H}\left(\frac{f_x}{f}\right)^{k^H/k^F} z. \tag{15}$$

We can summarize the results as follows:

[6]The equation (14a) is the same as (A.14) in Demidova (2008) (notation adapted).

[7]See Appendix A.

Lemma 2 *The equilibrium with two-way trade has $b^H > 1$ and $b^F > 1$, which corresponds to the following inequalities:*

$$A^{-k^H}\left(\frac{f_x}{f}\right)^{k^H/k^F} z < y < \frac{A^{k^H} f}{f_x} z, \tag{16}$$

Proof. From Appendix B, we can show that $b^H = \tau^{\sigma-1}A^{k^H+1-\sigma}\left(\frac{\varphi_d^{F*}}{\varphi_d^{H*}}\right)^{k^H}$ and $b^F = \tau^{\sigma-1}A^{k^F+1-\sigma}\left(\frac{\varphi_d^{H*}}{\varphi_d^{F*}}\right)^{k^F}$. It can easily be seen that $b^H > 1$ and $b^F > 1$ correspond to (16). ■

Figure 2(a) denotes the intersection between FEC-H and $b^H = 1$ line by the round point H^{bH} and between FEC-H and $b^F = 1$ by H^{bF}. Moreover, in Figure 2(b), the square point F^{bH} (F^{bF}) is the intersection between FEC-F and $b^H = 1$ ($b^F = 1$).[8] The z-intercepts at the four points (H^{bH}, H^{bF}, F^{bH}, and F^{bF}) in Figures 2(a) and 2(b) are given by:

$$z_{H^{bH}} = \frac{\Delta^H}{2f}, \quad z_{F^{bH}} = \left(\frac{\Delta^F}{f}\right)^{k^H/k^F}\Upsilon, \quad z_{H^{bF}} = \frac{\Delta^H A^{k^H}}{f_x\Lambda}, \quad z_{F^{bF}} = A^{k^H}\left(\frac{\Delta^F}{2f_x}\right)^{k^H/k^F}. \tag{17}$$

where $\Upsilon \equiv \left(\frac{A^{k^F}f}{f_x}(f_x/f)^{1-k^F/k^H} + \frac{f_x}{A^{k^F}f}\right)^{-k^H/k^F}$ and $\Lambda \equiv \frac{fA^{k^H}}{f_x} + \frac{f_x}{A^{k^H}f}(f_x/f)^{k^H/k^F-1}$.

Turning back to (14b), we can see that $\gamma^H > \beta > \gamma^F$ under $b^F > b^H$ and $\gamma^F > \beta > \gamma^H$ under $b^F < b^H$. Therefore, we depict the condition in which $b^F \lesseqgtr b^F$ by the $b^F = b^H$ line as in Figures 2(a) and 2(b), which can be written as:[9]

$$b^H = b^F\text{line}: \quad y = A^{\frac{(k^H-k^F)k^H}{k^H+k^F}} z, \tag{18}$$

If $b^F > b^H$, corresponding to $\gamma^H > \beta > \gamma^F$, then it holds that $y > A^{\frac{(k^H-k^F)k^H}{k^H+k^F}} z$. On the contrary, when $b^F < b^H$, we can show that $\gamma^F > \beta > \gamma^H$.

Comparing the slopes of the four lines ($b^H = b^F$, $b^F = 1$, $b^H = 1$, and the 45 degree lines), we obtain the following results.

Lemma 3 *We can show that the $b^H = b^F$ line is always in the upper of $b^F = 1$ and is in the lower of $b^H = 1$. Next, when $k^H = k^F$, the $b^H = b^F$ line corresponds to the 45 degree line. If $k^H > k^F$, then the $b^H = b^F$ line lies in the upper of the 45 degree line. On the contrary, if $k^H < k^F$, the $b^H = b^F$ line lies in the lower of the 45 degree line.*

[8]We omit FEC-F under $k^F \neq k^H$ in order not to confuse the readers with too many lines.

[9]Concretely, $b^F - b^H = \frac{f}{f_x}\left(A^{k^H}\left(\frac{\varphi_d^{H*}}{\varphi_d^{F*}}\right)^{k^F} - A^{k^F}\left(\frac{\varphi_d^{F*}}{\varphi_d^{H*}}\right)^{k^H}\right) = \frac{A^{k^H}fz}{f_x y}\left\{A^{k^F-k^H}\left(\frac{z}{y}\right)^{-\frac{k^F}{k^H}-1} - 1\right\}$. Based on the curly bracket in this equation, we obtain (18).

Proof. Comparing (15) with (18), we can show that:[10] $\frac{A^{k^H}f}{f_x} > A^{\frac{(k^H-k^F)k^H}{k^H+k^F}} > A^{-k^H}\left(\frac{f_x}{f}\right)^{\frac{k^H}{k^F}}$, which means that the positive slope of $b^H = b^F$ line is greater than that of $b^F = 1$, while it is lower than that of the $b^H = 1$ line. As a result, $b^H = b^F$ line lies in the upper of the $b^F = 1$ line and the lower of the $b^H = 1$ line. Next, when $k^H = k^F$, $b^H = b^F$ line in (18) can be reduced to the 45 degree line $y = z$. When $k^H > k^F$, the positive slope of the $b^H = b^F$ line is greater than the unity, which implies that the $b^H = b^F$ line is in the upper of the 45 degree line. When $k^H < k^F$, the reverse can be applied. That is, the $b^H = b^F$ line is in the lower of the 45 degree line under $k^H < k^F$. ■

From Lemma 2 and 3, we can conclude the following.

Lemma 4 *When the intersection between FEC-F and FEC-H lies in the upper of the* $b^F = b^H$ *line and the lower of the* $b^H = 1$ *line, it holds that* $b^F > b^H > 1$ *so that* $(\min\{2\beta, 1\} >)\gamma^H > \beta > \gamma^F(> 0)$. *In contrast, when it lies in the lower of the* $b^F = b^H$ *line and the upper of the* $b^F = 1$ *line, it holds that* $b^H > b^F > 1$ *so that* $(\min\{2\beta, 1\} >)\gamma^F > \beta > \gamma^H(> 0)$.

Proof. Using Lemma 2 and 3, from Figure 2(a) and 2(b) we obtain this result. ■

Finally, Figures 2(a) and 2(b) show four areas (A1)–(A4) in (z, y) graph that are separated by the $b^H = 1$, $b^H = b^F$, and $b^F = 1$ lines. In areas (A2) and (A3), two-way trade arises. Specifically, as in Lemma 4, we see that $(\min\{2\beta, 1\} >)\gamma^H > \beta > \gamma^F(> 0)$ in area (A2), and that $(\min\{2\beta, 1\} >)\gamma^F > \beta > \gamma^H(> 0)$ in area (A3). On the contrary, in areas (A1) and (A4), two-way trade cannot be seen.[11]

[10] More concretely, making use of (5), we can show that:

$$\frac{A^{k^H}f}{f_x} > A^{\frac{(k^H-k^F)k^H}{k^H+k^F}}, \Rightarrow 1 > \tau^{-\frac{2k^Hk^F}{k^H+k^F}}\left(\frac{f_x}{f}\right)^{\frac{-k^H(k^F+1-\sigma)-k^F(k^H+1-\sigma)}{(\sigma-1)(k^H+k^F)}},$$

$$A^{\frac{(k^H-k^F)k^H}{k^H+k^F}} > A^{-k^H}\left(\frac{f_x}{f}\right)^{\frac{k^H}{k^F}}, \Rightarrow \tau^{\frac{2k^H}{k^H+k^F}}\left(\frac{f_x}{f}\right)^{\frac{k^H(k^F+1-\sigma)+k^F(k^H+1-\sigma)}{k^F(\sigma-1)(k^H+k^F)}} > 1,$$

where we note that $\tau > 1$, $f_x > f$ and $k^i > \sigma - 1(> 0)$.

[11] We mention the trade specialization later.

4 Productivity cutoff levels

The purpose of this section is to determine graphically the productivity cutoff levels that satisfy FECs under two-way trade.[12] First, we can show the following:

Lemma 5 *In equilibrium with two-way trade, it always holds that* $\varphi_d^{i*} < \varphi_x^{i*}$ *$(i = H, F)$.*

Proof. Making use of Lemma 2, we can obtain the result. This is because $\varphi_d^{H*} \geq \varphi_x^{H*}$ and $\varphi_d^{F*} \geq \varphi_x^{F*}$ lead to $y \geq A^{k^H} z$ and $y \leq A^{-k^H} z$ respectively.[13] Then, the areas that satisfy $y \geq A^{k^H} z$ and $y \leq A^{-k^H} z$ are outside of the areas given (16). ■

Lemma (5) concludes that as long as two-way trade arises, there are no pure exporters that only export but do not produce for the domestic market. This finding is different from Demidova (2008) that shows that pure exporters may exist. In addition, using the Pareto productivity distribution function, we do not need to provide the extra assumption that Demidova (2008) provides in Assumption 1: *To ensure that in both countries only firms producing in the domestic market can export, assuming that $f_x/f > X$, where X depends on the difference in productivity distributions.* As mentioned in the Introduction, we cannot see what X is in a concrete expression.

4.1 Asymmetric lower bound of the support of the productivity distribution

We now assume $k \equiv k^H = k^F$ but $\hat{\varphi}^H \neq \hat{\varphi}^F$ so that $\Delta^H \neq \Delta^F$. The case of $\Delta^H < \Delta^F$ is shown in Figure 3(a), and $\Delta^H > \Delta^F$ in Figure 3(b). From Lemma 1, FEC-F graph is given by a straight line when $k^H = k^F$. There are then two important points that we should mention. First, this case does not satisfy HRSD of Demidova (2008) (See (6) in our paper); however, we emphasize that her findings can be obtained here. Second, although our focus is on the asymmetry of the productivity distributions, Δ^i includes f_e and δ. In sum, the

[12] In a *symmetric* Melitz model ($k \equiv k^H = k^F$ and $\Delta \equiv \Delta^H = \Delta^F$), from (9), we can analytically obtain $\varphi_d^{i*} = \left(\frac{f}{\Delta} \frac{\frac{fA^k}{f_x} - \frac{f_x}{fA^k}}{\frac{fA^k}{f_x} - 1} \right)^{1/k} (> 0)$, $\varphi_x^{i*} = A^k \varphi_d^{i*}$, $(i = H, F)$ where φ_d^{i*} has a positive sign due to (5). As $A > 1$, we find the symmetric Melitz (2003) result: $\varphi_d^{H*} = \varphi_d^{F*} < \varphi_x^{H*} = \varphi_x^{F*}$.

[13] For instance, taking account of $\varphi_d^{H*} \geq \varphi_x^{H*}$, we make use of (3), leading to $\varphi_d^{H*} \geq A\varphi_d^{F*}$. Therefore, we can show that $(\varphi_d^{H*})^{-k^H} \leq A^{-k^H} (\varphi_d^{F*})^{-k^H}$ (i.e., $z \leq A^{-k^H} y$). As a result, we can lead to $A^{k^H} z \leq y$.

asymmetry of Δ^i in this subsection can be applied to the asymmetry of death probability and entry cost.

We suppose that $\Delta^H < \Delta^F$ and $k^H = k^F$. From (11) and (12), we can show that

$$z_F > z_H, \quad z_{F^{45}} > z_{H^{45}}, \tag{19}$$

where we notice that $y_F > y_H$ or $y_F < y_H$. Based on (19), when the round point H^{bH} is the upper right of the square point of F^{bH} (that is, $z_{H^{bH}} > z_{F^{bH}}$), there exists equilibrium with two-way trade (see Figure 3(a)). Using (17), we can derive the condition that $z_{H^{bH}} > z_{F^{bH}}$:

$$\frac{\Delta^H}{\Delta^F} > 2\left(\frac{A^k f}{f_x} + \frac{f_x}{A^k f}\right)^{-1}. \tag{20}$$

Our findings can then be summarized in the following proposition:

Proposition 1 *Suppose that the assumption* (20) *is satisfied. There then exists equilibrium with two-way trade, where* $\varphi_x^{F*} > \varphi_x^{H*} > \varphi_d^{H*} > \varphi_d^{F*}$.

Proof. Making use of Figure 3(a), we find that the intersection S must lie above the $y = z$ line and hence, it holds that $\varphi_d^{H*} > \varphi_d^{F*}$. From (3) and Lemma 2, we can confirm that $\varphi_x^{F*} > \varphi_x^{H*} > \varphi_d^{H*} > \varphi_d^{F*}$. ■

We provide the intuition behind Proposition 1. Recall that we are assuming $\hat{\varphi}^H > \hat{\varphi}^F$ so that $\Delta^H < \Delta^F$. Since the lower bound of the support of the productivity distribution is higher in Home, operating home firms tend to have large productivities. Therefore, it is more difficult for new potential-operating firms to enter the home market and hence, home firms are less likely to be forced to exit. This corresponds to the order $\varphi_d^{H*} > \varphi_d^{F*}$ so that $\varphi_x^{F*} > \varphi_x^{H*}$ from (3). From these inequalities, it can be found that $\varphi_x^{F*} > \varphi_d^{H*} > \varphi_d^{F*}$, implying that foreign firms have difficulty with entry in the home market. Considering that two-way trade arises at the equilibrium, we must keep Lemma 5, leading to $\varphi_x^{H*} > \varphi_d^{H*}$. As a result, we lead to Proposition 1. Moreover, from (20), we find that when Δ^H is near to Δ^F such that the economy is symmetric, we can show the results in Proposition 1. The reason is that to satisfy (20), $\frac{\Delta^H}{\Delta^F}$ must be large but we must maintain the assumption that $\Delta^H < \Delta^F$.

Having analyzed the case of $\Delta^H < \Delta^F$, we now briefly study the opposite case of $\Delta^H > \Delta^F$ displayed in Figure 3(b). In this case, we can easily see that

$$y^H > y^F, \quad z_{H^{45}} > z_{F^{45}}. \tag{21}$$

In that case, as seen in Figure 3(b), it must hold that $z_{F^{bF}} > z_{H^{bF}}$ under the two-way trade, which corresponds to the following inequality:

$$\frac{1}{2}\left(\frac{A^k f}{f_x} + \frac{f_x}{A^k f}\right) > \frac{\Delta^H}{\Delta^F}. \tag{22}$$

The intersection S then uniquely exists in the lower right of the 45-degree line and in the upper left of the $b^F = 1$ line as in Figure 3(b). We summarize these results as follows:

Proposition 2 *Suppose that the assumption* (22) *is satisfied. There then exists the equilibrium with two-way trade, where the productivity cutoff levels are ordered by:* $\varphi_x^{H*} > \varphi_x^{F*} > \varphi_d^{F*} > \varphi_d^{H*}$.

Proof. See Proposition 1. ■

4.2 Asymmetric shape parameters

We now assume that the shape parameters are asymmetric: $k^H \neq k^F$ where we assume that $\hat{\varphi} \equiv \hat{\varphi}^H = \hat{\varphi}^F$. It can be observed that Δ^i includes the shape parameter k^i. Therefore, $k^H \neq k^F$ leads to $\Delta^H \neq \Delta^F$.[14] The asymmetry of the shape parameters k^i makes analytical inquiry difficult due to the highly nonlinear system of FECs. Therefore, we narrow down the possible cases under the asymmetric shape parameters. Concretely, we can rewrite FEC-H in (9) as $z = \frac{\Delta^H - f_x A^{-k^H} y}{f}$. Substituting it into FEC-F in (9) yields:

$$\Psi(y) \equiv f y^{\frac{k^F}{k^H}} + f_x A^{-k^F}\left(\frac{\Delta^H - f_x A^{-k^H} y}{f}\right)^{\frac{k^F}{k^H}} - \Delta^F = 0. \tag{23}$$

We can then show $\Psi'(y) = 0$ as follows:

$$\Psi'(y) = 0, \quad \Rightarrow \quad y = \left(\frac{A^{k^F} f}{f_x}\right)^{\frac{k^H}{k^H - k^F}} \left(\frac{A^{k^H} f}{f_x}\right)^{\frac{k^H}{k^H - k^F}} z. \tag{24}$$

We call (24) as $\Psi'(y) = 0$ line. We then find the following:

Lemma 6 *Assume that* $k^F > k^H$. *Then, the* $\Psi'(y) = 0$ *line is in the lower of the* $b^F = 1$ *line. Next, assume that* $k^F < k^H$. *Then, the* $\Psi'(y) = 0$ *line is in the upper of the* $b^H = 1$ *line.*

[14] We must observe that $k^i > k^j$ may lead to $\Delta^i > \Delta^j$ or $\Delta^i < \Delta^j$, which depends on the value of $\hat{\varphi}$. For instance, when $1 > \hat{\varphi} > 0$, we can always see that $\frac{\partial \Delta^i}{\partial k^i} > 0$. However, if $\hat{\varphi} > 1$, it may be held that $\frac{\partial \Delta^i}{\partial k^i} < 0$.

Proof. Assume that $k^F > k^H$. It then holds that $\big(\frac{f_x}{A^{k^F}f}\big)^{\frac{k^H}{k^F}} > \big(\frac{A^{k^F}f}{f_x}\big)^{\frac{k^H}{k^H-k^F}}\big(\frac{A^{k^H}f}{f_x}\big)^{\frac{k^H}{k^H-k^F}}$, meaning that the slope of the $b^F = 1$ line in (15) is large relative to that of the $\Psi'(y) = 0$ line in (24). That is, the $\Psi'(y) = 0$ line is in the lower of the $b^F = 1$ line. Next, assuming that $k^F < k^H$, we find that $\big(\frac{A^{k^F}f}{f_x}\big)^{\frac{k^H}{k^H-k^F}}\big(\frac{A^{k^H}f}{f_x}\big)^{\frac{k^H}{k^H-k^F}} > \frac{A^{k^H}f}{f_x}$, meaning that the slope of the $\Psi'(y) = 0$ line in (24) is larger than that of the $b^H = 1$ line. Hence, the $\Psi'(y) = 0$ line is in the upper of the $b^H = 1$ line. See Figures 4(a) and 4(b). ■

From Lemma 6, we lead to the following three findings. First, there are no multiple equilibria with two-way trade. Figures 4(a) and 4(b) show the cases in which there are two intersections S_1 and S_2. Figure 4(a) assumes that $k^H < k^F$, $y_H > y_F$, and $z_H > z_F$, while Figure 4(b) assumes that $k^H > k^F$, $y_H < y_F$, and $z_H < z_F$. In both cases, we can see that the square point F^{bF} lies in the upper right of the round point H^{bF} and that the round point H^{bH} is in the upper right of the square point F^{bH}. Then, Lemma 6 implies that the intersection S_2 must be in the lower of the $b^F = 1$ line in Figure 4(a) and that it must be in the upper of the $b^H = 1$ line in Figure 4(b). Therefore, the intersections S_2 in both figures lie in the outside of the area where $b^H > 1$ and $b^F > 1$, so that two-way trade cannot be seen at the intersection S_2. Therefore, the number of the equilibrium with two-way trade is at most one.

Second, when the intersection between FEC-H and FEC-F arises in the area where $b^H > 1$ and $b^F > 1$, which excludes the intersections S_2 in Figures 4(a) and 4(b), we find that the negative slope of the FEC-H line is always greater than that of the FEC-F graph from Lemma 6 (See S_1 in Figures 4(a) and 4(b)). That is, $\frac{\partial y}{\partial z}\big|_{\text{FEC-H}} < \frac{\partial y}{\partial z}\big|_{\text{FEC-F}} (< 0)$ in (10). This finding is plausible because at the intersection between FEC-F and the $\Psi'(y) = 0$ line (see the square point K in Figures 4(a) and 4(b)), the slope of the tangial line of FEC-F is the same as that of FEC-H.[15] This is important in analyzing the welfare impacts of trade liberalization in the next section.

Finally, suppose that $y^F > y^H$ and $z^H > z^F$ as in Figures 5(a) and 5(b). In that case, we can always see that the intersection is in the outside of the area in which $b^H > 1$ and $b^F > 1$

[15]This finding can be directly confirmed using (10). Assume that $k^H > k^F$. Then, if $\frac{\partial y}{\partial z}\big|_{\text{FEC-H}} > \frac{\partial y}{\partial z}\big|_{\text{FEC-F}}$ in (10), we can see that $\big(\frac{A^{k^H}f}{f_x} <\big)\big(\frac{A^{k^F}f}{f_x}\big)^{\frac{k^H}{k^H-k^F}}\big(\frac{A^{k^H}f}{f_x}\big)^{\frac{k^H}{k^H-k^F}} z < y$. This inequality indicates the area in which $b^H < 1$ and $b^F > 1$. In a similar manner, assuming that $k^F < k^H$, $\frac{\partial y}{\partial z}\big|_{\text{FEC-H}} > \frac{\partial y}{\partial z}\big|_{\text{FEC-F}}$ in (10), we show the area in which $b^H > 1$ and $b^F < 1$.

because of Lemma 6.

We can then summarize these findings:

Proposition 3 *Assume that $k^H \neq k^F$. First, there are no multiple equilibria with two-way trade. Second, at the equilibrium with two-way trade, it always holds that $\frac{dy}{dz}|_{FEC\text{-}H} < \frac{dy}{dz}|_{FEC\text{-}F} (< 0)$. Finally, there is no equilibrium with two-way trade under $y^F > y^H$ and $z^H > z^F$.*

Based on Proposition 3, the cases that we should consider are as follows: (i) $y^H > y^F$ and $z^H < z^F$; (ii) $y^H > y^F$ and $z^H > z^F$; (iii) $y^H < y^F$ and $z^H < z^F$.

Case (i): $y_H > y_F$ and $z_H < z_F$: Assuming that $y_H > y_F$ and $z_H < z_F$, from (11) we can find the following inequality:

$$\frac{f_x}{A^{k^H} f} f^{\frac{k^F - k^H}{k^F}} < \frac{\Delta^H}{(\Delta^F)^{k^H/k^F}} < \frac{A^{k^H} f}{f_x} f_x^{\frac{k^F - k^H}{k^F}}. \tag{25}$$

Let us focus on the shape parameters. When the value of $\frac{A^{k^H} f}{f_x}$ is sufficiently large, the inequality (25) tends to be met. Since $A > 1$, a larger k^H may lead to the inequality (25). Because the lower bounds of the productivity distribution are the same in both countries (i.e., $\hat{\varphi}^H = \hat{\varphi}^F$), a larger shape parameter in the home country means that home firms do not have the opportunity for large productivities due to less productivity dispersion. On the contrary, examining $f^{(k^F - k^H)/k^H}$ and $f_x^{(k^F - k^H)/k^H}$ in (25), the inequality tends to be satisfied under $k^F > k^H$ because the trade is costly under $f_x > f$. It can be concluded that the shape parameter k^H has a large value, and furthermore the parameter k^F has a larger value. As a result, there exists an asymmetry of productivity dispersion; however, in this case, less dispersion leads to equilibrium with two-way trade. It should be noticed again that a larger shape parameter corresponds to less dispersion of productivity distribution.

The fact that both FEC-H and FEC-F have a negative slope means that the number of the intersection between FEC-F and FEC-H is always one, which is the same as the equilibrium in Figures 3(a) and 3(b), except that FEC-F is given by a straight line in Section 4.1 and is a curve graph in this case. Taking account of (12) and (17), we find the following:

Proposition 4 *Assume that $y^H > y^F$ and $z^H < z^F$. (i) The equilibrium with two-way trade shows that $\varphi_x^{F*} > \varphi_x^{H*} > \varphi_d^{H*} > \varphi_d^{F*}$ if $z_{F^{45}} > z_{H^{45}}$ and $z_{H^{bH}} > z_{F^{bH}}$, that is,*

$\frac{f+f_x^{-k^H}}{\left(f+f_x A^{-k^F}\right)^{k^H/k^F}} > \frac{\Delta^H}{(\Delta^F)^{k^H/k^F}} > 2f^{\frac{k^F-k^H}{k^F}}\Upsilon$. *(ii) The equilibrium with two-way trade shows that* $\varphi_x^{H*} > \varphi_x^{F*} > \varphi_d^{F*} > \varphi_d^{H*}$ *if* $z_{F^{45}} < z_{H^{45}}$ *and* $z_{H^{bF}} < z_{F^{bF}}$, *that is,* $\frac{f+f_x^{-k^H}}{\left(f+f_x A^{-k^F}\right)^{k^H/k^F}} < \frac{\Delta^H}{(\Delta^F)^{k^H/k^F}} < 2^{-k^H/k^F} f_x^{\frac{k^F-k^H}{k^F}}\Lambda$.

Proof. The proof is fundamentally the same as that in Proposition 1. From (12) and (17), we can obtain the results. ■

Case (ii): $y^H > y^F$ and $z^H > z^F$: When $y^H > y^F$ and $z^H > z^F$, (11) leads to the following:

$$\frac{\Delta^H}{(\Delta^F)^{k^H/k^F}} > \frac{A^{k^H}f}{f_x} f_x^{\frac{k^F-k^H}{k^F}} \left(> \frac{f_x}{A^{k^H}f} f^{\frac{k^F-k^H}{k^F}} \right). \tag{26}$$

At first, when $k^H > k^F$ in case (ii), there is no intersection because FEC-F is convex to the origin. Therefore, assume that $k^H < k^F$ as in Figure 4(a). Unlike (25), when $\frac{A^{k^H}f}{f_x}$ has a small value, the inequality (26) tends to be met. That is, noting that $A > 1$, a small value of k^H may lead to (26), implying that home country has greater productivity dispersion. The symmetry of the productivity lower bound $\hat{\varphi} \equiv \hat{\varphi}^H = \hat{\varphi}^F$implies that home firms have a better chance of obtaining greater productivity. In case (ii), if the intersection between FEC-H and FEC-F exists, the number of the intersection is always two. However, as confirmed in Proposition 3, two-way trade does not arise at the intersection S_2 of Figure 4(a). On the contrary, at the intersection S_1, it is possible that two-way trade arises. More concretely, if $z_{F^{45}} > z_{H^{45}}$ and $z_{H^{bH}} > z_{F^{bH}}$ as in Proposition 4(i), there exists the equilibrium with two-way trade, which has $\varphi_x^{F*} > \varphi_x^{H*} > \varphi_d^{H*} > \varphi_d^{F*}$. Supposing that $z_{F^{45}} < z_{H^{45}}$ and $z_{H^{bF}} < z_{F^{bF}}$ as in Proposition 4(ii), we find the two-way trade equilibrium with $\varphi_x^{H*} > \varphi_x^{F*} > \varphi_d^{F*} > \varphi_d^{H*}$. These conditions are the same as

Case (iii): $y^H < y^F$ and $z^H < z^F$: The inequalities of $y^H < y^F$ and $z^H < z^F$ can be rewritten as:

$$\frac{\Delta^H}{(\Delta^F)^{k^H/k^F}} < \frac{f_x}{A^{k^H}f} f^{\frac{k^F-k^H}{k^F}} \left(< \frac{A^{k^H}f}{f_x} f_x^{\frac{k^F-k^H}{k^F}} \right). \tag{27}$$

Since the condition that $k^H < k^F$ leads to no intersection between FEC-F and FEC-H in (z, y) graph, we suppose that $k^H > k^F$ as in Figure 4(b). This is the opposite to case (ii) under $k^F > k^H$. In sum, it is likely to satisfy the inequality (27) when k^H has a small value, and furthermore, $k^H > k^F$ means that k^F takes a smaller value. Then, at the intersection S_1 in Figure 4(b), two-way trade may occur. Concretely, the inequalities $z_{H^bH} > z_{F^bH}$ and $z_{F^{45}} > z_{H^{45}}$ lead to the two-way trade equilibrium that $\varphi_x^{F*} > \varphi_x^{H*} > \varphi_d^{H*} > \varphi_d^{F*}$. When $z_{F^{45}} < z_{H^{45}}$ and $z_{H^bF} < z_{F^bF}$, we find equilibrium with the two-way trade where $\varphi_x^{H*} > \varphi_x^{F*} > \varphi_d^{F*} > \varphi_d^{H*}$.

5 Welfare and trade liberalization

In the last section, we determined the productivity cutoff levels that satisfy FECs under two-way trade φ_d^{i*} and φ_x^{i*} $(i = H,\ F)$. We now confirm the determination of the remaining variables. First, we can determine the weighted average productivities, $\tilde{\varphi}_d^i(\varphi_d^{i*})$ and $\tilde{\varphi}_x^i(\varphi_x^{i*})$, the probability of successful entry D^i and X^i, and the following price index:

$$\begin{aligned} P^i &= \left(\int_{\varphi^{i*}}^{\infty} \int_0^{M^i} p^i(z,\varphi)^{1-\sigma} dz \frac{g^i(\varphi)}{1 - G^i(\varphi^{i*})} d\varphi + \int_{\varphi_x^{j*}}^{\infty} \int_0^{M_x^j} p_x^i(z,\varphi)^{1-\sigma} dz \frac{g^j(\varphi)}{1 - G^j(\varphi_x^{j*})} d\varphi \right)^{\frac{1}{1-\sigma}} \\ &= \left(\frac{\sigma}{\sigma - 1} \right) \left(\frac{\beta L}{\sigma f} \right)^{\frac{1}{1-\sigma}} \frac{1}{\varphi^{i*}}, \end{aligned} \tag{28}$$

Next, from (13) and (14b), we can determine the fraction of γ^H and γ^F, and their values satisfy $\gamma^i \in (0, \min\{2\beta, 1\})$. Since two-way trade arises, we finally determine the mass of domestic and export firms as follows: $M_d^i = \frac{\gamma^H L}{\bar{r}^i}$ and $M_x^i = \frac{X^i}{D^i} M_d^i$, where $\bar{r}^i = r_d^i(\tilde{\varphi}_d^i(\varphi_d^{i*})) + \frac{X^i}{D^i} r_x^i(\tilde{\varphi}_x^i(\varphi_x^{i*}))$.

5.1 Bilateral trade liberalization

In this subsection, we examine the welfare impacts of trade liberalization. The welfare given in (1) can be rewritten as:[16]

$$W^i = (1-\beta)^{1-\beta} \beta^\beta \left(\frac{\beta L}{\sigma f} \right)^{\frac{\beta}{\sigma - 1}} (\rho \varphi_d^{i*})^\beta, \quad i = H, F. \tag{29}$$

Since the welfare level of the representative agent in a country i increases with φ_d^{i*}, it suffices to analyze the effects of trade liberalization (i.e., a decrease in τ) on the productivity cutoff

[16] We briefly show how to derive (29). First, maximizing the utility function (1) subject to a budget constraint $P^iC^i + N^i = L$ (recall $w = 1$), we have $P^iC^i = \beta L$ and $N^i = (1-\beta)L$. Substituting these solutions back into (1), we obtain $U^i = \beta^\beta(1-\beta)^{1-\beta}(P^i)^{-\beta}$. Inserting (28) into U^i derived above, we obtain (29).

level in the domestic market φ_d^{i*}. For this purpose, totally differentiating FEC-H in (9), we can show that

$$z = z(y, A), \text{where } \frac{\partial z}{\partial y} = -\frac{f_x}{f A^{k^H}}(< 0),\ \frac{\partial z}{\partial A} = \frac{k^H f_x y}{f A^{k^H+1}}(> 0). \tag{30a}$$

In what follows, substituting (30a) into FEC-F in (9) and totally differentiating it leads to the following:[17]

$$y = y(A),\ \text{where } \frac{\partial y}{\partial A} = B \times \frac{\frac{y}{z} - \frac{A^{k^H} f}{f_x}}{\frac{dy}{dz}\Big|_{\text{FEC-H}} - \frac{dy}{dz}\Big|_{\text{FEC-F}}}(> 0), \tag{30b}$$

and $B \equiv \frac{k^H f_x A^{-k^F-1} z}{f}\left(\frac{z}{y}\right)^{\frac{k^F}{k^H}-1} > 0$. We must observe that $y < \frac{A^{k^H} f}{f_x} z$ from (15). Furthermore, at the equilibrium with two-way trade, it holds that $\frac{dy}{dz}\big|_{\text{FEC-H}} < \frac{dy}{dz}\big|_{\text{FEC-F}} (< 0)$ in the denominator (See intersections S in Figures 3(a) and 3(b), intersection S_1 in Figure 4(a), and intersection S_1 in Figure 5(b)). As a result, $\frac{\partial y}{\partial A}$ has a positive sign.

Finally, when substituting (30b) into (30a) again, we obtain the following:

$$z = z(A),\quad \text{where } \frac{\partial z}{\partial A} = \Theta \times \frac{\frac{f_x}{f A^{k^F}} - \left(\frac{y}{z}\right)^{\frac{k^F}{k^H}}}{\frac{dy}{dz}\Big|_{\text{FEC-H}} - \frac{dy}{dz}\Big|_{\text{FEC-F}}}(> 0), \tag{30c}$$

and $\Theta \equiv \frac{k^H f_x A^{-1} z}{f}\left(\frac{z}{y}\right)^{\frac{k^F}{k^H}-1} > 0$. It should be observed that $\frac{f_x}{f A^{k^F}} < \left(\frac{y}{z}\right)^{\frac{k^F}{k^H}}$ from (15) and that $\frac{dy}{dz}\big|_{\text{FEC-H}} < \frac{dy}{dz}\big|_{\text{FEC-F}} (< 0)$. Therefore, it holds that $\frac{\partial z}{\partial A}$ has a positive sign.

We can then show the welfare effects of trade liberalization:

Proposition 5 *In the equilibrium with two-way trade, the effects of bilateral trade liberalization on welfare are always positive.*

Proof. The welfare effects can be calculated as follows:

$$\frac{\partial W^H}{\partial \tau} = \underbrace{\frac{\partial W^H}{\partial \varphi^{H*}}}_{(+)} \underbrace{\frac{\partial \varphi^{H*}}{\partial z}}_{(-)} \underbrace{\frac{\partial z}{\partial A}}_{(+)} \underbrace{\frac{\partial A}{\partial \tau}}_{(+)} < 0,\quad \frac{\partial W^F}{\partial \tau} = \underbrace{\frac{\partial W^F}{\partial \varphi^{F*}}}_{(+)} \underbrace{\frac{\partial \varphi^{F*}}{\partial y}}_{(-)} \underbrace{\frac{\partial y}{\partial A}}_{(+)} \underbrace{\frac{\partial A}{\partial \tau}}_{(+)} < 0, \tag{31}$$

[17] After we substitute (30a) into FEC-F and differentiate it totally, we can show that:

$$\left(\frac{k^F}{k^H} f_x A^{-k^F} z^{\frac{k^F}{k^H}-1} \frac{\partial z}{\partial A} - k^F f_x A^{-k^F-1} z^{\frac{k^F}{k^H}}\right) dA = -\left(\frac{f k^F}{k^H} y^{\frac{k^F}{k^H}-1} + \frac{k^F f_x A^{-k^F}}{k^H} z^{\frac{k^F}{k^H}-1} \frac{\partial z}{\partial y}\right) dy$$

When this equation is arranged, we can derive for (30b).

Therefore, a decrease in τ increases the welfare. ■

Our finding in Proposition 5 is obtained by Lemma 2 and $\frac{dy}{dz}\big|_{\text{FEC-H}} < \frac{dy}{dz}\big|_{\text{FEC-F}} (< 0)$. When the productivity distribution is given by Pareto type, even if the numeraire sector is additionally introduced and the asymmetry of the productivity distribution functions is assumed, our finding is consistent with that in the one-sector Melitz model as long as two-way trade can be seen. In other words, the bilateral trade liberalization under two-way trade leads to welfare gain through better resource allocation. On the contrary, our finding differs from Proposition 1 in Demidova (2008) that bilateral trade liberalization may lead to welfare loss in a country.

5.2 Unilateral trade liberalization

We now assume that the trade liberalization arises in a country i alone; that is, a unilateral trade liberalization under two-way trade. For simplicity, we consider a change in τ^H given τ^F. Then, (30b) and (30c) can be rewritten as:

$$\frac{\partial z}{\partial A^H} = \frac{-k^H (A^H)^{-1} y}{\frac{dy}{dz}\big|_{\text{FEC-H}} - \frac{dy}{dz}\big|_{\text{FEC-F}}} (> 0), \quad \frac{\partial y}{\partial A^H} = \frac{k^H (A^H)^{-1} y (A^F)^{-k^F} \frac{f_x}{f} \left(\frac{z}{y}\right)^{\frac{k^F}{k^H} - 1}}{\frac{dy}{dz}\big|_{\text{FEC-H}} - \frac{dy}{dz}\big|_{\text{FEC-F}}} (< 0), \quad (32)$$

where we note that $\frac{dy}{dz}\big|_{\text{FEC-H}} < \frac{dy}{dz}\big|_{\text{FEC-F}} (< 0)$. Then, we have the following proposition, the intuitive explanation for which is given in the next section.

Proposition 6 *Under the equilibrium with two-way trade, the unilateral trade liberalization by a decrease in τ^H improves welfare in the home country, but deteriorates welfare in the foreign country (that is, $\frac{\partial W^H}{\partial \tau^H} < 0$ and $\frac{\partial W^F}{\partial \tau^H} > 0$).*

Proof. Making use of (31), we can show the welfare effect of unilateral trade liberalization as follows:

$$\frac{\partial W^H}{\partial \tau^H} = \underbrace{\frac{\partial W^H}{\partial \varphi^{H*}}}_{(+)} \underbrace{\frac{\partial \varphi^{H*}}{\partial z}}_{(-)} \underbrace{\frac{\partial z}{\partial A^H}}_{(+)} \underbrace{\frac{\partial A^H}{\partial \tau^H}}_{(+)} (< 0), \quad \frac{\partial W^F}{\partial \tau^H} = \underbrace{\frac{\partial W^F}{\partial \varphi^{F*}}}_{(+)} \underbrace{\frac{\partial \varphi^{F*}}{\partial y}}_{(-)} \underbrace{\frac{\partial y}{\partial z} \frac{\partial z}{\partial A^H}}_{(-)} \underbrace{\frac{\partial A^H}{\partial \tau^H}}_{(+)} (> 0). \quad (33)$$

Therefore, we obtain the result in this proposition. ■

6 Specialization in the differentiated good sector

So far, we have considered the situation where two-way trade in the equilibrium appears; that is, it holds that $0 < \gamma^i < \min\{1, 2\beta\}$ $(i = H, F)$. In this section, we now consider that the equilibrium trade in a sector is specialized, and then examine the welfare effects of trade liberalization in the differentiated good sector. We now denote the fraction of labor employed in the homogenous good sector by Γ^i. Therefore, we can see that $\gamma^i + \Gamma^i = 1$ $(i = H, F)$. We can then easily obtain the following lemma.

Lemma 7 *Assume that* $\gamma^i = \min\{1, 2\beta\}$. *(i) Assume that* $\beta < 0.5$ *so that* $\gamma^i = 2\beta(< 1)$. *It then holds that* $\gamma^j = 0$, $\Gamma^i = 1 - 2\beta$, *and* $\Gamma^j = 1$. *(ii) Assume that* $1 > \beta > 0.5$ *so that* $\gamma^i = 1$. *It then holds that* $\gamma^j = 2\beta - 1$, $\Gamma^i = 0$ *and* $\Gamma^j = 2(1 - \beta)$.

Lemma 7 shows that whether or not a sector is specialized is determined by the value of β in the consumer preference (1). Concretely, assuming that $0.5 < \beta < 1$ and $\gamma^i = 1(< 2\beta)$ as in Lemma 7(ii), we can see the trade specialization in the homogeneous good sector; however, we cannot investigate the specialization in the homogeneous good sector in our model because of the model assumption that the homogeneous good is freely traded under imperfect specialization. On the contrary, assuming that $\beta < 0.5$ and $\gamma^i = 2\beta(< 1)$ as in Lemma 7(i), the trade specialization arises in the differentiated good sector. In summary, there are no firms that produce differentiated goods in a country j (i.e., $M_d^j = M_x^j = 0$); on the contrary, firms in a country i produce their goods. In that case, the price indexes in each country are $P^j = \left(\int_{\varphi_x^{i*}}^{\infty} \int_0^{M_x^i} p_x(\varphi)^{1-\sigma} \frac{g^i(\varphi)}{1-G^i(\varphi_d^{i*})} d\varphi \right)^{1/(1-\sigma)}$ and $P^i = \left(\int_{\varphi_d^{i*}}^{\infty} \int_0^{M_d^i} p_d(\varphi)^{1-\sigma} \frac{g^i(\varphi)}{1-G^i(\varphi_d^{i*})} d\varphi \right)^{1/(1-\sigma)}$, which can be rewritten as:

$$P^i = Z(M_d^i)^{1/(1-\sigma)}(\varphi_d^{F*})^{-1}, \quad P^j = Z\tau(M_x^i)^{\frac{1}{1-\sigma}}(\varphi_d^{i*})^{k^i/(1-\sigma)}(\varphi_x^{i*})^{(\sigma-1-k^F)/(1-\sigma)}, \tag{34}$$

where $Z \equiv \left(\frac{\sigma}{\sigma-1}\right)\left(\frac{k^i}{k^i+1-\sigma}\right)^{1/(1-\sigma)}$. We then obtain the following proposition:

Proposition 7 *Assume that* $\gamma^i = 2\beta$ *and* $\gamma^j = 0$. *We then obtain* $\varphi_d^{i*} = \varphi_x^{i*} = \left(\frac{f+f_x}{\Delta^i}\right)^{1/k^i}$.

Proof. See Appendix C. ■

Let us consider why Proposition 7 shows $\varphi_d^{i*} = \varphi_x^{i*}$ under the costly trade. For simplicity, suppose that $\gamma^F = 2\beta$ and $\gamma^H = 0$. Unlike two-way trade, we cannot use FEC in (8) in the home country because home firms do not enter the differentiated good sector, and furthermore we cannot use (3) under the specialization of the differentiated good sector. Now, since the trade specialization in this sector arises, the world expenditure on differentiated goods equals the total revenues in *foreign* country; that is, $R^F = E^H + E^F = 2\beta$. Then, observing that the expenditure on differentiated goods in each country is the same ($E^H = E^F$) and fixed, the entire exporting and non-exporting revenues are the same: $D^F r_d^F(\tilde{\varphi}_d^F) = X^F r_x^F(\tilde{\varphi}_x^F)$ as confirmed in Appendix C. Hence, as in Proposition 7, the productivity cutoff levels are the same.

Finally, let us consider the welfare impacts of trade liberalization (a decrease in the iceberg cost) under the specialization. Unlike two-way trade, the iceberg cost does not have any impacts on the productivity cutoff levels, as seen in Proposition 7. The reason is that foreign firms can enter the home market, irrespective of home firms under the specialization and therefore, the costly trade does not give any merits for the entry of domestic firms in the home country. Since $U^i = \beta^\beta(1-\beta)^{1-\beta}(P^i)^{-\beta}$ $(i = H, F)$ (See footnote 16 derived in (29)), from (34) we easily find the following:

Proposition 8 *Suppose that* $\gamma^F = 2\beta$ *and* $\gamma^H = 0$. *In that case, it holds that* $\frac{\partial W^H}{\partial \tau} < 0$ *and* $\frac{\partial W^F}{\partial \tau} = 0$.

We now explain the intuition in Proposition 6 and 8. First, when the trade specialization arises in the differentiated good sector, we cannot analyze the bilateral trade liberalization but the unilateral trade liberalization. We then find that the unilateral trade liberalization in Proposition 8 differs greatly from that in Proposition 6. Proposition 6 shows that the unilateral trade liberalization deteriorates welfare in an importing country. The reason is that less productive-exporting firms enter the market in the importing country, and hence unilateral trade liberalization does not induce reallocations of resources to contribute an aggregate productivity gain in the importing country.

When trade specialization in the differentiated good sector arises, as in Lemma 7, a decrease in the iceberg cost does not lead to the further entry of less-productive-exporting firms because home firms neither enter nor exit so that the demand on the differentiated goods always balances the supply on those produced by foreign firms. No entry of foreign firms with less productivities means that the reallocation of resources, which will be harmful to welfare in the home country, cannot be seen. On the contrary, when the iceberg cost decreases, from (2a) we can see that the price of the differentiated goods produced by foreign firms decreases, implying that consumers in the home country can spend on many differentiated goods and hence, this increases home welfare. Because the trade liberalization does not affect the productivity cut-off levels under the specialization, foreign welfare does not change because the price of the domestic goods does not change.

7 Concluding Remarks

Incorporating the homogeneous good (numeraire) sector into the Melitz model, we re-consider the findings in Demidova (2008), but, unlike her set-up, we assume the Pareto productivity distribution function. Then, even if the asymmetry of the productivity distribution functions is assumed, we conclude that the distortion caused by the numeraire sector is limited. Concretely, we show that there are no multiple equilibria and no pure exporters and that the welfare impacts of trade liberalization are qualitatively the same as those in the Melitz model in one sector.

Acknowledgements

We would like to thank Masakazu Emoto, Manabu Furuta, Yasuhiro Gintani, Hiroyuki Hashimoto, Yunfang Hu, Chihiro Inaba, Isao Kamata, Noritsugu Nakanishi, Hiroyuki Nishiyama, Takahiro Sato, Nobuyuki Takeuchi, Masao Yamaguchi, and participants at the 9th Spring Meeting of the Japan Society of International Economics, KIES 7th meeting, and Kobe Rokko Forum for their helpful comments. We are especially grateful to Yasukazu Ichino and Mizuki Tsuboi for their useful comments. All remaining mistakes are our own. This work was supported by JSPS KAKENHI Grant Number 19K01554.

References

[1] Demidova, S., 2008. Productivity improvements and falling trade costs: boon or bane? Int. Econ. Rev. 49(4), 1437-1462.

[2] Demidova, S., and Rodríguez-Clare, A., 2013. The simple analytics of the Melitz model in a small economy, J. Int. Econ. 90, 266-272.

[3] Helpman, E., Melitz, M.J., Yeaple, S.R., 2004. Export versus FDI with heterogeneous firms. Am. Econ. Rev. 94(1), 300-316.

[4] Melitz, M.J., 2003. The impact of trade on intra-industry reallocations and aggregate industry productivity. Econometrica. 71(6), 1695-1725.

[5] Melitz, M.J., and Ottaviano, G, I.P., 2008, Market size, trade, and productivity, *Review of Economic Studies* 75(1), 295–316.

[6] Melitz, M.J., Redding, S.J., 2014. Heterogeneous firms and trade. In: Gopinath, G., Helpman, E., Rogoff, K. (Eds.), Handbook of Int. Econ., vol 4, 1-54.

[7] Redding, S.J., 2011. Theories of heterogeneous firms and trade. Annual Rev. Econ. 3, 77-105.

[8] Spearot, A., 2016, Unpacking the long-run effects of tariff shocks: New structural implications from firm heterogeneity models, Am. Econ. J.: Microeconomics 8(2), 128–167.

Appendix A

Using (14a), we can derive (14b). Making use of $M_x^i = \frac{X^i}{D^i} M_d^i$ and $M_d^i = \frac{\gamma^i L}{\bar{r}^i}$ where M_d^i and M_x^i are the mass of domestic and export firms in a country i, (14a) can be rewritten as:

$$\frac{X^H \gamma^H}{D^H \bar{r}^H} r_x^H(\tilde{\varphi}_x^H) + (1-\gamma^H) - (1-\beta) = \frac{X^F \gamma^F}{D^F \bar{r}^F} r_x^F(\tilde{\varphi}_x^F). \tag{A.1}$$

In what follows, substituting $\bar{r}^i = r_d^i(\tilde{\varphi}_d^i) + \frac{X^i}{D^i} r_x^i(\tilde{\varphi}_x^i)$ and (13) into (A.1) and making use of b^H and b^F, we can show that (A.1) is $\frac{\gamma^H}{b^H+1} + \beta - \gamma^H = \frac{2\beta - \gamma^H}{b^F+1}$, which leads to (14b).

Appendix B

Taking account of (13) and $0 < \gamma^i < 1$, the fraction γ^i must be satisfied: $0 < \gamma^i < \min\{1, 2\beta\}$. First, suppose that $0.5 > \beta$ so that $0 < \gamma^i < 2\beta$. Substituting (14b) into this one, we can show that

$$-1 < \frac{b^F - b^H}{b^H b^F - 1} < 1. \tag{B.1}$$

We assume that $b^H b^F > 1$. From (B.1), we can show that $-(b^H b^F - 1) < b^F - b^H < b^H b^F - 1$. Therefore, to meet these inequalities, we conclude that $b^F > 1$ and $b^H > 1$. In what follows, assuming that $b^H b^F < 1$, from (B.1) we show that $-(b^H b^F - 1) > b^F - b^H > b^H b^F - 1$. To meet these inequalities, it must be held that $0 < b^H < 1$ and $0 < b^F < 1$.

Next, suppose that $0.5 < \beta$ so that $0 < \gamma^i < 1$. From (14b), the assumption $0.5 < \beta$ means that $\frac{b^F - b^H}{b^F b^H - 1}$ must be below the unity. We can then lead to the same inequality (B.1). Therefore, we can conclude: $b^F > 1$ and $b^H > 1$ or $0 < b^H < 1$ and $0 < b^F < 1$.

Finally, we can exclude the case that $0 < b^H < 1$ and $0 < b^F < 1$, which satisfy $0 < \gamma^i < \min\{1, 2\beta\}$. To be more concrete, we use $b^H = \frac{D^H}{X^H}\frac{r_d^H(\tilde{\varphi}_d^H)}{r_x^H(\tilde{\varphi}_x^H)}$. From (4), we can show that $\frac{D^H}{X^H} = \left(\frac{\varphi_x^{H*}}{\varphi_d^{H*}}\right)^{k^H}$. In what follows, the ratio of revenue is given by $\frac{r_d^H(\tilde{\varphi}_d^H)}{r_x^H(\tilde{\varphi}_x^H)} = \left(\frac{\varphi_d^{F*}}{\varphi_d^{H*}}\right)^{\sigma-1}\left(\frac{\tau\tilde{\varphi}_d^H}{\tilde{\varphi}_x^H}\right)^{\sigma-1}$, where $\tilde{\varphi}_d^H = \left(\frac{k^H}{1+k^H-\sigma}\right)^{1/(\sigma-1)}\varphi_d^{H*}$ and $\tilde{\varphi}_x^H = \left(\frac{k^H}{1+k^H-\sigma}\right)^{1/(\sigma-1)}\varphi_x^{H*}$. We can show that $b^H = \tau^{\sigma-1}A^{k^H+1-\sigma}\left(\frac{\varphi_d^{F*}}{\varphi_d^{H*}}\right)^{k^H}$. Similarly, we can lead to $b^F = \tau^{\sigma-1}A^{k^F+1-\sigma}\left(\frac{\varphi_d^{H*}}{\varphi_d^{F*}}\right)^{k^F}$. Therefore, $b^H = 1$ and $b^F = 1$ lines are shown by (15). When depicting $b^H = 1$ and $b^F = 1$ in Figure 2, we find that there is no area in which $0 < b^H < 1$ and $0 < b^F < 1$, while there is a (z, y)-area that satisfies $b^H > 1$ and $b^F > 1$. Therefore, we can obtain Lemma 2.

Appendix C

Without loss of generality, we suppose that $\gamma^H = 0$ and $\gamma^F = 2\beta$. Substituting $\gamma^H = 0$ into (14b), we show that $(b^H+1)(b^F-1) = 0$ so that $b^F = 1$ because b^H and b^F have positive signs. Noting that $b^F = \frac{D^F r_d^F(\tilde{\varphi}_d^F(\varphi_d^{F*}))}{X^F r_x^F(\tilde{\varphi}_x^F(\varphi_x^{F*}))} = 1$, we find that $\frac{D^F}{X^F}\left(\frac{p_x^F(\tilde{\varphi}_x^F(\varphi_x^{F*}))}{p_d^F(\tilde{\varphi}_d^F(\varphi_d^{F*}))}\right)^{\sigma-1} = \left(\frac{P^H}{P^F}\right)^{\sigma-1}$ where from (34) we can show that $\frac{P^H}{P^F} = \tau\left(\frac{X^F}{D^F}\right)^{1/(1-\sigma)}\left(\frac{\varphi_x^{F*}}{\varphi_d^{F*}}\right)^{\frac{\sigma-1-k^F}{1-\sigma}}$ and $\frac{p_x^F(\tilde{\varphi}_x^F(\varphi_x^{F*}))}{p_d^F(\tilde{\varphi}_d^F(\varphi_d^{F*}))} = \frac{\tau\varphi_d^{F*}}{\varphi_x^{F*}}$. We can show that this equation leads to $(\varphi_d^{F*})^{k^F} = (\varphi_x^{F*})^{k^F}$ and hence, $\varphi_d^{F*} = \varphi_x^{F*}$. Finally, substituting $\varphi_d^{F*} = \varphi_x^{F*}$ into $\left(f(\varphi_d^{F*})^{-k^F} + f_x(\varphi_x^{F*})^{-k^F}\right) = \Delta^F$ in FEC, we can obtain $\varphi_d^{F*} = \varphi_x^{F*} = \left(\frac{f+f_x}{\Delta^F}\right)^{1/k^F}$.

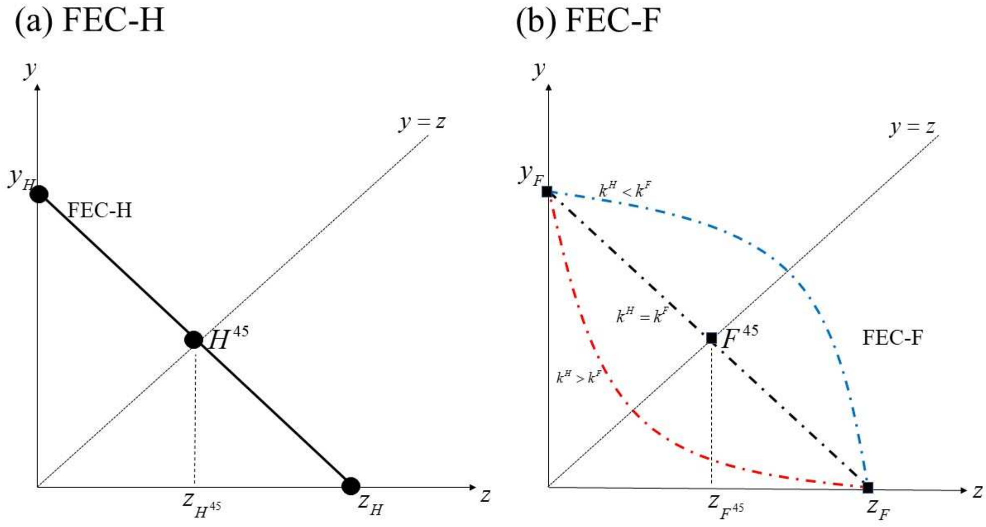

Figure 1: FEC-H and FEC-F: note that we omit FEC-F chain curves that $k^H < k^F$ and $k^H > k^F$.

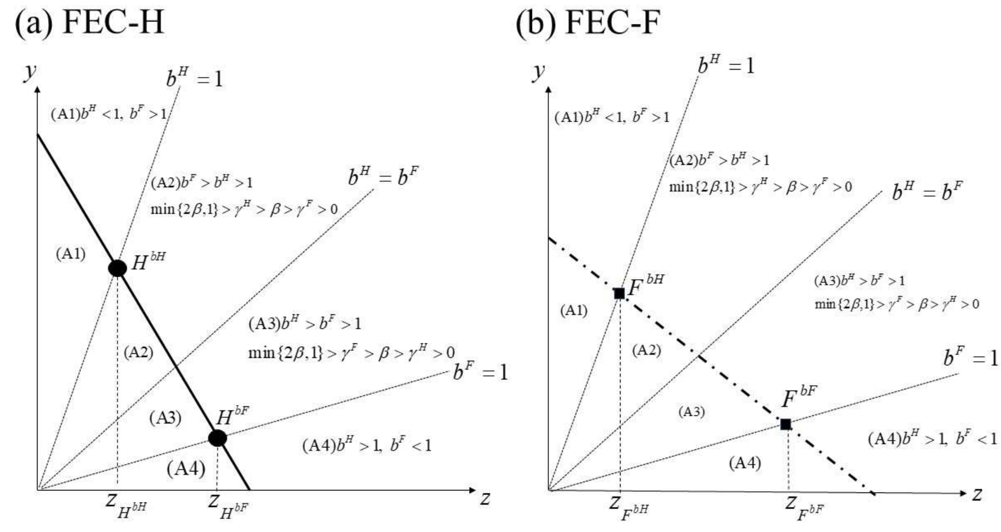

Figure 2: Trade balance conditions: note that we omit FEC-F chain curves that $k^H < k^F$ and $k^H > k^F$.

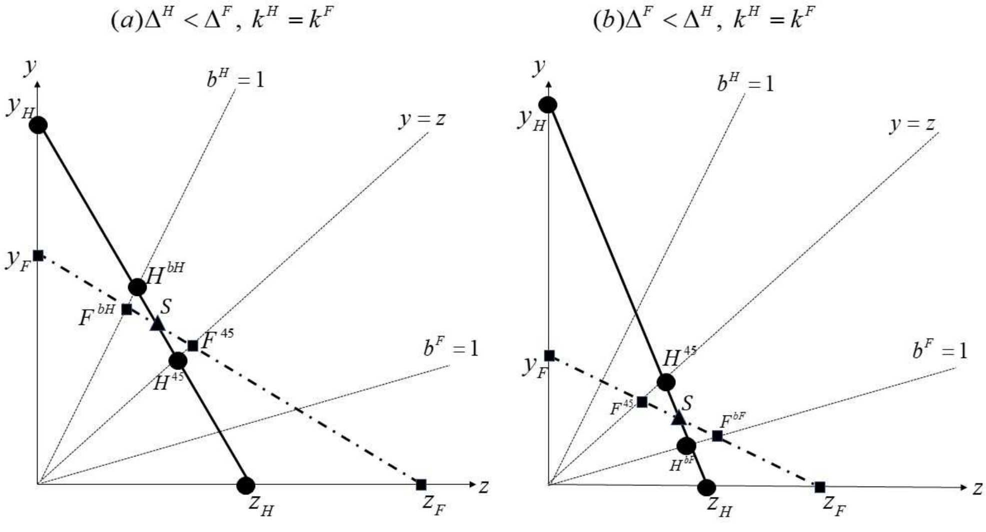

Figure 3: The cases of the unique equilibrium

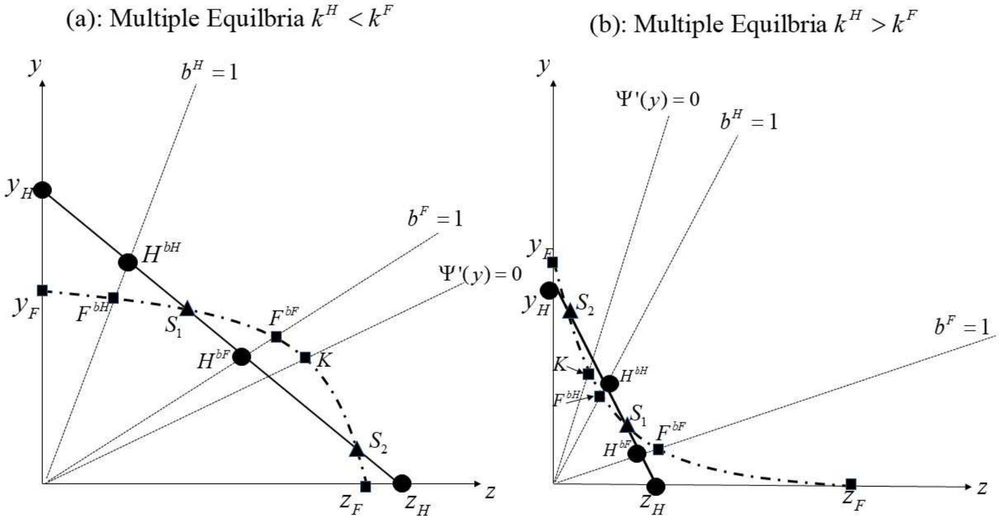

Figure 4: The cases of the multiple equilibria

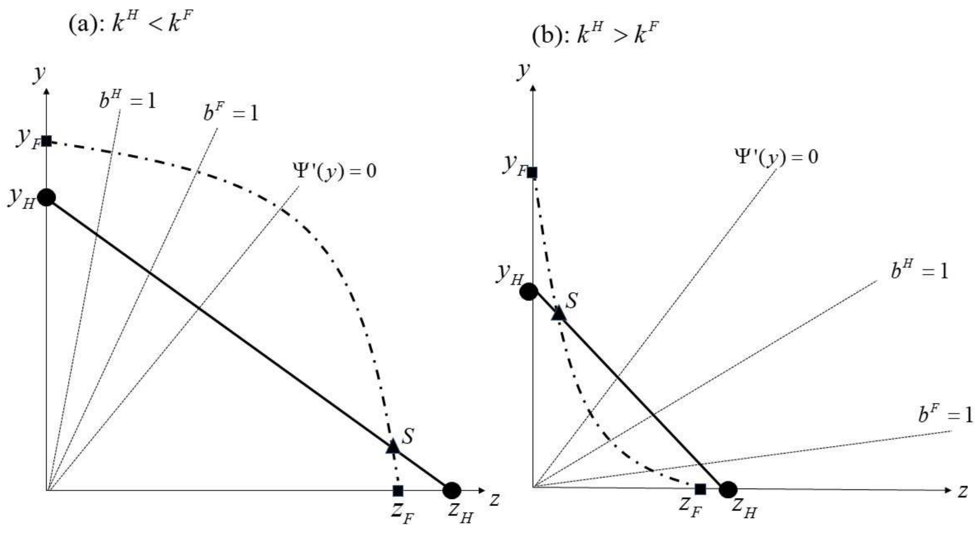

Figure 5: The cases of no equilibrium with two-way trade