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Partial privatization and subsidization in a time-consistent policy: output versus R&D subsidies

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Abstract

This study revisits welfare comparisons between output and R&D subsidies for a mixed duopoly with partial privatization in a time-consistent policy framework. We show that an output subsidy is welfare-superior to an R&D subsidy policy only when the degree of privatization is high. We also show that the government has a lower incentive to privatize the public firm under the R&D subsidy but full nationalization with an R&D subsidy can decrease the welfare than full privatization with an output subsidy.

JEL Classifications: L32; H21; L13

Keywords: Partial privatization; R&D subsidy; Output subsidy; Time-consistent policy

1. Introduction

In the context of mixed duopoly with R&D activities, a few studies have compared welfare effects of government subsidization policy.\(^1\) For instance, Gil Molto, et al. (2011) examined an R&D subsidy and showed that the subsidy leads to an increase total R&D and production, but not to an efficient distribution of production costs.\(^2\) They concluded that full privatization reduces R&D activities and welfare with or without subsidies. Haruna and Goel (2017) examined R&D activities under an output subsidy, and found that the relative output subsidy rankings are significantly affected by R&D spillovers, but the welfare with the output subsidy is the highest. Kesavayuth and Zikos (2013) then compared the relative welfare effects between R&D and output subsidies, and showed that an R&D subsidy is socially superior (inferior) to an output subsidy when R&D spillovers are high (low) in the presence of a full nationalized public

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\(^2\) Zikos (2007) analyzed the policy mix of output and R&D subsidies in a mixed duopoly with a fully nationalized public firm, while Lee and Tomaru (2017) examined partial privatization and showed that the first-best can be obtained irrespective of the degree of privatization policy.
firm. Incorporating partial privatization, Lee et al. (2017) investigated the welfare comparisons of the subsidy policies and showed that an output subsidy is always welfare-superior to an R&D subsidy policy, but the government has a higher incentive to privatize the public firm under the output subsidy than the R&D subsidy.

These studies confined their analyses into the committed policy environment where the government can credibly commit to its subsidy rate from the introduction stage of subsidy policy. In the process of policy-making, however, the ability of a government to commit credibly to a policy has significant implications to support the superior welfare properties associated with a committed policy. If the government cannot commit credibly to the stringency of the policy instrument, firms have strategic incentives because the government has an ex-post possibility to ratchet up future policy. Thus, the time-inconsistency problem can shed light on the ex post welfare effect of strategic behaviors of the firms.

This study revisits welfare comparisons between output and R&D subsidies in a mixed duopoly with partial privatization in a time-consistent policy framework. In the analysis of an output subsidy, we examine the model that the firms choose the cost-reducing R&D activities in the first stage and then the government rationally determines its welfare-maximizing subsidy rate in the second stage. We then compare the equilibrium outcomes with an R&D subsidy. We show that the public firm increases (decreases) the output production and R&D investment under output subsidy if the degree of privatization is high (low), while both output and R&D of private firm increase under output subsidy irrespective of the degree of privatization. It leads to the increases of market outputs and industry R&D investments irrespective of the degree of privatization, but the welfare is higher under output subsidy only when the degree of privatization is high. Hence, an output subsidy is welfare-superior to an R&D subsidy policy only when the degree of privatization is high. We also show that the government has a lower incentive to

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3 For a commitment issue on environmental regulations, for example, see Gersbach and Glazer (1999), Petrakis and Xepapadeas (1999), Poyago-Theotoky and Teerasuwannajak (2002), Moner-Colonques and Rubio (2015), Leal et al. (2018), and García et al. (2018) among others.

4 As a related work, Haruna and Goel (2019) considered the strategic relations between emission tax and firms’ emission-reducing R&D activities in a mixed duopoly. They examined the welfare effect of time-consistent emission tax and showed that privatization lead to reductions in R&D and output, but to an increase in overall emissions, which tends to make the environment worse.
privatize the public firm under the R&D subsidy, but full nationalization with an R&D subsidy can
decrease the welfare than full privatization with an output subsidy. Our findings highlight the significance
of the policy combination between partial privatization and subsidization policies in the time-consistent
framework for enhancing innovation activities under different ownership of organizations.

2. The Model and Analysis

Our model follows Lee et al. (2017) with a duopoly market in which two firms produce homogeneous
goods. Let the inverse demand function be \( P = a - Q \), where \( P \) is the market price, \( Q = q_0 + q_1 \) is the
market output, and \( q_i \) is the output of firm \( i = 0,1 \), respectively. The firms face identical costs functions
and marginal costs are increasing:

\[
C(q_i, x_i, x_j, x_i) = (c - x_i)q_i + x_i^2 \quad \text{and} \quad \Gamma(x_i) = x_i^2, \quad i, j = 0,1 \text{ and } i \neq j.
\]

where \( a > c > 0 \) and \( x_i \) denotes the amount of R&D investment for firm \( i \), which exhibits decreasing
returns to scale, i.e., the firm has to spend \( x_i^2 \) to implement cost-reducing R&D, \( x_i \). We also consider that
each firm might receive an output subsidy \( s q_i \) where \( s \) denotes the per-unit subsidy rate of output.

Then, the profit function of the firm is:

\[
\pi_i = (a - q_i - q_j)q_i - (c - x_i)q_i - x_i^2 + s q_i, \quad \text{for } i, j = 0,1 \text{ and } i \neq j.
\]

The welfare is defined as the sum of consumer surplus, \( CS = \frac{Q^2}{2} \), and both firms’ profits minus
total expenditures of subsidies:

\[
W = CS + \pi_0 + \pi_1 - s(q_0 + q_1).
\]

Note that the subsidies are financed from taxpayers in a lump sum manner, so that they cancel out when
aggregating. We assume that the government determines the subsidy rate to maximize the social welfare.

Firm 1 is a private firm that maximizes its own profit while firm 0 is a (partially privatized) public
firm. That is, we allow the government to have the shares in firm 0 and partially control the behavior of
the firm 0, and thus firm 0 is jointly owned by the government and private investors. Let \( \theta \in [0,1] \) be
the shares in firm 0 that private investors hold. Following Matsumura (1998), we assume that firm 0 maximizes the convex combination of its profit and welfare\(^5\):

\[ V = (1 - \theta)W + \theta \pi. \]

The timing of the game with output subsidy is as follows. Both firms decide R&D simultaneously in the first stage and then the government chooses the optimal level of subsidy in second stage. In the last stage, firms compete in the product market by setting quantities simultaneously. The game is solved by backward induction to obtain the following subgame perfect equilibrium outcomes.\(^6\)

**Lemma 1.** In a mixed duopoly with an output subsidy, the equilibrium outcomes are:

\[ s^* = \frac{12(a - c)(1 + 2\theta^2)(72 + 13\theta + 287\theta^2 + 4\theta^3 + 32\theta^4)}{2664 + 351\theta + 17529\theta^2 + 1052\theta^3 + 38252\theta^4 - 224\theta^5 + 27776\theta^6 - 1792\theta^7} \]

\[ x_0^* = \frac{(a - c)(360 + 81\theta + 2343\theta^2 + 516\theta^3 + 5076\theta^4 + 1376\theta^5 + 3584\theta^6 + 1280\theta^7)}{2664 + 351\theta + 17529\theta^2 + 1052\theta^3 + 38252\theta^4 - 224\theta^5 + 27776\theta^6 - 1792\theta^7} \]

\[ x_1^* = \frac{(a - c)(3 + 4\theta^2)(216 + 27\theta + 909\theta^2 + 8\theta^3 + 992\theta^4 - 64\theta^5)}{2664 + 351\theta + 17529\theta^2 + 1052\theta^3 + 38252\theta^4 - 224\theta^5 + 27776\theta^6 - 1792\theta^7} \]

\[ q_0^* = \frac{12(a - c)(1 + 2\theta^2)(60 + 13\theta + 283\theta^2 + 20\theta^3 + 32\theta^4)}{2664 + 351\theta + 17529\theta^2 + 1052\theta^3 + 38252\theta^4 - 224\theta^5 + 27776\theta^6 - 1792\theta^7} \]

\[ q_1^* = \frac{4(a - c)(1 + 2\theta^2)(216 + 27\theta + 909\theta^2 + 8\theta^3 + 992\theta^4 - 64\theta^5)}{2664 + 351\theta + 17529\theta^2 + 1052\theta^3 + 38252\theta^4 - 224\theta^5 + 27776\theta^6 - 1792\theta^7} \]

\[ \pi_1^* = \frac{(a - c)^2(23 + 104\theta^2 + 112\theta^4)(216 + 27\theta + 909\theta^2 + 8\theta^3 + 992\theta^4 - 64\theta^5)^2}{(2664 + 351\theta + 17529\theta^2 + 1052\theta^3 + 38252\theta^4 - 224\theta^5 + 27776\theta^6 - 1792\theta^7)^2} \]

\[ W^* = \frac{2(a - c)^2 \left\{ 984960 + 260496\theta + 13122351\theta^2 + 2537586\theta^3 + 72367623\theta^4 + 8985120\theta^5 + 211711448\theta^6 + 12143008\theta^7 + 346812272\theta^8 - 2393584\theta^9 + 301657344\theta^{10} - 20496384\theta^{11} + 108486656\theta^{12} - 13729792\theta^{13} - 524288\theta^{14} \right\}}{(2664 + 351\theta + 17529\theta^2 + 1052\theta^3 + 38252\theta^4 - 224\theta^5 + 27776\theta^6 - 1792\theta^7)^2} \]

A few remarks are in order. First, output subsidy rate is positive and it is non-monotone with the degree of privatization, i.e., \( \frac{\partial s^*}{\partial \theta} < 0 \) if \( 0.040 < \theta < 0.961 \) while \( \frac{\partial s^*}{\partial \theta} > 0 \) otherwise. Second, \( x_0^* < x_1^* \)

\(^5\) Since Matsumura (1998) examined partial privatization, studies on optimal privatization are increasingly popular and extensively used in many various contexts. For example, Ino and Matsumura (2010), Lee, et al. (2013) and Xu, et al. (2016) reviewed several research topics on optimal privatization.

\(^6\) It is easy and thus omitted. Detailed analysis of the equilibrium outcomes will be provided upon request.
and $q_0^* < q_1^*$ for $\theta \in [0,1)$ while $x_0^* = x_1^*$. but $q_0^* = q_1^*$ if $\theta = 1$. Thus, the public firm takes less R&D investments and produces less outputs. It also shows that both R&D and output of the private firm are decreasing in the degree of privatization while those of the public firm are increasing, i.e., $\frac{\partial x_1^*}{\partial \theta} < 0 < \frac{\partial x_0^*}{\partial \theta}$. As a result, total industry R&D and market outputs are also non-monotone with the degree of privatization, i.e., $\frac{\partial (x_0^* + x_1^*)}{\partial \theta} < 0$ when $0.021 < \theta < 0.840$ while $\frac{\partial (q_0^* + q_1^*)}{\partial \theta} < 0$ when $0.116 < \theta < 0.883$, but $\frac{\partial (x_0^* + x_1^*)}{\partial \theta} > 0$ and $\frac{\partial (q_0^* + q_1^*)}{\partial \theta} > 0$ otherwise. Third, the profit of the private firm is decreasing in the degree of privatization, i.e., $\frac{\partial \pi_1^*}{\partial \theta} < 0$. Finally, social welfare is increasing (decreasing) when the degree of privatization is high (low), i.e., $\frac{\partial W^*}{\partial \theta} > 0$ if $\theta < 0.933$. Then, the optimal degree of privatization is $\theta^* = 0.933$. Hence, partial privatization policy is optimal under the output subsidy policy. Note that $W^*(0) = 0.2776(a - c)^2 < W^*(1) = 0.284(a - c)^2$. Thus, full privatization policy under an output subsidy provides a higher welfare than that with full nationalization policy.

3. Comparison with R&D Subsidy

We now consider an R&D subsidy in a time-consistent policy framework. According to the timing of the game, if the government determines an R&D subsidy in the second stage, both firms’ R&D investments are already given in the first stage. That is, the choice of R&D subsidy in the second stage is not effective in the game since the levels of R&D are already fixed in the second stage. It implies that it is time-consistent if the government wants to increase welfare since she can set an R&D subsidy before the firms choose R&D investments.

Then, the analysis in a time-consistent framework is exactly the same with Lee et al. (2017) where the government chooses an R&D subsidy in the first stage and then firms choose their R&D investments in the second stage.\footnote{In Lee et al. (2017), we obtain the optimal R&D subsidy from Eq. (4) in p.170. Then, putting it into the equilibrium outcomes in p.167~168, we can obtain the outcomes in Lemma 2.} Let $s_x$ be the per-unit subsidy rate of R&D output and thus each firm might receive an R&D subsidy $s_x x_i$. 
Lemma 2. In a mixed duopoly with an R&D subsidy, the equilibrium outcomes are:

\[
x^R_x = \frac{2(a-c)(6501 + 8434\theta + 19596\theta^2 + 16202\theta^3 + 4219\theta^4 - 80\theta^5 - 112\theta^6)}{(11 + 4\theta)(31914 + 47095\theta + 63193\theta^2 + 59653\theta^3 + 30521\theta^4 + 7784\theta^5 + 784\theta^6)}
\]

\[
x^R_0 = \frac{4(a-c)(1182 + \theta(1614 + \theta(2124 + \theta(2195 + 7\theta(137 + 20\theta)))))}{31914 + \theta(47095 + \theta(63193 + \theta(59653 + \theta(30521 + 56\theta(139 + 14\theta))))})
\]

\[
x^R_1 = \frac{(a-c)(4137 + \theta(6218 + \theta(8178 + \theta(3975 + 8\theta(129 + 14\theta)))))}{31914 + \theta(47095 + \theta(63193 + \theta(59653 + \theta(30521 + 56\theta(139 + 14\theta))))})
\]

\[
q^R_0 = \frac{(a-c)(10047 + \theta(10973 + \theta(15497 + \theta(13083 + 40\theta(115 + 14\theta)))))}{31914 + \theta(47095 + \theta(63193 + \theta(59653 + \theta(30521 + 56\theta(139 + 14\theta))))})
\]

\[
q^R_1 = \frac{(a-c)(6501 + \theta(10585 + \theta(14225 + \theta(13687 + 2\theta(3737 + 8\theta(129 + 14\theta)))))}{31914 + \theta(47095 + \theta(63193 + \theta(59653 + \theta(30521 + 56\theta(139 + 14\theta))))})
\]

\[
\pi^1_R = \frac{(a-c)^2\{(795312837 + 2882124972\theta + 6552824598\theta^2 + 11092883204\theta^3 + 14442900700\theta^4 + 14932457392\theta^5 + 12409742562\theta^6 + 8152426596\theta^7 + 4100806823\theta^8 + 1524028700\theta^9 + 401496224\theta^{10} + 70596736\theta^{11} + 7413504\theta^{12} + 351232\theta^{13}\}\}}{31914 + \theta(47095 + \theta(63193 + \theta(59653 + \theta(30521 + 56\theta(139 + 14\theta))))})
\]

where superscript $R$ denotes the equilibrium outcomes under the R&D subsidy. Lemma 2 shows contrasting results with Lemma 1. First, R&D subsidy rate is positive but it is monotonically decreasing in the degree of privatization, i.e., $\frac{\partial s^R}{\partial \theta} < 0$. Thus, a higher privatization reduces R&D subsidy rate.

Second, $x^R_0 > x^R_1$ and $q^R_0 > q^R_1$ for $\theta \in [0,1)$ while $x^R_0 = x^R_1$ and $q^R_0 = q^R_1$ if $\theta = 1$. Thus, the public firm takes more R&D investments and produces more outputs. It also shows that both R&D and output of the private firm are increasing in the degree of privatization while those of the public firm are decreasing, i.e., $\frac{\partial x^R_0}{\partial \theta} < 0 < \frac{\partial x^R_1}{\partial \theta}$ and $\frac{\partial q^R_0}{\partial \theta} < 0 < \frac{\partial q^R_1}{\partial \theta}$. However, total industry R&D and market outputs are decreasing in the degree of privatization, i.e., $\frac{\partial (x^R_0 + x^R_1)}{\partial \theta} < 0$ and $\frac{\partial (q^R_0 + q^R_1)}{\partial \theta} < 0$. Third, the profit of the private firm is increasing in the degree of privatization, i.e., $\frac{\partial \pi^R_1}{\partial \theta} > 0$. Finally, social welfare is increasing (decreasing) when the degree of privatization is low (high), i.e., $\frac{\partial W^R}{\partial \theta} > 0$ if $\theta \leq 0.175$. Thus, the optimal degree of privatization is $\theta^R = 0.175$. Hence, partial privatization policy is still optimal under the R&D
subsidy policy. Note that \( W^R(0) = 0.2778(a-c)^2 > W^R(1) = 0.273(a-c)^2 \). Thus, full privatization policy under the R&D subsidy provides a lower welfare than that with full nationalization policy.

**Lemma 3.** Comparing the equilibrium outcomes between output and R&D subsidies,

1. \( x_0^R > x_0^* \) if \( \theta < 0.408 \); \( x_1^R < x_1^* \) and \( X^R < X^* \) for any \( \theta \in [0,1) \)
2. \( q_0^R > q_0^* \) if \( \theta > 0.331 \); \( q_1^R < q_1^* \) and \( Q^R < Q^* \) for any \( \theta \in [0,1) \)
3. \( \pi_1^R < \pi_1^* \) for any \( \theta \in [0,1] \)

Lemma 3 provides the effect of partial privatization on the comparisons of R&D investment and output production. It states that the public firm invests on more R&D and produces more output with a higher degree of privatization under the output subsidy while the private firm’s R&D and industry R&D are always higher with output subsidy. Thus, as the privatized public firm becomes a more profit-oriented firm, it behaves more aggressively invest in R&D and produce output under the output subsidy. As a result, total industry R&D and market outputs are higher under the output subsidy than R&D subsidy, which increases consumer surplus and the private firm’s profit as well. However, when the degree of privatization is low, the public firm behaves more aggressively invest in R&D and produce output under the R&D subsidy. It implies that the degree of privatization is critical to assess the welfare comparisons between the two subsidies.

**Proposition 1.** Comparing the welfare levels between the two subsidies yields \( W^R \geq W^* \) if \( \theta \geq 0.127 \).

It states that the output subsidy can increase the welfare only when the degree of privatization is very low. This result is contrast with the previous analysis which compared the relative welfare effects between R&D and output subsidies. For example, Kesavayuth and Zikos (2013) considered a full nationalized public firm and showed that an R&D subsidy is socially superior to an output subsidy in the absence of R&D spillovers, while Lee et al (2017) showed that an output subsidy is welfare-superior to an R&D subsidy irrespective of the degree of privatization. However, in a time-consistent framework where the
commitment is not credible and thus firms and the government are able to behave in a rational manner, the welfare effect of partial privatization is crucial in the subsidy policies.

**Proposition 2.** Comparing the welfare levels between the two subsidies under the optimal privatization policies yields $W_R(\theta^R) < W^*(\theta^*)$ where $\theta^R = 0.175 < \theta^* = 0.933$.

This implies that if the government determines the optimal degree of privatization policies under the time-consistent framework, it has a higher incentive to privatize the public firm under the output subsidy than the R&D subsidy. Then, a higher privatization policy accompanying with an output subsidy can provide a higher welfare than an R&D subsidy.  

**Proposition 3.** Comparing the welfare levels between the two subsidies under full nationalization and full privatization yields $W^*(1) > W^R(0) > W^*(0) > W^R(1)$.

It states that social welfare in a private duopoly with an output subsidy is higher than that in a mixed duopoly under the output subsidy while reverse is true under the R&D subsidy. Thus, the government has a lower incentive to privatize the public firm under the R&D subsidy, but full nationalization with an R&D subsidy can decrease the welfare than full privatization with an output subsidy. This also contrast to Gil Molto et al. (2011) who showed that privatization would result in a reduction in welfare with or without subsidization. (See their findings in Proposition 6 and 9, respectively). Our findings suggest that full nationalization is effective for increasing welfare as long as the R&D subsidy is implemented while full privatization is effective for maintaining higher welfare under an output subsidy.

**Reference**


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8 This is the same with Lee et al. (2017) who showed that an output subsidy is welfare-superior to an R&D subsidy irrespective of the degree of privatization, but the government has a higher incentive to privatize the public firm under the output subsidy than the R&D subsidy when the government can commit to the subsidy rate.


