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Australia's Underground Economy – Redux?

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Abstract: Bajada (2006) recognises that his earlier books and papers used a faulty method for measuring the underground economy in Australia. He also reports finding a new “more serious problem” in the method. All of these failures can be avoided, it is claimed, by reduced use of currency modelling and more reliance on outside estimates. Despite delivering estimates up to two-thirds less than before, the revised method involves substantial double counting. Ironically, these problems are found only in Bajada’s particular method, not in currency modelling generally.

Author’s note: This paper was written at the invitation of an Associate Editor of the *Economic Record* in response to a comment by Christopher Bajada (2006) called “Australia’s Underground Economy Revisited”. Unfortunately Bajada withdrew his submission at a late stage of the editorial process – after he received this response.

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JEL Classification: E26, E41, E51, E62

1. Introduction

Christopher Bajada (2006) recognises that his earlier books and papers used a faulty method for measuring the underground economy. In Bajada (1999, 2001, 2002, 2003) a long time series from the late 1960s to date shows unrecorded income in Australia roughly constant at 15% of GDP. Now it is acknowledged that (i) the calculations are extremely sensitive to changes in units of measurement (when logically they should be invariant), and (ii) the answers are directly related to an assumed value for the income velocity of currency that is implausibly large. These are the main criticisms of Breusch (2005).

In reconsidering his previous use of currency modelling, Bajada also identifies a new “more serious problem”. He argues that it is not possible to disaggregate the level of currency usage into legal and underground sectors, as required in the method, because the two components of the constant term in the aggregate model are not separately identifiable. His solution employs the currency model for the more modest purpose of estimating the growth rate of the underground economy rather than its absolute size. The overall level of underground incomes is established from two outside “independent” estimates at one point in time. This reduced purpose of currency modelling also appears to avoid, or at least ameliorate, the earlier criticisms of sensitivity to units of measurement and dependence on extreme assumptions about velocity.

However, we will see that the problems are all due to contradictions in Bajada’s method. The mathematical model in logarithms is inconsistent with the interpretation he gives to addition as the aggregation of physical quantities. Also, the intercept cannot be constant if the model is interpreted as an aggregate, so it is unhelpful to describe the problem as separately identifying two components of a constant term. Once the intercept is recognised as time-varying, the differencing operation that reduces the model to growth rates will no longer remove the troublesome part of the equation.

Ironically, Bajada’s identification problem is seen to be a feature of *his* particular method, not of currency modelling generally, as he suggests.

The new results are questionable in their making, even if we accept the growth rate estimate derived from currency modelling. The overall size of the underground economy is found by adding together the two outside estimates. These sources of information overlap substantially in their coverage, so the calculation involves a large degree of double counting. The new results are also remarkably different from those reported earlier. In particular, the size of the underground economy in recent years is much smaller, in one case down to around 5% of GDP, or two-thirds less than the previous estimate. The time shape of the new series is also very different. Rather

than constant over time, the new estimates show the underground economy on a long-term decline as a percentage of official GDP.

2. The “more serious problem”

The approach to measuring the size of the underground economy in Bajada (2006) starts with the currency demand model in his equation (4):

$$\ln C_t = \alpha_0 + \beta_1 \ln YD_t + \beta_2 \ln R_t + \beta_3 \ln TD_t + \gamma_1 \ln T_t + \gamma_2 \ln W_t + u_t. \quad (4)$$

Here C is currency holdings, YD is disposable income, R is an interest rate, TD is a technology variable, T is an average tax rate, and W is the ratio of welfare benefits to disposable income. The model represents *total* currency demand, which arises partly from the legitimate observed economy and partly from the underground economy. Underlying the model in (4) is the idea that the earlier variables (YD , R , TD) are the determinants of “legitimate currency” use, while the later variables (T , W) are the determinants of “illegal currency.”

After the aggregate model is estimated, legitimate currency is predicted by dropping the variables (T , W). As Bajada (2006, p.3) puts it, more clearly than in any of his earlier published works, “A standard way of estimating the holdings of legal currency is ... by setting $\hat{\gamma}_1 = \hat{\gamma}_2 = 0$.” Herein lies the new problem he identifies: the intercept α_0 in the aggregate model is presumably a composite of two intercept parameters from two separate underlying sub-models for legitimate and illegal currency use. Since these intercepts appear in the estimated model only as their sum, there is no way to identify separately the two contributions. The prediction of legitimate currency that is made by dropping the variables (T , W) will leave behind part of the intercept that should also be dropped, so the prediction will be in error by an unknown amount. This error will carry forward to the calculation of illegal currency as the difference between total and predicted legitimate currency, and hence to the estimate of the size of the underground economy, which is obtained from illegal currency and some assumption about the income velocity of illegal currency.

His conclusions from this reasoning are twofold. First, “it will be impossible to estimate the *absolute* size of the underground economy using the currency demand approach without knowing more on the decomposition of [the intercept]” (Bajada, 2006, p.4; italics in original). Second, because differencing the equation will remove any constants such as the intercept, “the currency demand approach is still quite useful for estimating *changes* (or the growth rate) in the size of the underground economy over time” (Bajada, 2006, p.4; italics and parentheses in original).

Unfortunately neither of these conclusions is sound, because the mathematical structure of the model is at violence with empirical reality. To accept Bajada's diagnosis that the problem lies in our inability to disaggregate the intercept when the model is estimated in aggregate form, we would need to be convinced of two things. The first is that the individual sub-models portray the phenomena they represent in meaningful ways, at least in principle (that is, if the variables could be observed). The second is that the aggregate model is a mathematically valid representation of the aggregation of the two separate measurements.

The second of these points is more easily seen. An aggregate of currency held in two sectors of the economy is the simple sum of the two stocks. This idea is expressed in Bajada's equations (1) to (3) and the surrounding text, where the separate and aggregated models are written on the original scale, and where it is natural to add the two components of currency together to form the total over the two sectors of the economy. Aggregation in this case is represented mathematically by addition on the raw scale of the variables (in units of dollars). But in equation (4) – which is the model estimated, and from which the inferences are drawn – the two components of currency stock are added together *after they have been transformed by taking logarithms*. That mathematical operation does not represent physical aggregation, because adding the logarithms is equivalent to multiplying the components on the original scale (and the result has units of dollars *squared*). Since the aggregate model does not represent the total stock as the sum over the two sectors, it is incorrect to describe the disaggregation problem as one of identifying the two additive components in the intercept. Even if we knew the partition of the intercept between the two sources, that would tell nothing useful about decomposing total currency into its components. The move to logarithms between his equations (3) and (4) is made without explanation and apparently without recognising that addition on the log scale has different meaning from addition on the original scale.

The interpretation of the model as aggregation implies sub-models for the components of the aggregate. As described in Bajada (2006, pp.2-3), the sub-model for legitimate currency in the observed economy is a fairly standard money demand function in which currency holding depends on a scale variable, YD , a technology proxy, TD , and an opportunity cost, R . By contrast with this standard relationship, currency in the underground sector is made a function of just two “incentive” ratios, T and W . The absence of any measure of the opportunity cost of holding currency is understandable if there are few alternatives to dealing in cash in the underground economy. But there is nothing here to allow for scale, which leaves this sub-model with the feature that it will predict no change in currency demand when the economy expands, as long as the ratios T and W remain constant. Put differently, the intercept in this sub-model has to absorb all of the effects of a change in income. That is an untenable restriction to place on a model that

will be used to represent 25 years of Australian economic history over which real GDP increased by 230% and nominal GDP increased by 750%. It is unconvincing to describe the problem as identifying the two constants that comprise the intercept, when one of those components cannot be constant.

On the other hand, the model might be expanded to allow the intercept to have a time-varying component related to underground income. But in that case, the differencing operation that reduces the model to a relationship in growth rates would no longer remove the troublesome intercept and along with it the identification problem.

3. Endemic in the literature?

An interesting feature of the “more serious problem” identified by Bajada is that it occurs only in *his* particular approach and not, as he suggests, in other literature that uses currency modelling to estimate the underground economy.

The difference between Bajada and others lies not so much in the form of currency equation that is estimated. Most other applications in fact estimate a log-linear relationship that is at least superficially similar to equation (4). There are differences in the definitions of variables: in some cases the variables are real or per capita, while in others they are nominal aggregates at national level. Most researchers will include the interest rate in its level, R , rather than the logarithm form that is used here, $\ln R$, on the sensible grounds that the interest rate arises as the discount rate in the exponent of a multiplicative theory model and hence goes into its natural form when the model is linearized. Others will include as stimulants of an underground economy some measures of the extent or impact of economic regulation. These variables often appear instead of the welfare rate variable that is included here, although some measure of tax rates is almost always present. In one leading form of currency modelling for estimating the underground economy, introduced by Tanzi (1983) and cited by Bajada, the tax rate variable is incorporated in the form $\ln(1+T)$ rather than the form $\ln T$ that is used here.

With broadly similar currency models being used all round, it might be assumed that the inference methodology is the same in all cases. The crucial point of departure of Bajada from the other literature is the counter-factual that he invokes and the associated interpretation he gives to his calculations on the estimated model. The key is found in the passage quoted earlier “A standard way of estimating the holdings of legal currency is ... by setting $\hat{\gamma}_1 = \hat{\gamma}_2 = 0$ ” (Bajada, 2006, p.3). That is a restriction on the coefficients of the estimated model and it is intended, as Bajada’s new paper makes abundantly clear, to represent a disaggregation of the observed total of currency stocks into its two components. When the restriction is imposed, what remains is supposed to be that part of currency not stimulated by tax or welfare rates, that is currency used

in the legitimate economy (or at least it would be so, if only the two components of the intercept could be disentangled).

The other literature in this area describes a different counter-factual, namely one where tax rates are either zero or at some conceptual or historical minimum. That is not a restriction on the *coefficients* of the estimated model, but instead a hypothesis about the *variables* entering the model. In the cases of Tanzi (1982), Schneider (1986) and Hill and Kabir (2000), for example, the experiment consists of holding a tax variable at some low level that was in force at a time when the underground economy was thought to be non-existent, or before some major change was made to the tax regime. An alternative approach, used by Tanzi (1983), Gadea and Serrano-Sanz (2002) and Schneider (2005), for example, allows the tax rate to enter as $\ln(1+T)$ and considers the effects of a hypothetical tax rate of zero. In this approach, the included variable drops out of the regression equation because $\ln(1) = 0$, but again the hypothesis concerns the magnitude of the stimulus variable not the coefficient of that variable in the model.

Thus Bajada is wrong when he describes setting $\hat{\gamma}_1 = \hat{\gamma}_2 = 0$ as “a standard way of estimating the holdings of legal currency.” That way is his and his alone.

None of the other literature seeks to interpret the counter-factual as disaggregation. They simply have a model for aggregate currency holdings over both sectors of the economy, and they speculate on how much less currency might be used if tax rates were as low as they once were, or in other cases, if tax rates were zero. Bajada, on the other hand, proposes a counter-factual world in which tax rates (and welfare rates) are whatever they actually are, but *responses* to those tax and welfare rates are different from what has been observed. To clarify his reasoning for this approach, he introduces disaggregation as his motive and finds problems with that explanation because he cannot disentangle the intercept. The rest of the literature has no interest in, nor any need for, the disaggregation explanation. Bajada’s problem with the intercept does not trouble them.

4. The new methodology

Bajada’s solution to the weakness he has found in his currency model approach is to eschew estimates of the *level* of the underground economy in favour of less ambitious estimates of its *growth rate*. In equation (4) when sequential differences are taken, all of the variables will be converted to the form $\Delta \ln X_t \approx (X_t - X_{t-1})/X_{t-1}$, which is the instantaneous proportional rate of growth of the original variable X_t . Most important, any additive constants such as the intercept will vanish when the equation is differenced. With the troublesome intercept gone, the growth rate in currency demand can be predicted both with and without the tax and welfare variables. This partition gives a disaggregation of the total growth rate into respective rates in the

legitimate and underground sectors. The approach is described in the text that accompanies Bajada's equations (13) and (14).

Estimating the growth rate of the underground economy is also less problematic for the additional reason that it largely sidesteps the two problems described in Breusch (2005). With all of the variables in growth rates, it makes no difference whether the tax and welfare rates are calculated as proportions or percentages of their income bases, because the same growth rates are calculated in both cases. (Alternatively this invariance can be explained as another implication of removing the intercept in the equation by differencing.)

In the currency demand approach, estimates of currency in the underground economy are converted to estimates of income levels by an assumption about the income velocity of currency in circulation. That velocity assumption is less crucial when the inference is about growth rates, because the link from growth in currency to growth in income requires only an assumption about growth in velocity, not about the level of velocity itself. Thus the extreme and untenable assumption about the level of velocity made in Bajada's previous estimates can largely be avoided.

This apparently neat solution is only as good as the model that supports it. The problem with the growth rate equations, Bajada's (13) and (14), is that adding growth rates is inconsistent with aggregation in the levels of the original variables. This is the same point I made earlier when I observed that aggregation of physical quantities is not represented by an additive model on the log scale (or, equivalently, by a multiplicative model on the original scale, as in Bajada's equation (12)). The matter is obscured somewhat by the improper piece of mathematics in which his (12) is "rewritten" as (13), where the latter is a different relationship entirely. But even with the correct mathematics to linearize (12) and convert it to growth rates, adding logarithms or growth rates does not represent physical aggregation.

5. Empirical results

In an application of the new method to Australia for the period 1979-2004, currency modelling is used to partition the observed growth rate of currency use into its legal and underground components. The growth of illegal currency is converted into the growth of underground economy income by assuming a growth rate of the income velocity of currency. This income growth rate is converted in turn into the absolute level of underground economy income by the use of "two independent estimates" for the year 2000.

Velocity calculations

Bajada employs two alternative velocity measures, which he reports as having “quite similar” growth rates. Neither of these velocity measures is very convincing for the purpose, because they are ratios of recorded national income (thus the legal part of the total) to recorded currency or broad money stocks (which are totals of legal and underground uses). Again, on the argument that growth rates are less sensitive than levels to the precise definitions, it is possible that the differences are slight.

Are the alternative growth rates really “quite similar”? We are invited to compare Bajada’s Figures 1a and 1b, which are superficially similar in their high-frequency variation from one quarter to the next. But in the eventual inference that is made, the data are aggregated from the quarterly frequency to annual, and the long-run picture over 25 years is drawn. What really matters with these velocity measures is their sustained growth rates over the long run, and in this respect the measure based on currency is *more than twice as large* as the one based on broad money. A sharp eye may be able to detect that the average in his Figure 1a is much higher than the average in Figure 1b, although the difference in the averages is largely obscured by the high variability. The divergence is more readily apparent from Table 2 where the columns based on currency grow by 37% over 25 years, while the columns based on broad money grow by less than 16%. For some reason, only one velocity assumption is depicted in Bajada’s Figure 2. If both assumptions were plotted on the same chart, while the lines would necessarily cross, the other line would have so much stronger growth that it would start the period off the bottom of the scale!

Outside estimates

One of the “independent” estimates of the underground economy in the year 2000 is the range of \$29.5 to \$53.6 billion derived by Schneider, Braithwaite and Reinhart (2001) from a survey of households. The other range of \$8.5 to \$25.3 billion comes from a discussion paper of the Australian Bureau of Statistics (ABS, 2004) describing treatment of the underground economy in Australia’s national accounts. What is most remarkable about these figures (apart from the wide ranges provided by both sources and their dissimilarity from each other) is the treatment by Bajada in which they are *added together* to form his baseline estimate of the absolute size of the underground economy.

It is never stated explicitly that the two sets of estimates are added, but that is what gives the figures reported in his Table 2. The reasoning is apparently that the former is an estimate of *household* contribution to the underground economy, while the latter is an estimate of the

contribution by the *business sector*. But this argument misinterprets the data: apart from some slippages in their coverage, these two calculations are largely estimates of the *same* quantity.

In the survey on which the calculation by Schneider *et al* is based, householders were asked the question, “Have you worked for cash-in-hand payments in the last 12 months? By cash-in-hand we mean cash money that tax is not paid on.” Respondents who answered “yes” were then asked “How much did you earn in this way in the last 12 months?” and asked to provide their best estimate in dollar terms. Schneider *et al* then aggregate the survey results to a national total, with weighting of the various occupational groups such as tradespeople who are found to be major participants. They also include some escalation factors to allow for the additional opportunity of unemployed people and early retirees to participate in the underground economy (and they exaggerate by double counting, but those excesses will be documented in another place).

An examination of the ABS publication reveals the following. The lower bound of \$8.5 billion (or 1.3% of GDP) reported by Bajada is actually the explicit adjustment to the income side of the GDP account that is already included in the published accounts for 2000-01. That adjustment is based in part on a study of aggregated income tax audits but mostly it arises from the balancing of the income side of the account with production and use data that the ABS holds to be more reliable. Bajada’s upper bound of \$25.3 billion is not a figure reported anywhere in the ABS paper. It is most likely calculated by adding the ABS upper bounds for possible understatement in two income categories ‘Gross operating surplus of private, non financial corporations’ (\$5.5 billion) and ‘Gross mixed income’ (\$19.8 billion). The latter is by far the largest sector of adjustment in the ABS figures, because:

Some studies have indicated that householders participate actively in the underground economy on own-account. Such activities, commonly referred to as ‘moonlighting’, generally involve individuals who are employees in the observed economy but who, outside of official working hours, participate in the underground economy. They do so by offering goods and services directly to consumers (sometimes referred to as ‘consumer to consumer’ transactions) for ‘cash in hand’ and then not reporting this activity. It is important to note that in the national accounts this type of activity would be classified as gross mixed income (GMI) rather than employee income.
(Australian Bureau of Statistics, 2004, pp.2-3)

Corporate profits are unlikely to be paid in cash, even if they are earned in unreported dealings, so it seems plausible to add them to the cash-in-hand income estimated by Schneider *et al*. However, the majority of what Bajada calls “business sector contribution” to the underground economy is income derived by householders when working for cash on their own account. Such income is already included in the Schneider *et al* estimate, because the survey question covered

own-account receipts as well as employee payments. Adding the ABS and Schneider *et al* figures together is substantially double counting.

Absolute dollar size

It is interesting to compare the results in Bajada's Table 2 with the estimates published in his earlier books and papers. Apart from short-run variability, the previous estimates are nearly constant at 15% of recorded GDP from the mid-1960s forward. By contrast, the range reported here for recent years is from 5% to 11% of GDP, which is up to two-thirds less than previously claimed. The reporting of absolute dollar amounts in constant prices in Table 2 obscures another major difference from the previous estimates (which is mentioned in a footnote). Instead of a constant percentage over time, these new estimates decline steadily in relation to official GDP, in one case to the extent that the percentage is little more than half of its original size by the end of the 25-year period. Thus the new results are markedly different in both level and trend from those reported earlier.

6. Conclusions

There have been many attempts to measure the size of the underground economy by monitoring movements in public holdings of currency. None of these efforts is particularly convincing, although some are less persuasive than others. The worst are fraught with illogical methods or require extreme and implausible assumptions to obtain interesting results. Both of these features are found in the earlier claims by Christopher Bajada of an underground economy in Australia of a persistent 15% of recorded GDP.

The previous method has been abandoned by its creator, along with the headline-grabbing claim of 15%. Now the currency model is used to estimate only the growth rate of the underground economy, while the overall level is derived from the results of others. The logic is no better than before, with some old errors retained (addition on a log scale to represent aggregation) and some new ones introduced (double counting of outside data).

The picture of underground activity is much revised from the previous claims, in places shrunken by as much as two-thirds of the earlier estimate for the same period. The time shape is also very different, with the underground economy as a proportion of recorded income now shown to be falling steadily over the years.

No comfort can be derived from this rosier view of a smaller and relatively declining underground economy in Australia, just as no concern was needed for the earlier one. The new numbers are nonsense, just like the abandoned ones.

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