Demand Risk and Diversification through Trade

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Abstract

I develop a theory of risk diversification through geography, where risk-averse entrepreneurs exploit the spatial correlation of demand across countries to lower the variance of sales. Both the probability of entry and trade flows to a market are increasing in the "Diversification Index", which depends on the multilateral covariance of a country's demand with all other markets. The firms' risk diversification behavior may imply higher welfare gains from trade than in standard trade models. Risk-augmented gravity regressions show that demand risk significantly affects trade patterns. The risk diversification channel increases welfare gains from trade by 16% relative to models with risk neutrality. The quantitative application highlights the role of demand uncertainty in shaping the economic consequences of the recent Chinese boom.

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1 Introduction

Recent empirical evidence has shown that demand shocks explain a large fraction of the variation in firm sales across countries (see Eaton et al. (2011), Di Giovanni et al. (2014), Munch and Nguyen (2014) and Hottman et al. (2015) among others). When selling to a market, firms may not be able to perfectly insure against unexpected demand fluctuations. The role of demand uncertainty is particularly important in the case of costly irreversible investments, such as producing a new good or selling to a new destination (see Handley and Limao (2015)). In fact, according to a survey among 350 leading companies across the world, dealing with demand risk is the most important business challenge for global firms.[1] Therefore, it is crucial to understand how demand risk affects firms behavior across markets, and evaluate its economic implications.

I provide a theoretical characterization and an empirical assessment of the importance of demand risk for firms behavior on global markets. I argue that exporting to foreign countries is an opportunity to diversify demand risk. Selling to destinations with imperfectly correlated demand can hedge firms against idiosyncratic shocks hitting sales, in the spirit of classical portfolio theory (Markowitz (1952) and Sharpe (1964)). While this is an intuitive mechanism, it has not been fully explored by the macro and international trade literature. In a multi-country general equilibrium model of trade with spatially correlated demand, I characterize how firms' risk diversification behavior affects trade patterns and study its general equilibrium implications. I quantify that risk diversification explains 15% of trade patterns, and increases welfare gains from trade by 16% relative to models with risk neutrality.

In the first tier of my analysis, I develop a general equilibrium trade model with monopolistic competition and heterogeneous firms, along the lines of Melitz (2003). The model is characterized by two novel elements. First, the demand for a differentiated variety is subject to country-variety shocks, which I allow to be imperfectly correlated across countries. This is the only source of uncertainty in the economy, and it can reflect shocks to tastes, consumers confidence, regulation, firm reputation, etc.[2] Second, firms are owned by risk-averse entrepreneurs. This assumption is motivated by the evidence, discussed in Section 2, that the volatility of cash-flows is a primary concern for

[1] This survey was conducted in 2012 by the consulting firm Capgemini: https://www.capgemini.com/wp-content/uploads/2017/07/The_2012_Global_Supply_Chain_Agenda.pdf

[2] For tractability, I rule out aggregate shocks that affect all varieties, as in Eaton et al. (2011) and Nguyen (2012). This is consistent with the recent empirical evidence that firm-destination specific shocks, rather than aggregate shocks, account for the overwhelming majority of the variation in firms' sales across countries (see e.g. Di Giovanni et al. (2014) and Hottman et al. (2015)). Nevertheless, when I allow for aggregate demand shocks in a robustness exercise, the results do not substantially differ from the baseline.

many companies across the globe, especially if the managers' compensation is tied to the performance of the firm (see Ross (2004) and Panousi and Papanikolaou (2012)).[3]

The entrepreneurs' problem consists of two stages. In the first stage, entrepreneurs do not know the realization of the demand shocks, but make an irreversible investment: they choose in which countries to operate, and in these markets perform costly marketing and distributional activities. After the investment is made, in the second stage firms learn the consumers demand and produce. The spatial correlation of demand across countries implies that, in the investment stage, entrepreneurs face a combinatorial problem, since both the extensive and the intensive margin decisions are interdependent across markets.

I overcome this challenge by assuming that firms send costly ads in each country where they want to sell. These activities allow firms to reach a fraction n of the consumers in each location, as in Arkolakis (2010). This implies that the firm's choice variable is continuous rather than discrete, and thus firms simultaneously choose *where* to sell (depending on whether n is optimally zero or positive) and *how much* to sell (firms can choose to sell to some or all consumers). Therefore, the firm's extensive and intensive margin decisions are not taken market by market, but rather performing a global diversification strategy, along the lines of the "portfolio analysis" pioneered by Markowitz (1952) and Sharpe (1964).[4]

I characterize the model-consistent exogenous measure of risk, which I name "Diversification Index." This variable measures the diversification benefits that a market can provide to firms exporting there, and it depends on the entire pattern of spatial covariance of demand across countries.[5] I show that the probability of entering a market and the intensity of trade flows are increasing in the market's Diversification Index. If demand in a country is relatively stable and negatively/mildly correlated with demand in the other countries, then entrepreneurs optimally choose, *ceteribus paribus*, to export more there to hedge their business risk. This implies a fundamental trade-off: selling to a "remote" destination may require higher trade and marketing costs, but it may also hedge firms against domestic fluctuations in demand.

I then evaluate the aggregate implications of the firms' risk diversification behavior.

[3]In the special case of no risk aversion, my model is isomorphic to standard gravity models with monopolistic competition and Pareto distributed firms' productivities, as in Arkolakis et al. (2008) and Chaney (2008).

[4]The concavity of the firm's problem that arises from the presence of risk also implies that, if the determinant of the covariance matrix is strictly positive, the optimal solution is unique.

[5]The Diversification Index can be seen as a multivariate generalization of the classical Sharpe Ratio, proposed by Sharpe (1966) and used in finance to assess the risk-return profile of an asset. In fact, in the limit case in which all demand correlations are zero, the Diversification Index equals the simple ratio between mean and variance, similarly to the Sharpe Ratio, which is computed as an asset's "excess average return" divided by its standard deviation.

In a two-country version of the model, I provide an analytical characterization of the welfare gains from trade. When the covariance of demand between the two countries is sufficiently low, firms use more intensively international trade to hedge their domestic demand risk. This implies a stronger competitive pressure among firms, which in general equilibrium generates a "pro-competitive" effect which can lead to higher welfare gains from trade than in standard models with risk neutrality, such as the class of models considered in Arkolakis et al. (2012) (ACR henceforth).[6]

To assess the quantitative relevance of the risk diversification benefits of international trade, and test the model's predictions, I rely on a panel dataset of Portuguese manufacturing firms' domestic and international sales, from 1995 to 2005. Portugal exports to a wide range of destinations, and therefore is a good laboratory to study the implications of risk diversification for international trade.

First, I structurally recover the unobserved demand shocks from the observed yearly firm-destination sales. I identify the demand shocks as innovations from the growth rate of domestic and international sales, similarly to Di Giovanni et al. (2014). The empirical methodology controls for unobserved firm and destination characteristics, and for firm-destination supply responses.[7] Under the assumption of log-normally distributed demand, routinely maintained by the literature (see Cochrane (2009) and Eaton et al. (2011) among others), I estimate the country-level means and covariance matrix of the demand shocks, and compute the Diversification Index. The results show that countries more geographically distant from each other have significantly lower bilateral covariance of demand, suggesting the existence of a trade-off between distance and risk diversification. In addition, countries with lower GDP per capita tend to have a lower Diversification Index, consistent with the evidence shown in Koren and Tenreyro (2007) with very different measures of risk.

Using the Portuguese firm-level data, I then estimate "risk-augmented" gravity regressions at the firm-destination level, which document that both the extensive and intensive margins of trade, even after controlling for standard gravity variables, are significantly affected by the model-consistent measure of risk. In addition, I document that both the mean/standard deviation ratio and the average covariance across countries, the key components of the Diversification Index, significantly affect trade flows. As an external validation, I also run "risk-augmented" gravity regressions at the country and sector

[6] These include the models in Krugman (1980), Eaton and Kortum (2002), Melitz (2003), and Chaney (2008).

[7] In particular, I include firms' investment rates, capital intensity and productivity, interacted with country dummies, to control for the endogenous response of these variables to foreign demand shocks, as documented by Mayer et al. (2016) and Friedrich et al. (2018), which could affect the observed sales.

level, using bilateral manufacturing trade flows in 2005, which confirm the firm-level findings.

In the second part of the empirical analysis, I calibrate the remaining parameters of the model. Using the firm's first order conditions, I estimate the entrepreneurs's risk aversion in the cross-section of Portuguese firms. The implied "risk premium" is quite large, lending support to the assumption that firms are risk averse. The structural calibration suggests that, overall, the risk diversification motive explains 15% of the observed variation in trade flows. In addition, I show that my model reproduces the existence of many small exporters observed in the data (see e.g. Arkolakis (2010) and Eaton et al. (2011)), since risk averse firms may optimally reach only few consumers in a given market.[8]

I use the structural model to quantify the risk diversification benefits of international trade. Specifically, I follow Costinot and Rodriguez-Clare (2013) and compute the welfare gains of going from autarky, i.e. a world where trade costs are infinitely high, to the observed trade equilibrium in 2005. The results illustrate that countries, like for instance Ireland, Singapore and Germany, that have cheaper access to destinations that are a good hedge against risk (i.e. destinations with a lower covariance with domestic demand), are able to significantly improve their risk-return profile upon a trade liberalization. In general, an improvement of the average risk-return profile is typically reflected into higher welfare gains, which are, for the median country, 16% higher than ACR. Therefore, the "pro-competitive" effects of the firms' risk diversification behavior are quantitatively relevant.

Moreover, I explore the welfare implications of the recent integration of China in the world economy. When I shock the calibrated model with the 30% increase in Chinese productivity observed in the data between 2005 and 2015, real wages rise in all countries, due to the availability of cheaper goods, but there is a reduction in the entrepreneurs' welfare. The reason is that the Chinese productivity boom lowers the export activity of firms from other countries, reducing their profits but also their ability to diversify risk with international trade. This worsens their risk-return profile, exacerbating the negative effects of the shock on entrepreneurs, and implying an aggregate welfare effect, for the median country, of only 0.2%.

This paper contributes to the literature studying the importance of uncertainty for international trade and welfare, such as Handley and Limao (2015), Fillat and Garetto (2015) and Allen and Atkin (2016), as well as the literature that investigates more directly

[8] This stands in contrast with trade models with fixed costs, such as Melitz (2003) and Chaney (2008), which require large fixed costs to match the observed firm entry patterns, contradicting the existence of many small exporters. In addition, while fixed costs models always imply "strict sorting" of exporters into foreign destinations, my framework does not, consistent with the data.

the behavior of exporters under risk, such as Rob and Vettas (2003), Ramondo et al. (2013), and Impullitti et al. (2013).[9] My contribution relative to these works is threefold. First, while they typically focus on the volatility as measure of risk, I generalize their approach and highlight the importance of the spatial correlation of demand, as captured by the Diversification Index, in shaping the firms' export behavior. Second, I provide a novel characterization of the welfare gains from trade as a function of demand risk, and determine, after a structural estimation, its quantitative importance.[10] Third, I document the role of demand uncertainty in shaping the economic consequences of the recent integration of China in the global economy, contributing also to the quantitative literature on the economic effects of the recent Chinese boom (see e.g. Caliendo et al. (2015), Galle et al. (2017) and Adao et al. (2019)).

There is a vast literature, across several fields, that has proposed different measures of uncertainty and studied their impact on the economy. These include proxies based on, among others, stock prices (Bloom (2009)), newspaper coverage (Baker et al. (2016)), tariff gaps (Pierce and Schott (2016)), GDP volatility (Koren and Tenreyro (2007)), consumption volatility (Boguth and Kuehn (2013)). The Diversification Index proposed in this paper captures a distinct aspect of uncertainty compared to existing measures, as it takes into account for the entire pattern of spatial covariance of demand across countries. Therefore, it can be used as a model-consistent proxy for demand risk in any cross-country empirical analysis.

The insights provided in this paper apply also to other contexts characterized by uncertainty. For instance, if there is uncertainty about trade policy in a given country (e.g. as for the US-China trade relationships before 2001, as in Pierce and Schott (2016), Handley and Limão (2017), Bianconi et al. (2019) and Alessandria et al. (2019)), firms may diversify such risk by exporting to other countries where trade policy is more predictable. My results suggest that failing to take into account for the risk diversification behavior of firms may imply a severe miscalculation of the welfare impact of uncertainty on the economy.

Risk diversification is a well-known mechanism studied by the finance literature, and it has been applied also in other settings. However, the novelty of this paper is to show that, in the context of a trade model, risk diversification can generate "pro-competitive" effects through general equilibrium forces. Therefore, the paper contributes also to the

[9] Other related works are Riaño (2011) Nguyen (2012), Vannoorenberghe (2012), Vannoorenberghe et al. (2014), De Sousa et al. (2015), Gervais (2016), Conteduca et al. (2017). For earlier contributions, see Helpman and Razin (1978), Kihlstrom and Laffont (1979), Newbery and Stiglitz (1984), Eaton and Grossman (1985) and Maloney and Azevedo (1995).

[10] For this reason, the paper also innovates in respect to the vast literature that examines the determinants of the welfare gains from trade, see e.g. Arkolakis et al. (2012) and Melitz and Redding (2015).

literature studying the different sources of pro-competitive effects of international trade (see e.g. Edmond et al. (2015), De Blas and Russ (2015), Feenstra and Weinstein (2017) and Arkolakis et al. (2018)).

The paper also relates to the broad literature that studies the determinants of trade patterns. Theoretically, previous models of firms' export decision have typically assumed that exporters make independent entry decisions for each destination market - see Melitz (2003), Chaney (2008) and Helpman et al. (2008a) among others. In contrast, in my model entry in a market depends on the global diversification strategy of the firm, which I characterize despite its inherent complexity. Other existing works, such as Morales et al. (2015), Ahn and McQuoid (2017) and Lind and Ramondo (2018), feature interdependence of exporting decisions across markets, but arising from supply-side forces, rather than from demand linkages. Empirically, while the existing literature has highlighted the importance of firm-destination shocks for the cross-sectional variation of sales (see e.g. Di Giovanni et al. (2014), Hottman et al. (2015), and Eaton et al. (2011)), a distinct contribution of this paper is to document that the spatial correlation of such shocks has important consequences for trade patterns.

Lastly, the paper complements the strand of literature that studies the effect of international trade on macroeconomic volatility. Di Giovanni et al. (2014) and di Giovanni et al. (2018) investigate the role of individual firms in international business cycle comovement and aggregate volatility. di Giovanni and Levchenko (2009) and Caselli et al. (2015) study the effect of trade openness on aggregate output volatility. My paper, in contrast, sheds light on the other direction, i.e. how demand risk affects international trade patterns through the firms' risk diversification behavior.[11]

The remainder of the paper is organized as follows. Section 2 presents the general equilibrium model with risk averse entrepreneurs. In Section 3 I estimate the relevant parameters of the model and, in Section 4, test its predictions in the data. In Section 5 I perform the counterfactual exercises and investigate their robustness, while Section 6 concludes.

[11]Note also that the risk diversification mechanism highlighted in my paper is consistent with the empirical evidence that countries trading more with each other have more correlated business cycles (Frankel and Rose (1998), Kose and Yi (2006), Burstein et al. (2008) and Kleinert et al. (2015)). Indeed, this evidence typically refers to bilateral correlations in aggregate output at the business cycle frequency, while my theoretical and empirical results show the importance of the long-run *multilateral* covariance of demand in shaping trade patterns.

2 A trade model with risk-averse entrepreneurs

In this section, I first propose a static multi-country general equilibrium trade model with N asymmetric countries, featuring stochastic demand and risk averse entrepreneurs. Within the general framework, I characterize the optimal solution of the firms' production problem, and show that both the extensive and intensive margins of trade depend on a model-consistent measure of demand risk. I then focus on the more tractable case of two symmetric countries, and show how the spatial correlation of demand affects the welfare gains from international trade.

2.1 Environment

Throughout the paper, I will denote the importing market by j, and the exporting market by i, where $i, j = 1, ..., N$. Each country j is populated by a continuum of workers of measure $\tilde{L}_j$, and a continuum of risk-averse entrepreneurs of measure M_j. Each entrepreneur owns a non-transferable technology to produce, with a given productivity, a differentiated variety under monopolistic competition, as in Melitz (2003) and Chaney (2008). To focus only on international trade as a mechanism firms can use to diversify their risk, I assume that financial markets are absent.[12]

Consumption. Both workers and entrepreneurs have access to a potentially different set of goods Ω_{ij}. Each agent v in country j chooses consumption by maximizing a CES aggregator of a continuum number of varieties, indexed with ω:

$$\max U_j(v) = \left(\sum_i \int_{\Omega_{ij}} \alpha_j(\omega)^{\frac{1}{\sigma}} q_j(\omega, v)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}} \tag{1}$$

$$\text{s.to} \sum_i \int_{\Omega_{ij}} p_j(\omega) q_j(\omega, v) d\omega \leq y_j(v) \tag{2}$$

where $y(v)$ is agent v's income, and $\sigma > 1$ is the elasticity of substitution across varieties. Workers earn the same non-stochastic wage w by working (inelastically) for the entrepreneurs. In contrast, entrepreneurs' only source of income are the profits they obtain from operating their firm.

The term $\alpha_j(\omega)$ reflects an exogenous demand shock specific to good ω in market j. It is the only source of uncertainty in the economy, and it can reflect shocks to tastes, cli-

[12] This assumption captures in an extreme way the incompleteness of financial markets, and the fact that often firms have limited access to financial derivatives to hedge risks (see Hentschel and Kothari (2001) and Guay and Kothari (2003). An interesting avenue for future research would be to introduce complete markets and compare the economic outcomes with those of the baseline model.

matic conditions, consumers confidence, regulation, firm reputation, etc. Define $\alpha(\omega) \equiv \alpha_1(\omega), ...\alpha_N(\omega)$ to be the vector of realizations of the demand shock for variety ω. I assume that:

Assumption 3a $\alpha(\omega) \sim H(\bar{\alpha}, \Sigma)$, *i.i.d. across* ω

Assumption 3a states that the demand shocks are drawn, independently across varieties, from a multivariate distribution characterized by a N-dimensional vector of means $\bar{\alpha}$ and a $N \times N$ variance-covariance matrix Σ, where N is the number of countries. Given the interpretation of $\alpha_j(\omega)$ as a consumption shifter, I assume that the distribution has support over $\mathbb{R}^+$.

Assumption 3a implies that the demand shocks are destination-variety specific, thus ruling out any aggregate shock that would affect the demand for all varieties in a given destination. This is appealing on both theoretical and empirical grounds. From a theoretical standpoint, I impose this assumption because, given the continuum of varieties, the demand shocks average out by the Law of Large Numbers and thus I can treat aggregate variables, such as wages and price indices, as non stochastic. In addition, this restriction has been typically imposed by the literature that incorporates demand shocks into an international trade model, such as Eaton et al. (2011), Crozet et al. (2012) and Nguyen (2012). Besides for its tractability, Assumption 3a can be justified by the recent empirical evidence that firm-destination specific shocks account for the overwhelming majority of the variation in firms' sales across countries (see e.g. Di Giovanni et al. (2014) and Hottman et al. (2015)).[13] Nevertheless, in the empirical analysis I perform a robustness exercise and allow for aggregate demand shocks, and show that the resulting estimated $\bar{\alpha}$ and Σ do not substantially differ from the baseline.[14]

Production. Entrepreneurs are the only owners and managers of their firms, and produce a unique variety ω using only labor, with a productivity z drawn from a known distribution, as highlighted in Assumption 3b:

[13]Hottman et al. (2015) have shown that 50-70 percent of the variance in firm sales can be attributed to differences in firm appeal. Eaton et al. (2011) and Kramarz et al. (2014) with French data and Munch and Nguyen (2014) with Danish data have instead estimated that firm-destination idiosyncratic shocks drive around 40-45% percent of sales variation. Di Giovanni et al. (2014) show that firm-specific components account for the vast majority of the variation in sales growth rates across firms, the remaining being sectoral and aggregate shocks. Recent contributions also include Bricongne et al. (2012), Nguyen (2012), Munch and Nguyen (2014), Berman et al. (2015a) and Armenter and Koren (2015).

[14]Since the baseline model does not feature multiple sectors, I am also effectively assuming that different sectors have the same degree of risk. However, in the empirical analysis I show that the predictions of the model regarding the impact of demand uncertainty on trade patterns hold also at the sector level.

Assumption 3b *A firm producing variety ω draws a productivity z from a known distribution $G(\cdot)$, independently from other firms and from the demand shocks $\alpha_j(\omega)$.*

Since each firm with a given productivity z produces a unique variety ω, to simplify notation I will use z to identify both. Entrepreneurs choose how to operate their firm in country i by maximizing the following indirect utility in real income:

$$\max V\left(\frac{y_i(z)}{P_i}\right) = E\left(\frac{y_i(z)}{P_i}\right) - \frac{\gamma}{2} Var\left(\frac{y_i(z)}{P_i}\right), \tag{3}$$

where $y_i(z)$ equals net profits. The mean-variance specification above can be derived assuming that the entrepreneurs maximize the expectation of a CARA utility in real income.[15] Such specification has been widely used in the portfolio allocation literature (see, for example, Markowitz (1952), Sharpe (1964) and Ingersoll (1987)), and has the advantage of having a *constant* absolute risk aversion, given by the parameter $\gamma > 0$, which gives a lot of tractability to the model.[16]

The assumption of risk-averse entrepreneurs is the main departure of the model from the existing literature. There is a recent literature supporting this assumption. Empirically, Cucculelli et al. (2012) survey several Italian entrepreneurs in the manufacturing sector and document that 76.4% of them are risk averse. A survey promoted by the consulting firm Capgemini reveals that, among 350 managers and CEO of leading companies across several countries, 52% of them believes that demand volatility is the most important challenge for their firm, as shown in Figure 1. Further evidence that entrepreneurs have a risk-averse behavior has been recently provided, in different contexts, by Herranz et al. (2015) and Allen and Atkin (2016).

Theoretically, an extensive literature has shown that risk aversion arises if corporate management seeks to avoid the costs of financial distress, when these costs are increasing in the cash-flows volatility (see Froot et al. (1993) and Allayannis et al. (2008)), and if the value of managers' compensation is tied to the value of the firms, exposing them to firm-specific risk (see Murphy (1999), Ross (2004), Parrino et al. (2005) and Panousi and

[15] If the entrepreneurs have a CARA utility with parameter γ, a second-order Taylor approximation of the expected utility leads to the expression in 3 (see Eeckhoudt et al. (2005) and De Sousa et al. (2015) for a standard proof). If the demand shocks are normally distributed, the expression in 3 is exact (see Ingersoll (1987)). Maloney and Azevedo (1995), in the context of a trade model, also assume that firms maximize a CARA utility.

[16] One shortcoming of the CARA utility is that the absolute risk aversion is independent from wealth. In Appendix 8.1.15, I consider a variation of the model where the entrepreneurs maximize a CRRA utility, which features a decreasing absolute risk aversion. Quantitatively, in Section 5.1.1 I show that the overall implications for welfare gains do not change substantially.

Papanikolaou (2012)).

2.2 The production problem

The production problem consists of two stages. In the first, firms know only the distribution of the demand shocks, $H(\bar{\alpha}, \Sigma)$, but not their realization. Under uncertainty about demand, firms make an irreversible investment: they choose in which countries to operate, and in these markets perform costly marketing and distributional activities. After the investment in marketing costs, firms learn the realized demand. I assume that the investment decision cannot be changed after the demand is observed. Therefore, firms cannot change neither the set of destinations, nor the marketing investment made in such countries.[17] Finally, firms produce with a production function linear in labor and entrepreneurs spend their realized profits across different consumption goods, according to their sub-utility function in (1).

The fact that demand is correlated across countries implies that, in the first stage, entrepreneurs face a combinatorial problem. Indeed, both the extensive margin (whether to export to a market) and the intensive margin (how much to invest in marketing) decisions are intertwined across markets: any decision taken in a market affects the outcome in the others. Then, for a given number of potential countries N, the choice set includes 2^N elements, and computing the indirect utility function corresponding to each of its elements would be computationally unfeasible.[18]

I deal with such computational challenge by assuming that, in the first stage, firms send costly ads in each country where they want to sell. These activities allow firms to reach a fraction $n_{ij}(z)$ of consumers in location j, as in Arkolakis (2010). This implies that the firm's choice variable is continuous rather than discrete, and thus firms simultaneously choose *where to sell* (if $n_{ij}(z)$ is optimally zero, firm z does not sell in country j) and *how much to sell* (firms can choose to sell to some or all consumers).

The consumers' maximization problem implies that the agent v's demand for variety ω is:

$$q_{ij}(\omega, v) = \alpha_j(\omega) \frac{p_{ij}(\omega)^{-\sigma}}{P_j^{1-\sigma}} y_j(v), \tag{4}$$

[17]This assumption captures the idea that marketing and distributional activities present irreversibilities that make reallocation too costly after the shocks are realized. An alternative interpretation of this irreversibility is that firms sign contracts with buyers before the actual demand is known, and the contracts cannot be renegotiated. For a similar assumption, but in different settings, see Ramondo et al. (2013), Albornoz et al. (2012) and Conconi et al. (2016).

[18]Other works in trade, such as Morales et al. (2014), Blaum et al. (2015), Antras et al. (2017) and Tintelnot et al. (2018) deal with similar combinatorial problems, but in different contexts.

where $p_{ij}(\omega)$ is the price of variety ω produced in i and sold in j, and P_j is the standard Dixit-Stiglitz price index.

The fact that the ads are sent independently across firms and destinations, and the existence of a continuum number of consumers, imply that the total demand in country j for the variety produced by firm with productivity z is:

$$q_{ij}(z) = \alpha_j(z) \frac{p_{ij}(z)^{-\sigma}}{P_j^{1-\sigma}} n_{ij}(z) Y_j, \tag{5}$$

where Y_j is the total income spent by consumers in j, and P_j is the Dixit-Stiglitz price index:

$$P_j^{1-\sigma} \equiv \sum_i M_i \int_0^\infty \int_0^\infty \alpha_j(z) n_{ij}(z) \left(p_{ij}(z)\right)^{1-\sigma} g_i(z) h(\alpha) d\alpha dz, \tag{6}$$

with $g_i(z)$ being the probability density of productivities of firms in i and $h(\alpha)$ the probability density of the demand shocks.

Therefore, the first stage problem consists of choosing $n_{ij}(z)$ to maximize the following objective function:

$$\max_{\{n_{ij}\}} \sum_j E\left(\frac{\pi_{ij}(z)}{P_i}\right) - \frac{\gamma}{2} \sum_j \sum_s Cov\left(\frac{\pi_{ij}(z)}{P_i}, \frac{\pi_{is}(z)}{P_i}\right) \tag{7}$$

$$\text{s. to } 1 \geq n_{ij}(z) \geq 0 \tag{8}$$

where $\pi_{ij}(z)$ are net profits from destination j:

$$\pi_{ij}(z) = q_{ij}(z) p_{ij}(z) - q_{ij}(z) \frac{\tau_{ij} w_i}{z} - f_{ij}(z), \tag{9}$$

and $\tau_{ij} \geq 1$ are iceberg trade costs and f_{ij} are marketing costs.[19] In particular, I assume that there is a non-stochastic cost, $f_j > 0$, to reach each consumer in country j, and that this cost is paid in foreign labor, as in Arkolakis et al. (2008).[20] Thus, total marketing costs are:

$$f_{ij}(z) = w_j f_j L_j n_{ij}(z). \tag{10}$$

[19] I normalize domestic trade barriers to $\tau_{ii} = 1$, and I further assume $\tau_{ij} \leq \tau_{iv}\tau_{vj}$ for all i, j, v to exclude the possibility of transportation arbitrage.

[20] This is without loss of generality, as in Arkolakis et al. (2008) and Arkolakis et al. (2012).

where $L_j \equiv \tilde{L}_j + M_j$ is the total measure of consumers in country j.

The bounds on $n_{ij}(z)$ in equation (8) are a resource constraint: the number of consumers reached by a firm cannot be negative and cannot exceed the total size of the population. Using finance jargon, a firm cannot "short" consumers ($n_{ij}(z) < 0$) or "borrow" them from other countries ($n_{ij}(z) > 1$). This makes the maximization problem in (7) more challenging, because it is subject to $2N$ *inequality* constraints.[21]

The assumption that the shocks are independent across a continuum of varieties implies that aggregate variables w_j and P_j are non-stochastic. Therefore, plugging into $\pi_{ij}(z)$ the optimal consumers' demand from equation (5), I can write expected profits more compactly as:

$$E\left(\pi_{ij}(z)\right) = \bar{\alpha}_j n_{ij}(z) r_{ij}(z) - \frac{1}{P_i} f_{ij}(z), \tag{11}$$

where $\bar{\alpha_j}$ is the expected value of the demand shock in destination j, and

$$r_{ij}(z) \equiv \frac{1}{P_i} \frac{Y_j p_{ij}(z)^{-\sigma}}{P_j^{1-\sigma}} \left(p_{ij}(z) - \frac{\tau_{ij} w_i}{z} \right). \tag{12}$$

Similarly, since marketing costs are non-stochastic, the covariance between $\pi_{ij}(z)$ and $\pi_{is}(z)$ is simply:

$$Cov\left(\frac{\pi_{ij}(z)}{P_i}, \frac{\pi_{is}(z)}{P_i} \right) = n_{ij}(z) r_{ij}(z) n_{is}(z) r_{is}(z) Cov(\alpha_j, \alpha_s), \tag{13}$$

where $Cov(\alpha_j, \alpha_s)$ is the covariance between the shock in country j and in country s.

Although there is no analytical solution to the first stage problem, because of the presence of inequality constraints, we can take a look at the firm's interior first order condition:

$$\underbrace{r_{ij}(z)\bar{\alpha}_j - \gamma r_{ij}(z) \sum_s n_{is}(z) r_{is}(z) Cov(\alpha_j, \alpha_s)}_{\text{marginal benefit}} = \underbrace{\frac{1}{P_i} w_j f_j L_j}_{\text{marginal cost}} . \tag{14}$$

Equation (14) equates the real marginal benefit of adding one consumer to its real marginal cost. While the marginal cost is constant, the marginal benefit is decreasing in $n_{ij}(z)$. In particular, it is equal to the marginal revenues minus a "penalty" for risk, given by the

[21] In finance, it is well known that there is no closed form solution for a portfolio optimization problem with lower and upper bounds (see Jagannathan and Ma (2002) and Ingersoll (1987)).

sum of the profits covariances that destination j has with all other countries (including itself). The higher the covariance of demand, and thus profits, in market j with the other countries, the smaller the diversification benefits the market provides to a firm exporting there.[22]

To find the general solution for n_{ij} and p_{ij}, I make the following assumption, which I assume will hold throughout the paper:

Assumption 3c *The determinant of the covariance matrix is strictly positive, i.e. $det(\Sigma) > 0$.*

Assumption 3c is a necessary and sufficient condition to have uniqueness of the optimal solution. Since Σ is a covariance matrix, which by definition always has a non-negative determinant, this assumption simply rules out the knife-edge case of a zero determinant.[23] Defining μ to be the vector of Lagrange multipliers associated with the upper bound, and λ the one associated with the lower bound, I prove in Appendix 8.1.1 that the optimal solution is:

Proposition 1. *For firm z from country i, the unique vector of optimal* $\mathbf{n}$ *satisfies:*

$$\mathbf{n} = \frac{1}{\gamma}\tilde{\Sigma}^{-1}\left[\pi - \mu + \lambda\right], \tag{15}$$

where $\tilde{\Sigma}$ is firm z's matrix of profits covariances and π is the vector of expected net profits. Moreover, the optimal price charged in destination j is a constant markup over the marginal cost:

$$p_{ij}(z) = \frac{\sigma}{\sigma - 1}\frac{\tau_{ij}w_i}{z}. \tag{16}$$

Proposition 1 shows that the optimal solution, as expected, resembles the standard mean-variance optimal rule, which dictates that the fraction of wealth allocated to each asset is proportional to the inverse of the covariance matrix times the vector of expected

[22]Note the difference in the optimality condition with Arkolakis (2010). In his paper, the marginal benefit of reaching an additional consumer is constant, while the marginal penetration cost is increasing in $n_{ij}(z)$. In my setting, instead, the marginal benefit of adding a consumer is decreasing in $n_{ij}(z)$, due to the concavity of the utility function of the entrepreneur, while the marginal cost is constant.

[23]A zero determinant can happen only in the case where all pairwise correlations are exactly 1.

excess returns (see Ingersoll (1987) and Campbell and Viceira (2002)). The entrepreneurs, rather than solving a maximization problem country by country, as in traditional trade models, perform a *global* diversification strategy: they trade off the expected global profits with their variance, the exact slope being governed by the absolute degree of risk aversion $\gamma > 0$.

This implies that the firm's entry decision in a market (that is, whether $n > 0$) does not depend on a market-specific productivity cutoff and, upon entry, firms may optimally choose to reach only a fraction of consumers, rather than the entire market. This feature stands in contrast with traditional trade models with fixed costs, such as Melitz (2003) and Chaney (2008), and is consistent with the empirical evidence, as shown in Section 4.

Finally, since the pricing decision is made after the uncertainty is resolved, and conditional on the number of consumers chosen in the first stage, the optimal price follows a standard constant markup rule over the marginal cost, shown in equation (16). This is because the realization of the shock in market j only shifts upward or downward the demand curve without changing its slope, as shown in equation (5), and any realized shock is absorbed by a change in the firm's labor demand.[24] In Appendix 8.1.2, I consider an alternative production setting, in which also the pricing decision is made under uncertainty, and show that the aggregate implications of the model are unchanged.

A limit case. It is worth looking at the optimal solution in the special case of risk neutrality, i.e. $\gamma = 0$. In Appendix 8.1.4 I show that, in this case, a firm sells to country j only if its productivity exceeds an entry cutoff:

$$\left(\bar{z}_{ij}\right)^{\sigma-1} = \frac{w_j f_j L_j P_j^{1-\sigma} \sigma}{\bar{\alpha}_j \left(\frac{\sigma}{\sigma-1}\tau_{ij} w_i\right)^{1-\sigma} Y_j}, \tag{17}$$

and that, whenever the firm enters a market, it sells to all consumers, so that $n_{ij}(z) = 1$. This case is isomorphic (with $\bar{\alpha}_j = 1$) to the firm's optimal behavior in trade models with risk-neutrality and fixed entry costs, such as Melitz (2003) and Chaney (2008). In

[24] A demand shock shifts up or down the quantity demanded, as shown in equation (5). This in turn affects the demand for labor:

$$L_i(z) = \sum_j \mathbb{I}_j(z) \frac{q_{ij}(z)}{z} = \frac{1}{z} \sum_j \mathbb{I}_j(z) \alpha_j(z) \frac{p_{ij}(z)^{-\sigma}}{P_j^{1-\sigma}} n_{ij}(z) Y_j,$$

where $\mathbb{I}_j(z)$ equals 1 if firm z enters j. Intuitively, after learning the demand shocks, a firm can reallocate resources toward destinations that had positive shocks, but it cannot i) either sell to countries in which it did not make the marketing investment, ii) or change the number of consumers to sell to. Once firm-level labor demand is aggregated in the labor market clearing condition, as shown below in equation (26), the firm-level demand shocks are averaged out by the Law of Large Numbers.

these models, firms enter all profitable locations, i.e. the markets where the revenues are higher than the fixed costs of production, and upon entry they serve all consumers. The case of $\gamma = 0$ constitutes an important benchmark, as I will compare the welfare impact of counterfactual policies in my model with a positive risk aversion versus models with risk neutrality.

2.3 Trade patterns

Proposition 1 implies that the sales of firm z to country j are given by:

$$x_{ij}(z) = p_{ij}(z)q_{ij}(z) = \alpha_j(z)\left(\frac{\sigma}{\sigma-1}\frac{\tau_{ij}w_i}{z}\right)^{1-\sigma} Y_j P_j^{\sigma-1} n_{ij}(z). \tag{18}$$

Equation (18) suggests that the unobserved demand shocks $\alpha_j(z)$ can be structurally recovered from the observed firm-level trade flows, upon controlling for the other determinants of trade flows that appear in equation (18). In Section 3, I will use this feature of the model to back out the demand shocks and estimate their moments.

I now investigate how trade patterns are affected by risk. To this end, I define the following ex-ante measure of risk:

Definition 1. *Given a covariance matrix Σ and a vector of expected values $\bar{\alpha}$, the Diversification Index is defined as*

$$D \equiv \Sigma^{-1}\bar{\alpha}. \tag{19}$$

The Diversification Index is an ex-ante measure of risk at the country-level. For example, with two symmetric countries, it simply equals:

$$D = \frac{\bar{\alpha}}{\sigma^2(1+\rho)}, \tag{20}$$

where σ^2 and $\bar{\alpha}$ denote the variance and the mean of the demand shocks, respectively, and ρ is the cross-country correlation. Equation (20) shows that the Diversification Index is increasing in the mean, decreasing in the variance, and decreasing in the correlation of demand with the other country. It is easy to verify that the same holds for the general case with N countries in equation (19). Therefore the Diversification Index summarizes the ex-ante diversification benefits that a country provides to firms selling there, since it is inversely proportional to the overall riskiness of its demand.[25]

At this stage, it is useful to define the auxiliary matrix A:

Definition 2. *Given a covariance matrix Σ, the cofactor matrix associated with Σ has elements equal to $C_{kj} \equiv (-1)^{k+j} M_{kj}$, where M_{kj} is the (k, j) minor of Σ. Define A the matrix whose i, j element equals $A_{ij} \equiv -\sum_{k \neq i} C_{ik} \Sigma_{kj}$ for $i \neq j$, and $A_{ij} = 1$ for $i = j$.*

Recall also the definition of an M-matrix (see Berman and Plemmons (1994)):

Definition 3. *A matrix A is an M-matrix if and only if: i) the off-diagonal entries are less than or equal to zero, ii) A is nonsingular, iii) A^{-1} is nonnegative.*

In the following Proposition, I characterize how trade patterns depend on the Diversification Index in partial equilibrium, i.e. holding constant aggregate prices and wages:

Proposition 2. *If A is a M-matrix, then the probability of exporting and the amount exported to a market are increasing in its Diversification Index.*

Proposition 2 suggests that risk affects trade patterns not only through the mean and variance of profits, but through the multilateral covariance summarized by D, i.e. how much the demand in a market co-varies with demand in all other countries. The sufficient condition to have a positive effect of the Diversification Index on $n_{ij}(z)$ is A to be a M-matrix, i.e. all off-diagonal elements of A must be negative. It is easy to verify that A is a M-matrix whenever *some* demand covariances are negative.[26] Intuitively, the pattern of demand covariances across countries has to give enough diversification benefits in order for firms to engage in international trade. This will turn out to be important also in shaping the welfare gains from trade, as discussed in Proposition 5.

Propositions 1 and 2 highlight that firms may optimally reach only few consumers in a given market, and thus export small amounts. This stands in contrast with trade models with fixed costs, such as Melitz (2003) and Chaney (2008), which require *large* fixed

[25]The Diversification Index can be seen as a generalization of the Sharpe Ratio typically used in finance to rank assets by their riskiness (see Sharpe (1966)). In fact, in the limit case in which all demand correlations are zero, the Diversification Index equals the simple ratio between mean and variance, similarly to the Sharpe Ratio, which is computed as an asset's "excess average return" divided by its standard deviation.

[26]This can be seen, for example, for the case $N = 4$, where a typical element of the matrix A is:

$$A_{21} = \rho_{12}\sigma_1^3\sigma_2\sigma_3^2\sigma_4^2(1 - \rho_{13}^2 - \rho_{14}^2 - \rho_{34}^2 + 2\rho_{13}\rho_{14}\rho_{34}).$$

where ρ_{ij} is the demand correlation between j and i, and σ_i is the standard deviation of i. Then, to have $A_{21} < 0$, at least one correlation must be negative.

costs to explain firm entry patterns, contradicting the existence of many small exporters observed in the data. In the empirical section, I will use this feature to test the model's fit to the data.

Having characterized the exporting behavior of risk averse firms, I now define the world equilibrium and discuss its properties.

2.4 General equilibrium

I now describe the equations that define the trade equilibrium of the model. Following Helpman et al. (2004), Chaney (2008) and Arkolakis et al. (2008), I assume that the productivities are drawn, independently across firms and countries, from a Pareto distribution with density:

$$g(z) = \theta z^{-\theta-1}, \ z \geq \underline{z}, \tag{21}$$

where $\underline{z} > 0$. Aggregate trade flows from i to j are:

$$X_{ij} = M_i \int_{\underline{z}}^{\infty} \bar{\alpha}_j p_{ij}(z)^{1-\sigma} \frac{Y_j}{P_j^{1-\sigma}} n_{ij}(z) g(z) dz. \tag{22}$$

The price index is:

$$P_i^{1-\sigma} = \sum_j M_j \int_{\underline{z}}^{\infty} \bar{\alpha}_i n_{ji}(z) p_{ji}(z)^{1-\sigma} g(z) dz, \tag{23}$$

where $n_{ji}(z)$ and $p_{ji}(z)$ are shown in Proposition 1.[27] Since the optimal fraction of consumers reached, $n_{ij}(z)$, is bounded between 0 and 1, a sufficient condition to have a finite integral is that $\theta > \sigma - 1$, as also discussed in Section 2.6. As in Chaney (2008), the number of firms is fixed to M_i, implying that in equilibrium there are profits, which equal:[28]

$$\Pi_i = M_i \sum_j \left(\frac{1}{\sigma} \int_{\underline{z}}^{\infty} \bar{\alpha}_j p_{ij}(z)^{1-\sigma} Y_j P_j^{\sigma-1} n_{ij}(z) g(z) dz - \int_{\underline{z}}^{\infty} f_{ij}(z) g(z) dz \right). \tag{24}$$

I impose a balanced current account, thus the total expenditures in each country must equal to labor income plus business profits:

[27] The assumption that the demand shocks are i.i.d. across varieties implies that, in (23), $\bar{\alpha}_i = \bar{\alpha}_i(z) \equiv \int_0^{\infty} \alpha_i(z) h_i(\alpha) d\alpha$.

[28] Note that, even in a model with free-entry of firms, there are profits in equilibrium, because entrepreneurs need to be compensated for taking on business risk. See Appendix 8.1.10 for details.

$$\sum_k X_{ki} \equiv Y_i = w_i \tilde{L}_i + \Pi_i. \tag{25}$$

Finally, the labor market clearing condition states that in each country the supply of labor must equal the amount of labor used for production and marketing (paid by foreign firms exporting to the domestic country):

$$M_i \sum_j \int_{\underline{z}}^{\infty} \frac{\tau_{ij}}{z} \tilde{\alpha}_j p_{ij}(z)^{-\sigma} Y_j P_j^{\sigma-1} n_{ij}(z) g(z) dz + \sum_j M_j \int_{\underline{z}}^{\infty} f_i n_{ji}(z) L_i g(z) dz = \tilde{L}_i, \tag{26}$$

The trade equilibrium in this economy is characterized by a vector of wages $\{w_i\}$, price indexes $\{P_i\}$ and income $\{Y_i\}$ that solve the system of equations (23), (25), (26), where n_{ij} is given by equation (15).[29]

2.5 Welfare gains from trade

I define welfare in country i as the equally-weighted sum of the welfare of workers and entrepreneurs. Since workers maximize a CES utility, their welfare is simply the real wage $\frac{w_i}{P_i}$, as in ACR. In contrast, the entrepreneurs' indirect utility is given by equation (3).[30] Then, aggregate welfare equals:

$$W_i = \frac{w_i \tilde{L}_i}{P_i} + \frac{\Pi_i}{P_i} - \tilde{R}_i, \tag{27}$$

where $\tilde{R}_i \equiv M_i \int_{\underline{z}}^{\infty} \frac{\gamma}{2} Var\left(\frac{\pi_i(z)}{P_i}\right) dG(z)$ is the aggregate "risk premium".

A limit case. As shown earlier, when the risk aversion is zero the firm optimal behavior is the same as in standard monopolistic competition models, as Melitz (2003). It is easy to show that, in the special case of $\gamma = 0$, the welfare gains after a reduction in trade costs from τ to τ' are given by:

$$\ln \hat{W}_i|_{\gamma=0} = -\frac{1}{\theta} \ln \hat{\lambda}_{ii} \tag{28}$$

where $\hat{x} \equiv \frac{x'}{x}$, λ_{ii} denotes the domestic trade share in country i and θ equals the trade elasticity. As shown by ACR, several trade models predict the welfare gains from trade

[29] Given the analytical complexity of the firm problem, finding sufficient conditions that guarantee the uniqueness of the equilibrium is difficult. However, when solved numerically, the model does not display the occurrence of multiple equilibria.

[30] This also corresponds to the Certainty Equivalent, the correct *money-metric* measure of welfare in presence of uncertainty (see Pratt (1964) and Pope et al. (1983)).

to be equal to equation (28), such as Eaton and Kortum (2002), Melitz (2003), Arkolakis et al. (2008) and Chaney (2008). Therefore, the case of $\gamma = 0$ provides the theoretical benchmark to study how departures from risk-neutrality affect the welfare gains from trade liberalization.

2.6 Two symmetric countries

To illustrate some properties of the model and to obtain a closed-form expression for the welfare gains from trade, I study the special case where there are two perfectly symmetric countries, home and foreign. Define $\bar{\alpha}$ to be the expected value of the demand shock, $Var(\alpha)$ its variance and ρ the cross-country correlation of shocks. For simplicity, I assume that $\bar{\alpha} = Var(\alpha) = 1$, and I also set $\underline{z} = 1$. I consider two opposite equilibria: one in which there is autarky, and one in which there is free trade, so $\tau_{ij} = 1$ for all i and j.[31]

2.6.1 The firm problem

Under autarky, the Diversification Index is simply the ratio between the mean and the variance of the demand shocks:

$$D_A = \frac{\bar{\alpha}}{Var(\alpha)} = 1. \tag{29}$$

Instead, under free trade the Diversification Index is

$$D = \frac{\bar{\alpha}}{Var(\alpha)\,(1+\rho)} = \frac{1}{1+\rho}. \tag{30}$$

Notice that the Diversification Index is decreasing in the cross-country correlation of demand: the larger this correlation, then the smaller the diversification benefits from selling abroad.

In Appendix 8.1, I show that in both equilibria the firm's optimal solution is:

Proposition 3. *The unique optimal $n(z)$ satisfies:*

[31] Throughout this sub-section I assume that $\gamma > \tilde{\gamma}$ (where $\tilde{\gamma}$ depends only on parameters), so that $n(z) < 1$ always for all z. This eliminates the multiplier of the upper bound. Intuitively, if the entrepreneurs are sufficiently risk averse, they always prefer to *not* reach all consumers. See Appendix 8.1.5 for more details.

$$n(z) = 0 \qquad\qquad \textit{if } z \le z^*$$
$$n(z) = \frac{D}{\gamma}\frac{\left(1-\left(\frac{z^*}{z}\right)^{\sigma-1}\right)}{r(z)} \; \textit{if } z > z^*$$

where $r(z)$ are real gross profits, as in equation (12), and the entry cutoff is:

$$z^* = \left(\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1}\frac{fP^{1-\sigma}\sigma}{\bar{\alpha}Y}\right)^{\frac{1}{\sigma-1}}. \tag{31}$$

In both equilibria, the Diversification Index is a sufficient statistics for risk in the entrepreneur's optimal decision.[32] Under free trade, D is decreasing in the cross-country correlation of demand: the more correlated is demand with the foreign country, the "riskier" the world and thus the lower the number of consumers reached. The existence of a single entry cutoff means that there is strict sorting of firms into markets, as in Melitz (2003). However, that happens only because of the perfect symmetry between the two countries, which implies that $n(z)$ is not affected by the Lagrange multipliers of the other location. In the general case of N asymmetric countries, firms do not strictly sort into foreign markets, as explained in the previous section.

2.6.2 The effect of free trade on risk diversification

I now study the effects of going from autarky to a free trade equilibrium with a symmetric country. I first investigate how free trade affects the risk-return profile of each firm. In order to evaluate the level of risk diversification attained by each firm, I follow the finance literature (see e.g. Cochrane (2009)) and compute the Sharpe Ratio, i.e. the mean/standard deviation ratio of real profits:

$$SR(z) = \frac{E\left(\frac{\pi(z)}{P}\right)}{Std\left(\frac{\pi(z)}{P}\right)}. \tag{32}$$

[32] The perfect symmetry and the absence of trade costs imply that any firm will choose the same $n(z)$ in both the domestic and foreign market. This means that either a firm enters in both countries, or in neither of the two. This feature is the reason why perfect symmetry and free trade is the only case in which I can derive an analytical expression for $n(z)$. If there were trade costs $\tau_{ij} > 1$, the optimal $n(z)$ would still depend on the Lagrange multiplier of the other destination.

Intuitively, the higher this ratio, the higher are expected real profits of firm z relative to their volatility, proxied by their standard deviation, and therefore the better the risk-return profile of the firm.[33] In the following proposition, I characterize how the firm-level Sharpe Ratio changes from autarky to free trade in general equilibrium:[34]

Proposition 4. *In going from autarky to free-trade:*

i) Conditional on surviving, the percentage change in the Sharpe Ratio of firm z is:

$$\hat{SR}(z) = \left(\frac{2}{1+\rho}\right)^{\frac{1}{2}} \frac{\left(1-(1+\rho)^{-\frac{\sigma-1}{\theta+1}}(2)^{\frac{\sigma-1}{\theta+1}}\psi\,(z)^{1-\sigma}\right)}{\left(1-(z)^{1-\sigma}\,\psi\right)} - 1 \tag{33}$$

where $\psi \equiv \gamma^{\frac{1-\sigma}{\theta+1}} M^{\frac{\sigma-1}{\theta+1}} \left(\tilde{L}\right)^{\frac{-\sigma}{(1+\theta)}} f^{1+\theta}\tilde{\sigma}$ *and* $\tilde{\sigma} > 0$ *depends only on* σ *and* θ*.*

ii) The change in the Sharpe Ratio is always increasing in z, while the effect of ρ is ambiguous.

Equation (33) highlights three effects of free trade on the ex-post Sharpe ratio. First, there is a direct diversification effect given by $\left(\frac{2}{1+\rho}\right)^{\frac{1}{2}}$: free trade increases $SR(z)$ since entrepreneurs can hedge domestic risk by exporting to the foreign country. Second, there is a competition effect, given by $(2)^{\frac{\sigma-1}{\theta+1}}$: free trade implies more competition among firms and thus lower prices, which translates into a higher entry cutoff, lower expected profits and a lower $SR(z)$. Third, there is an indirect diversification effect, given by $(1+\rho)^{-\frac{\sigma-1}{\theta+1}}$: a higher correlation of demand lowers $n(z)$, from Proposition 3, which dampens the competition effect of trade on prices, implying a smaller increase in the entry cutoff and an increase in the Sharpe Ratio.

This discussion suggests that, while a lower demand correlation improves the risk-return profile of a firm through the direct diversification effect, the indirect effect that works through the price index and the entry cutoff may actually lower the Sharpe Ratio. When the competition effect is stronger, i.e. ψ is high (e.g. when $\tilde{L}$ is small or M is high), free trade can actually backfire and *lower* the ex-post Sharpe Ratio. In other words, conditional on surviving (i.e. $n(z) > 0$ under both autarky and free trade), real profits may decrease by more than the reduction in their volatility. Not surprisingly, this is more likely to happen to smaller firms, as the Sharpe Ratio is strictly increasing in the firm productivity.

[33] Note the difference between evaluating the ex-ante riskiness of a set of potential destinations, for which one should take into account for the full covariance structure, as in equation (19), and evaluating the ex-post riskiness of a portfolio, as in equation (32).

[34] Equations (80)-(81) and (89)-(90) in Appendix (8.1) solve for the equilibrium endogenous variables in both autarky and free trade. I use that result to incorporate the general equilibrium response of prices to free trade into the Sharpe Ratio of the firms.

2.6.3 Welfare gains from trade

I now characterize the effect of free trade on aggregate welfare. Recall from equation (27) that welfare can be written as real income minus the aggregate risk premium. In the Appendix I prove the following result:

Proposition 5. *Welfare gains of going from autarky to free trade are given by:*

$$\ln \hat{W}_i = \ln \hat{W}_i|_{\gamma=0} \cdot \frac{\theta}{\theta+1} - \frac{1}{\theta+1} \ln(1+\rho) \tag{34}$$

Therefore, welfare gains are higher than in a risk neutral model if and only if $\rho < (2)^{-\frac{1}{\theta}} - 1$.

Proposition 5 shows that free trade has three distinct effects on welfare.[35] The first is given by the reduction in the price index due to more competition from foreign firms, the standard channel present in trade models with risk neutrality, i.e. ACR. Since with autarky the trade share is 1, with two symmetric countries $\ln \hat{W}_i|_{\gamma=0}$ simply equals $\frac{1}{2}$. The second effect, given by $\frac{1}{\theta+1} \ln(1+\rho)$, is the general equilibrium counterpart of the direct diversification effect highlighted in Proposition 4. When the correlation of demand with the foreign country is low, firms increase their exports in order to hedge their domestic demand risk, according to Proposition 3. This implies tougher competition among firms, which leads to lower prices and higher welfare gains. The third channel, given by $\frac{\theta}{\theta+1}$, dampens the competition effect of trade on welfare. Lower prices induced by free trade increase the expected real profits, but also their variance, generating a feedback effect that lowers the number of consumers reached by the firms, reducing the competition and increasing the price index.

The combination of these general equilibrium effects implies that my model predicts larger welfare gains from trade than models with risk neutral firms as long as the correlation of demand is sufficiently low. For example, setting $\theta = 5$, a typical value for this parameter, welfare gains in my model are higher than in ACR as long as $\rho > -0.13$.[36]

[35] Note that both the risk aversion and the mean and variance of the shocks do not affect the welfare gains from trade only because of the perfect symmetry assumed between the two countries. In the general case, they do affect the gains, as shown in Section 5.

[36] In Appendix 8.1.10, I also solve an extension of the model that allows for free entry of firms, similarly to Melitz (2003). Since entrepreneurs are risk-averse, they enter until the expected utility of running a firm equals a fixed entry cost. In such setting, the gains from variety brought by the change in the number of entrants push the aggregate welfare gains up and higher than in the model with fixed entry. Specifically, the welfare gains equal $\ln \hat{W} = \ln \hat{W}_i|_{\gamma=0} - \frac{1}{\theta} \ln(1+\rho)$, which are higher than in Proposition 5 since $|\rho| < 1$. Therefore, also with free-entry the welfare gains from trade are higher than in ACR as long as the demand correlation is sufficiently low.

To sum up, the theoretical results discussed in this section highlight the importance of the sign and magnitude of the cross-country covariance of demand in shaping i) the direction of trade flows, ii) the risk diversification benefits of international trade, and iii) the welfare gains from trade. Therefore, the estimation of the covariance matrix of demand is crucial for the quantification of the effective benefits of international trade on risk diversification and welfare. To this end, in the following section I develop a methodology to estimate the first and second order moments of the demand shocks, which will be used, together with the other parameters, to assess the relevance of the risk diversification channel in Section 5.

3 Estimation

The analysis mostly relies on a panel dataset on domestic and international sales of Portuguese firms to 210 countries, between 1995 and 2005. These data come from Statistics Portugal and roughly aggregate to the official total exports of Portugal. I merged this dataset with data on some firm characteristics, such as number of employees, total sales and equity, which I extracted from a matched employer–employee panel dataset called Quadros de Pessoal. I also merged the trade data with another dataset, called Central de Balancos, containing balance sheet information, for all Portuguese firms from 1995 to 2005.[37] I also use data on manufacturing trade flows from the UN Comtrade database as the empirical counterpart of aggregate bilateral trade in the model, complemented with data on manufacturing production from WIOD and UNIDO (see Dietzenbacher et al. (2013)), all for 2005.[38]

From the Portuguese trade dataset, I consider the 10,934 manufacturing firms that, between 1995 to 2005, were selling domestically and exporting to at least one of the top 34 destinations served by Portugal.[39] Trade flows to these countries accounted for 90.56% of total manufacturing exports from Portugal in 2005. I exclude from the analysis foreign firms' affiliates, i.e. firms operating in Portugal but owned by foreign owners, since their exporting decision is most likely affected by their parent's optimal strategy. The universe of Portuguese manufacturing exporters is comprised of mostly small firms, the average number of destinations served was 5 in 2005, and the average export share was 30%.

[37] I describe these datasets in more detail in Appendix 8.2.

[38] I use data on bilateral trade flows and domestic sales from the TradeProd database assembled by CEPII. See De Sousa et al. (2012).

[39] I first select the top 45 destinations from Portugal by value of exports, and then I keep the countries for which there is data on manufacturing production, in order to construct bilateral trade flows. The remaining set consists of 34 countries. See the list of countries in Table 3 in Appendix 8.2.

3.1 Estimation of the Diversification Index

The first step of the empirical analysis is to estimate the country-level measure of risk, the Diversification Index, which requires estimating the means and covariance matrix of the demand shocks. I use the years 1995-2004 to estimate these moments. This follows a long tradition in empirical finance that uses historical data to identify the moments of risky assets (see e.g. Cochrane (2009)), and a recent empirical literature that identifies demand shocks as innovations from the growth rate of sales (see e.g. Di Giovanni et al. (2014)). Using time-series variation of firms' sales also enhances the power of the estimation, resulting in tighter confidence intervals of the point estimates. I then treat the sample means and covariances of the estimated shocks as estimates of the corresponding population moments, which firms take as given when they choose their risk diversification strategy in 2005.

The estimation of the moments of the demand shocks entails two steps. First, I use the firm-destination level equation for sales to recover the unobserved demand shocks in each year of the sample. Then I use these shocks to compute the means and covariance matrix of demand across countries. For brevity, I define the variables $\tilde{x} \equiv \ln(x)$ and $\Delta x \equiv x_t - x_{t-1}$.

3.1.1 Step 1: Identification of demand shocks

I assume that the structural model in Section 2 is the Data Generating Process:

Assumption 4a *In every year t of the sample period, the world economy is generated by the model of Section 2, and all parameters are constant throughout the sample period.*

Assumption 4a and equation (18) imply that, in every year t, the log-sales of Portuguese firm s to country j (including Portugal itself) can be written as:

$$\tilde{x}_{js}^t = \delta^t + \delta_s^t + \delta_j^t + \varepsilon_{js}^t \tag{35}$$

where $\delta^t \equiv (1-\sigma)\ln\left(\frac{\sigma}{\sigma-1}w\right)$, $\delta_s^t \equiv (\sigma-1)\ln(z_s)$, $\delta_j^t \equiv (1-\sigma)\ln\left(\tau_j^t\right) + \ln\left(\frac{Y_j^t}{P_j^{t1-\sigma}}\right)$, and

$$\varepsilon_{js}^t = \tilde{\alpha}_{js}^t + \tilde{n}_{js}^t. \tag{36}$$

Under Assumption 4a, and the fact that $\tilde{n}_{js}^t$ does not depend on the realization of the shocks as shown in Proposition 1, taking a first-difference across time eliminates $\tilde{n}_{js}^t$:

$$\varepsilon_{js}^t - \varepsilon_{js}^{t-1} = \tilde{\alpha}_{js}^t + \tilde{n}_{js}^t - \tilde{\alpha}_{js}^{t-1} - \tilde{n}_{js}^{t-1} = \Delta\tilde{\alpha}_{js}^t \tag{37}$$

Lastly, Assumption 3b, i.e. the orthogonality between firm-level productivity and demand shocks, implies that I can estimate the structural sales equation as a fixed-effect OLS regression:

$$\Delta\tilde{x}_{js}^{t} = \delta_s^t + \delta_j^t + \mathbf{Z}_{js}^{t}{}'\beta + \eta_{js}^t \tag{38}$$

where $\mathbf{Z}_{js}^t$ is a set of firm-destination specific variables. The structural specification in (38) controls for firm-specific productivity shocks that are common to all destinations, by means of the firm fixed-effect, and by destination-specific characteristics, such as trade costs and real income, by means of the destination fixed effect. Accordingly, the structural residual η_{js}^t in equation (38) identifies $\Delta\tilde{\alpha}_{js}^t$, the *log-change* in demand shocks. Effectively, this approach identifies demand shocks as innovations from the growth rate of domestic and international sales, similarly to Gabaix (2011), Castro et al. (2010) and Di Giovanni et al. (2014).

The vector $\mathbf{Z}_{js}^t$ includes log changes in firm-year-specific investment, capital intensity, and productivity (proxied by revenues per worker), all interacted with country-specific dummies.[40] Including the firms' investment rates and capital intensity controls for the possibility that firms endogenously respond to demand shocks in a market by changing their capital structure, as highlighted in Friedrich et al. (2018), thus affecting $\Delta\tilde{x}_{js}^t$.[41] Controlling for the firm productivity interacted with country dummies accounts for the evidence, shown in Mayer et al. (2016), that foreign demand shocks may induce changes in firms' productivity, thus affecting sales.

Lastly, note that the specification in (38) controls for some features of the firms' behavior that the model does not capture explicitly. In fact, the firm and destination fixed effects control for endogenous markups (see De Loecker and Warzynski (2012)), while the first differencing absorbs the time-invariant component of pricing-to-market, which is firm-destination specific, and any time-invariant trade costs that are firm-destination specific.[42]

[40] These are the variables that have the best coverage across the firms' balance sheets from the Central de Balancos dataset. Given the dimensionality of the data, replacing $\mathbf{Z}_{tjs}$ with an interaction of fixed effects would not be computationally feasible.

[41] Note that adding investment interacted with destination dummies should also control for the spending in marketing and distributional activities that exporters make in order to learn the demand in each market. While in principle these should be absorbed, given Assumption 4a, by the year-to-year first difference, any time-varying component of that may affect sales. Using investment data at the firm level is an imperfect proxy for these expenditures, but is the best I can do given the data availability.

[42] De Loecker and Warzynski (2012) show that, in a very general environment, markups can be written as a function of destination-level revenue shares and industry-level output elasticity. In logs, these are absorbed in the destination and firm fixed effects.

3.1.2 Step 2: Estimation of $\tilde{\alpha}$ and Σ

Once the demand shocks are structurally recovered, in order to estimate the covariance matrix I make the following identifying assumption.

Assumption 4b *Demand shocks are independently and identically distributed across firms and time.*

The i.i.d assumption is useful because it allows to exploit both time-series and cross-sectional variation in the residuals. Since firms independently draw the shocks from the same distribution, to compute the covariance matrix of the *log-changes* of the shocks, for every pair of destinations j and k, I stack the residuals $\hat{\eta}^t_{js}$ and $\hat{\eta}^t_{ks}$ for each Portuguese firm s that was selling to both markets j and k in year t. Effectively, each firm-year pair is a vector of (at most) N correlated demand shocks draws. I compute an unbiased estimate of the covariance between country j and k as:

$$Cov\left(\Delta\tilde{\alpha}_j, \Delta\tilde{\alpha}_k\right) = \frac{1}{S_{jk}-1}\sum_{s=1}^{S_{jk}}\left(\hat{\eta}_{js} - \bar{\eta}_j\right)\left(\hat{\eta}_{ks} - \bar{\eta}_k\right) \tag{39}$$

where $\hat{\eta}_{ks}$ and $\hat{\eta}_{js}$ are the residuals from equation (35), S_{jk} is the number of observations (total number of firms-year pairs that sell to both markets j and k) and $\bar{\eta}_i \equiv \frac{1}{S_{jk}}\sum_{s=1}^{S_{jk}}\hat{\eta}_{is}$ for all i.

Note that, since the expectation and the covariance are linear operators, an alternative approach would be to compute the covariances for each Portuguese exporter using only time-series variation, and then, for each bilateral pair, compute the average covariance across firms (see Appendix 8.1.12 for a proof). In the robustness section, I show that the resulting point estimates are indeed very similar.[43]

To compute the covariance matrix of the *log* of the shocks, I use the assumption that the shocks are i.i.d. across time to obtain (see Appendix 8.1.11 for the proof):

$$\tilde{\Sigma}_{jk} \equiv Cov\left(\tilde{\alpha}_j, \tilde{\alpha}_k\right) = Cov\left(\Delta\tilde{\alpha}_j, \Delta\tilde{\alpha}_k\right)/2. \tag{40}$$

Finally, in order to recover the covariance matrix from the covariance of the log shocks, $\tilde{\Sigma}_{jk}$, I have to make a functional form assumption about the distribution of the demand shocks:[44]

[43]However, since the number of years is much smaller than the number of Portuguese exporters selling to each bilateral pair, such alternative approach lowers the degrees of freedom and inflates the standard errors.

[44]The need to make a functional form assumption on the distribution of the shocks arises from the fact that I recover the growth rates of the demand shocks, and not directly their levels. If I were to estimate the

Assumption 4c *Demand shocks are distributed as*

$$log\alpha(z) \sim N\left(\mathbf{0}, \tilde{\Sigma}\right)$$

The log-normality assumption has been traditionally used in empirical asset pricing to model asset returns (see Singleton (2006) and Cochrane (2009)), but recently also in the quantitative trade literature to model demand shocks (see Eaton et al. (2011), Nguyen (2012) and Crozet et al. (2012)), for its tractability and because it has a positive domain (demand shocks cannot be negative).[45]

Using the properties of the normal distribution, I obtain the covariance of the *level* of the shocks as:

$$\Sigma_{jk} \equiv Cov\left(\alpha_j, \alpha_k\right) = \exp\left(\frac{1}{2}\left(Var\left(\tilde{\alpha}_j\right) + Var\left(\tilde{\alpha}_k\right)\right)\right)\left[\exp\left(Cov\left(\tilde{\alpha}_j, \tilde{\alpha}_k\right)\right) - 1\right] \tag{41}$$

for all j and k, and the expected value as:

$$\bar{\alpha}_k \equiv E\left[\alpha_k\right] = \exp\left(\frac{1}{2}Var\left(\tilde{\alpha}_k\right)\right) \tag{42}$$

for all k.

3.1.3 Results and descriptive statistics

I implement the estimation methodology described above using data on Portuguese firm-level domestic and international sales from 1995 to 2004. Given the several fixed effects and interaction terms, I do not report the results of the regressions for brevity, but it is interesting to note that the R^2 is around 0.5. This suggests that 50% of the variation in firms sales is driven by firm-destination demand shocks, a striking number but in line with the existing empirical evidence (see e.g. Eaton et al. (2011), Di Giovanni et al. (2014) and Hottman et al. (2015)).

Table 1 reports some summary statistics on the estimated moments and their bootstrapped standard errors, while Figure 2 plots the distribution of the estimated covari-

structural equations for sales in levels, I could not identify the demand shocks separately from $\tilde{n}^t_{js}$.

[45]Note that in the data I cannot back out the mean of $\tilde{\alpha}_j$, because I only observe the residuals ε_{js}, and $\varepsilon_{js} = \tilde{\alpha}_{js} + \tilde{n}_{js}$. For this reason, I impose $E\left[\tilde{\alpha}_j\right] = 0$ in Assumption 4d. However, as shown in equation (42), the mean of α_j is *not* equal to zero.

ances.[46] The covariances range from -1.73 to 2.49, with a median of -0.04, while variances are typically larger, and standard errors are quite small. Such vast heterogeneity in the variance and covariances across countries suggests the potential for risk diversification. In fact, Table 2 documents that more remote destinations offer better risk diversification benefits, since bilateral demand covariances are significantly negatively correlated with bilateral distance. This highlights the trade-off that exporters face. On one hand, the traditional profit maximization motive gives firms incentives to sell to nearby or "similar" destinations, because of lower trade costs. On the other hand, the risk minimization motive gives firms incentives to sell to remote countries, which could hedge against domestic fluctuations in demand.

Using the estimated Σ and $\bar{\alpha}$, I compute the country-level Diversification Index, using equation (19). Table 3 reports the Diversification Indexes for the countries in the sample, together with their bootstrapped standard errors. Column 1 in Table 4 instead reports some summary statistics. Standard errors are relatively small, suggesting that the Diversification Indexes are quite precisely estimated. Figure 3 shows that countries with a lower GDP per capita in 2005 had a lower Diversification index, consistent with evidence in Koren and Tenreyro (2007) that poorer countries are typically riskier than richer countries.

3.1.4 Robustness

I now examine the robustness of the estimates to some of the assumptions made for the empirical methodology just described, and show that they do not significantly affect the results.

No supply shocks. I first investigate how controlling for the firm-destination supply shocks affects the estimates. Since data on capital intensity and investment rates is missing for some firms in the sample, the baseline approach could generate a selection bias. To this end, I re-estimate equation (38) without the controls $\mathbf{Z}_{js}^{t}$, and compute the Diversification Index as explained above. Reassuringly, Figure 4 documents that omitting the controls for supply shocks does not significantly alter the estimated Diversification Index.

Firm-level covariances. As discussed in the previous section, an alternative approach would be to compute the covariances for each Portuguese exporter using only time-series variation, and then, for each bilateral pair, compute the average covariance across firms.

[46] Specifically, for each iteration, I sample with replacement for each pair of countries from the existing structurally recovered demand shocks and use the steps shown in the previous section to compute the covariance matrix (and the Diversification Index). I repeat this procedure 1,000 times and compute the standard deviation.

Figure 4 highlights that such approach would produce a Diversification Index that is highly correlated with the baseline one.[47]

Aggregate shocks. The empirical methodology described above rules out the presence of country-specific demand shocks, such as a monetary tightening or exchange rate fluctuations, that affect the sales of all exporters to a certain destination. As discussed in Section 2, introducing aggregate shocks would significantly complicate the computational effort needed to solve the model. However, here I empirically investigate whether allowing for aggregate shocks substantially alters the riskiness of a country. In Appendix 8.1.13, I show how to use the structural equation (38) to back out aggregate shocks. Figure 4 shows that the baseline DI and the one augmented with macro shocks are highly correlated. Therefore, while macro shocks are certainly an important component of the overall demand that exporters face, the baseline setting seems to be a good approximation of the demand risk faced by exporters.[48]

Learning. There is evidence that, in the short run, firms sequentially enter different markets to learn their demand behavior, and often exit very soon (see Albornoz et al. (2012), Ruhl and Willis (2014) and Berman et al. (2015b) among others). This short-run learning behavior may contaminate the estimation of the moments. For this reason, I re-estimate the moments of the distribution considering only "established" firm-destination pairs, i.e. exporters selling to a certain market for at least 5 years, for which the learning process is most likely over. Figure 4 shows that using only established firm-destination pairs does not significantly affect the estimates.[49]

Sectoral covariances. In order to maximize the number of observations used to compute the moments, and therefore enhance the precision of the estimates, I have assumed that the distribution of the shocks is the same for all sectors. However, this assumption may be too restrictive. To compromise between the level of disaggregation and the credibility of the estimates, I compute the moments of the distribution separately for 7 manufacturing sectors.[50] Columns 2-8 in Table 4 report some summary statistics, showing the

[47] Using this approach, however, leads to higher standard errors, since each firm-level covariance is computed with fewer observations. This is the reason why I do not use these estimates as baseline.

[48] This result is consistent with the recent empirical evidence that firm-destination specific shocks, rather than aggregate shocks, account for a large fraction of the variation in firms' sales across countries (see e.g. Di Giovanni et al. (2014) and Hottman et al. (2015)).

[49] On one hand, having at least 5 years of observations for each firm-destination pair allows to have more accurate estimates of the moments of the distribution. On the other hand, this approach selects firms that have most likely faced positive demand shocks, generating a selection bias. Sales by established exporters represent 89% of total exports volume throughout the sample period.

[50] Specifically, I assign each Portuguese firm to a given sector based on the main product the firm produces. The sectors are: Food, Beverages and Tobacco; Textile Products; Wood and Products of Wood and Cork; Non-Metallic Minerals; Metals; Machinery; Others.

heterogeneity of the riskiness across sectors and the larger magnitude of the bootstrapped standard errors.[51]

3.2 Estimation of risk aversion

To estimate the firms' risk aversion, I follow Allen and Atkin (2016) and directly use the firms' first order conditions. For simplicity, I assume that marketing costs are sufficiently high so that there is no Portuguese firm selling to the totality of consumers in any country, implying that $\mu_j(z) = 0$ for all j and z.[52] For each destination j where firm z is selling to (and thus for which $\lambda_j(z) = 0$), the FOC can be written as:

$$\underbrace{\bar{\alpha}_j r_j(z)}_{\text{marginal expected gross profits}} = \underbrace{w_j f_j L_j / P}_{\text{marketing cost}} + \gamma \underbrace{\sum_s r_j(z) n_s(z) r_s(z) Cov(\alpha_j, \alpha_s)}_{\text{marginal variance of gross profits}} \tag{43}$$

where the left hand side is the *marginal* expected benefit of adding one additional consumer in country j, while the right hand side is its marginal cost, which is the sum of the marketing costs plus the *marginal* variance. The intuition for equation (43) is that the risk aversion regulates the slope of the relationship between the marginal expected profits and their marginal variance. The higher γ, the more entrepreneurs want to be compensated for taking additional risk, and thus higher marginal variance of profits must be associated with higher marginal expected profits.

I exploit this relationship to identify the risk aversion in the data. In Appendix 8.1.14, I show how to recover the unobserved *marginal* moments from the observed actual moments of gross profits, without requiring any further assumption. Then I can write equation (43) as a fixed-effect OLS regression:

$$MargE[GrossProfits_{jz}] = d_j + \gamma \; MargVar[GrossProfits_{jz}] + \varepsilon_{jz} \tag{44}$$

where $d_j \equiv w_j f_j L_j / P$ is a destination fixed effect that controls for country-specific marketing costs, and where ε_{jz} is not a structural residual, but simple econometric error.

[51] In addition, the sparsity of the firm-country trade matrix implies that, for some country pairs within a sector, there is no Portuguese firm selling to both markets. In such cases, I set the covariance to be the same as the corresponding aggregate covariance computed in the baseline. The presence of missing covariances, together with less precise estimates, is the reason why I do not use sectoral covariances in the baseline.

[52] Given the median size of Portuguese firms, this seems a reasonable assumption.

I use data on observed gross profits of Portuguese firms from 1995 to 2004 to compute the variables in equation (44), which is then implemented with an OLS regression. The regression is weighted by firms' total sales in 2005, both to minimize the influence of outliers and to estimate the appropriate average effect for the counterfactual analysis. Table 5, Column 1, shows that there is a positive and statistically significant relationship between the expected marginal profits and their variance, with a risk-aversion parameter of 0.41. The point estimate is a bit lower - 0.28 - if I exclude domestic sales (Column 2), and it remains the same if the sample includes only established exporters (Column 3).[53]

It is important to note that the magnitude of the risk aversion parameter *per se* is not sufficient to assess the level of risk aversion of firms. Instead, it is appropriate to look at the implied *risk premium*, which expresses the risk aversion in relation to the magnitude of the risk taken, i.e. the so-called "size of the gamble". For a CARA utility, the risk premium, expressed as percentage of the size of the gamble, can be approximated as

$$\phi \approx \frac{ln(0.5(e^{-\gamma g} + e^{\gamma g}))}{\gamma g} \tag{45}$$

where γ is the risk aversion and g is the size of the gamble.[54] Following the empirical literature, I proxy g by the standard deviation of firm sales. Then, a risk aversion coefficient of 0.41 implies an average risk premium of $\phi = 74\%$, which suggests a high level of risk aversion for exporters.[55]

3.3 Calibration

I calibrate the model in the year 2005. For the full calibration, I introduce a non-tradeable good produced, under perfect competition, with labor and unitary productivity. Consumers spend a constant share ξ of their income on the manufacturing tradeable goods, and a share $1 - \xi$ on the non-tradeable good. I assume that the demand for the non-tradeable good is non-stochastic. I set $\xi = 0.23$, which is the median value of the consumption shares on manufacturing estimated by Caliendo and Parro (2014).

[53] This number is in the same ballpark of the estimate provided in Allen and Atkin (2016), which use the returns of crops to estimate Indian farmers' risk aversion.

[54] In other words, the risk premium is the fraction of the potential loss in a gamble that an agent is willing to pay to avoid facing that gamble. See Babcock et al. (1993) for a discussion.

[55] Note that if firms hedged profits fluctuations with financial derivatives, they would be able to reduce the volatility of their cash-flows, implying an upward bias for the risk aversion. However, this concern is mitigated by the evidence that hedging practices are not widespread among Portuguese firms (see Iyer et al. (2014)), and by the fact that the sample is composed mostly by small firms, whose access to financial markets is limited (see Gertler and Gilchrist (1994), Hoffmann and Shcherbakova-Stewen (2011)).

I set the elasticity of substitution to $\sigma = 4$, consistent with estimates of an average mark-up of 33% in the manufacturing sector (see Christopoulou and Vermeulen (2012) and Broda and Weinstein (2006)). I set the technology parameter θ to 5, which is around the value found by a large empirical literature (see Head and Mayer (2013) and Simonovska and Waugh (2014)). I proxy $\tilde{L}_j$ with the total number of workers in the manufacturing sector, while M_j is the total number of manufacturing firms, both from UNIDO.

I follow Arkolakis (2010) and assume that the cost to reach a certain number of consumers is lower in markets with a larger population, so the per-consumer cost equals:

$$f_j = L_j^{\alpha-1}. \tag{46}$$

I set $\alpha = 0.42$, as calibrated in Arkolakis (2010).

Finally, I calibrate the matrix of trade costs in order to match the observed matrix of international trade shares in the manufacturing sector. Appendix 8.3 describes in detail the Simulated Method of Moments algorithm used. Figure 5 shows that the model correctly reproduces the heterogeneity in trade shares that we observe in the data in 2005.

Since the model reasonably matches the amount of trade observed in the data, I can compute the fraction of trade flows that is explained by risk diversification. To this end, I set the risk aversion to zero and, holding constant the other parameters, I compute the predicted trade shares, λ_{ij}^N. Then, 1 minus the R^2 of a regression of the calibrated trade shares on λ_{ij}^N represents the fraction of variation in the trade flows explained by risk diversification.[56] Figure 6 plots λ_{ij}^N against the calibrated trade shares, and the associated R^2 of 0.85 suggests that risk diversification explains 15% of the observed trade flows.

4 Testing the model predictions

Armed with the estimated Diversification Index, I first perform reduced-form tests to investigate whether the predictions of the model hold in the data. In particular, I run a set of "risk-augmented" gravity regressions at the firm and country level. Then, I use the fully calibrated model to perform a structural test of the model predictions.

[56] Using the actual trades shares or the calibrated ones does not make a difference, since the R^2 between them is 0.996.

4.1 Reduced-form evidence

Firm-level extensive margin. Proposition 2 states that the probability that a firm enters a market, i.e. that $n_{ij}(z) > 0$, is increasing in the market's Diversification Index, conditional on trade barriers and country characteristics. To test this prediction, I run the following "gravity" regression:

$$Pr\left(x_{sj} > 0\right) = \beta_0 + \delta_s + \mathbf{X}_j'\beta_1 + \beta_2 ln\left(D_j\right) + \varepsilon_{js}, \tag{47}$$

where x_{sj} are the sales of Portuguese firm s to foreign country j in 2005, δ_s is a firm fixed effect, X_j is a set of destination-specific controls, which include standard gravity variables, as well as log of GDP, log of population, an index of remoteness that proxies for multilateral resistance (as in Bravo-Ortega and Giovanni (2006) and Frankel and Romer (1999)), and trade openness.[57] Standard errors are clustered at the country level, to control for unobserved correlation across firms within a destination. All regressions are weighted by firms total sales in 2005, to minimize the influence of outliers.

Column 1 in Table 6 shows that, as predicted by the model, Portuguese firms are more likely to enter in countries with higher D, even after controlling for trade barriers and destination specific characteristics. The coefficient β_2 is statistically significant at the 1% level, and it implies that one standard deviation increase in the Diversification Index raises the probability of exporting by 3%. Column 2 estimates a Probit model that confirms the findings of Column 1. Moreover, since I cannot directly control for destination fixed effects, given the presence of $ln(D_j)$, I follow Head and Mayer (2013) and Baker and Fortin (2001) and use a more efficient two-step methodology, in which I first regress $Pr\left(x_{sj} > 0\right)$ on firm and destination fixed effects, and then regress the estimated destination fixed effect on $ln(D_j)$ and the other controls using feasible generalized least squares (FGLS). Column 3 shows that the coefficient of $ln(D_j)$ in the second stage is similar to that of Column 1 but it has a much lower standard error.

I also investigate how the different components of the Diversification Index affect the results. Column 4 shows that both the Sharpe Ratio, i.e. the mean/standard deviation ratio, and the average covariance with all other markets significantly affect the firms' exporting decision to a destination, suggesting that both are key drivers of the extensive margin of trade.

[57]The gravity controls include the distance from Portugal, average ad-valorem tariff rate (from UNCTAD), dummies for common language, common legal origins, contiguity, common currency, WTO membership, regional trade agreement, and a dummy for European countries to control for the effect of the European Union on trade flows.

Firm-level intensive margin. Proposition 2 states that, conditional on entering a market, firms tend to sell more to countries with a high Diversification Index, conditional on trade barriers and country characteristics. To test this prediction, I run the following "gravity" regression:

$$ln\left(x_{sj}\right) = \beta_0 + \delta_s + \mathbf{X}_j'\beta_1 + \beta_2 ln\left(D_j\right) + \varepsilon_{js}, \tag{48}$$

where the controls are the same as for the extensive margin regressions. Column 1 in Table 7 shows that results from the OLS regression, and confirms the prediction of the model: conditional on entry, firms export more to countries with higher Diversification Index, even after controlling for trade barriers and destination specific characteristics. Column 2 performs, instead, a Poisson Pseudo-Maximum Likelihood estimation to control for heteroskedasticity as in Silva and Tenreyro (2006), while Column 3 performs a two-stage Heckman estimation to control for selection of firms into exporting.[58] Columns 4 performs the two-step methodology implemented also for the extensive margin. In all specifications, results are similar to the baseline. Column 5 shows that both the destination's Sharpe Ratio and the average covariance with all other markets significantly affect the intensive margin of trade.

Country-sector intensive margin. As an external validation of the model's predictions, I also estimate the intensive margin regression using trade flows at the country level. Specifically, I run the following gravity regression

$$ln\left(x_{ij}\right) = \beta_0 + \delta_i + \mathbf{X}_{ij}'\beta_1 + \mathbf{X}_j'\beta_2 + \beta_3 ln\left(D_j\right) + \varepsilon_{ij}. \tag{49}$$

where x_{ij} are aggregate manufacturing trade flows from country i to j, with $i \neq j$, in 2005, from UN Comtrade. δ_i is an exporter fixed effect, $\mathbf{X}_j$ is a set of destination-specific controls, such as log of GDP, log of population, remoteness, trade openness, while $\mathbf{X}_{ij}$ is a set of bilateral gravity variables, including distance, log of bilateral tariffs, dummies for common language, common legal origins, contiguity, colonial links, common currency, WTO membership.[59] Column 1 in Table 8 estimates equation (49) with OLS, Column 2 with PPML, Column 3 implements the two-steps methodology as in Head and Mayer (2013).[60]

[58] As in Helpman et al. (2008b), I use the number of procedures to start a business, from Djankov et al. (2002), as the variable that satisfies the exclusion restriction, i.e. it affects selection into foreign markets but does not affect the second stage regression.

[59] For each pair of countries, I compute an average of the ad-valorem tariffs applied in 2005 within the manufacturing sector. Data are from UNCTAD.

[60] Following Head and Mayer (2013) and Baker and Fortin (2001), I additionally control for $\bar{\mathbf{X}}_j \equiv \frac{1}{N}\sum_i \mathbf{X}_{ij}$, the average across exporters of each export-importer variable. Standard errors are clustered at the importer level.

All three columns display a positive and significant coefficient of $ln\left(D_j\right)$, suggesting that the Diversification Index affects trade flows not only for Portuguese firms, but instead for firms from all origins. The elasticity of trade flows with respect to D_j is around 0.15, while the elasticity with respect to distance of -1, as typical found in the empirical literature (see Head and Mayer (2013)). Lastly, Column 4 shows that both the destination's Sharpe Ratio and the average covariance with all other markets significantly affect trade flows at the country level.

One drawback of the previous regressions is that I cannot directly for destination fixed effects, given the presence of $ln\left(D_j\right)$. This issue is however overcome when I exploit the heterogeneity of the Diversification Index estimated at the sector-country level to implement the following gravity regression:

$$ln\left(x_{ijk}\right) = \beta_0 + \delta_i + \delta_j + \mathbf{X}_{ij}'\beta_1 + \beta_2 ln\left(D_{jk}\right) + \varepsilon_{ijk}, \tag{50}$$

where i is the exporter, j is the importer, k is the manufacturing sector, and $\mathbf{X}_{ij}$ is the same set of variables as before. Column 5 in Table 8 shows that the coefficient on $ln\left(D_{jk}\right)$ is similar to the baseline coefficient and is statistically significant at the 1% level, with an elasticity of trade flows with respect to the Diversification Index of 0.11. Lastly, Column 6 documents that the Sharpe Ratio and the average covariance with other destinations also significantly determine the intensity of trade flows across countries.

4.2 Untargeted moments

Having tested the predictions of the model with a reduced-form approach, I now use the structure of the model more intensely, to test its ability to reproduce some salient features of the data.

Entry of firms. As discussed in the theoretical section, Proposition 1 implies that the firm's entry decision in a market (that is, whether $n > 0$) does not depend on a market-specific entry cutoff. Therefore, firms' sorting into exporting is not strictly hierarchical, as in traditional trade models with fixed costs, such as Melitz (2003) and Chaney (2008). In these models, as well as in my model with no risk aversion, any firm selling to the $k+1$ most popular destination necessarily sells to the k most popular destination as well, since that has a lower entry cutoff.

Table 9 reports, for each of the top destinations from Portugal, the fraction of exporters that are strictly sorted. Specifically, for each destination j, I consider all Portuguese exporters selling there, and compute the fraction of firms selling to all destinations more popular than j. For instance, the table suggests that only 23% of Portuguese firms that in 2005 were exporting to US, the 7th most popular destination, were also exporting to all the top 6 destinations. Across the top 7 destinations, the fraction of strictly sorted firms in the data is 54%, it is 76% in my model with risk aversion, and it is by construction 100% in a Melitz (2003) model.[61] Therefore, my model is able to somewhat reproduce this feature of the data, even without assuming that marketing costs are heterogeneous across firms, as in Eaton et al. (2011).

Sales distribution. Propositions 1 and 2 suggest how firms, following their global diversification strategy, may optimally reach only few consumers in a given market, and thus export small amounts. I test this feature of the model by plotting, in Figure 7, the distribution of sales, across all destinations, of Portuguese small firms. The graph suggests that the model with risk aversion is able to reproduce very well the left tail of the observed distribution. In contrast, the model with $\gamma = 0$, which corresponds to a standard fixed cost model such as Melitz (2003) and Chaney (2008), cannot predict the existence of small exporters, because of the presence of fixed costs of entry.[62]

5 Counterfactual Analysis

Armed with the calibrated model, I now conduct two counterfactual exercises to understand better the economic mechanisms of the model and to quantify the impact of risk diversification on welfare.

5.1 Welfare Gains from Trade

Following Costinot and Rodriguez-Clare (2013), I focus on an important counterfactual exercise: moving from autarky. Formally, starting from the calibrated trade equilibrium in 2005, I assume that variable trade costs in the new equilibrium are such that $\tau_{ij} = +\infty$ for all pair of countries $i \neq j$. All other structural parameters are the same as in the initial equilibrium. Once I solve the equilibrium under autarky, I compute the welfare gains associated with moving from autarky to the observed equilibrium. The presence of a non-tradeable sector in the calibrated model implies that welfare is slightly different

[61] Evidence that exporters are not strictly sorted has been shown also by Eaton et al. (2011) and Armenter and Koren (2015), among others.

[62] Arkolakis (2010) also shows that a model with convex marketing costs is able to replicate the observed left tail of the sales distribution, while a fixed cost model cannot.

from the baseline model:

$$W_i = \tilde{\xi} \left(\frac{w_j}{P_j} \right)^{\xi} \tilde{L}_i + \frac{\Pi_i}{P_i} - RP_i \tag{51}$$

where $\tilde{\xi} \equiv (\xi)^{\xi} ((1-\xi))^{1-\xi}$. In addition, I compute the welfare gains in the limit case of no risk aversion, $\gamma = 0$, which is isomorphic to the gains predicted by the large class of models considered in ACR.[63]

Figure 8 shows that aggregate welfare gains are typically decreasing in the domestic trade share, as in ACR, but they can deviate substantially from risk neutral models. The overall numbers are small because the model includes a large non-tradeable sector, while the trade shock is only in the manufacturing sector. Some countries, like Ireland, Singapore and Germany, gain substantially more from trade than in ACR, while for other countries, such as Canada and Czech Republic, the gains are very close to ACR. In relative terms, welfare gains in my model are, for the median country, 16% higher than ACR.[64] Therefore, the "pro-competitive" effect of risk diversification on welfare is quantitatively relevant.

I now use the insights provided by the theory to investigate the determinants of the welfare gains from trade. Using equation (51), I decompose the welfare gains in my model as

$$d\ln W_i = \underbrace{\frac{\tilde{\xi} \left(w_j/P_j \right)^{\xi} \tilde{L}_i}{W_i} d\ln \left(\left(w_j/P_j \right)^{\xi} \right)}_{\text{workers' gains}} + \underbrace{\frac{\Pi_i/P_i}{W_i} d\ln \left(\Pi_i/P_i \right) - \frac{\tilde{R}_i}{W_i} d\ln \tilde{R}_i}_{\text{entrepreneurs' gains}}. \tag{52}$$

The first term reflects the gains that are accrued by workers, since their welfare is simply given by the real wage. The second term in (52) represents the entrepreneurs' welfare gains, which in turn are the sum of the percentage change in real profits and in the aggregate risk premium. Figure 9 plots the workers' and the entrepreneurs' gains against the total welfare changes. While the change in the real wage is positive for all countries, reflecting the effect of trade competition on prices, the change in the gains accrued by firms is negative or close to zero for some markets.

To explain the heterogeneity in the entrepreneurs gains, it is useful to resort to the

[63] In the limit case of $\gamma = 0$, welfare gains can be computed simply as a function of the change in domestic trade shares, the trade elasticity θ, and the share of consumption in manufacturing ξ. Since in autarky domestic trade shares are by construction equal to 1, it suffices to know the domestic trade shares in the initial calibrated equilibrium to compute the welfare gains under risk neutrality.

[64] Specifically, for each country I compute $\frac{\ln \hat{W}_i - \ln \hat{W}_i|_{\gamma=0}}{\ln \hat{W}_i|_{\gamma=0}}$, and the median value is 16%.

intuition provided by Proposition 4. In the symmetric countries case, the ability of firms to exploit international trade to diversify risk depends crucially on the covariance of demand with the foreign country. In particular, the firm-level Sharpe Ratio, i.e. the ratio between expected and standard deviation of real profits, increases with free trade as long as the firm is sufficiently productive, and the covariance of demand is sufficiently low (and the competition effect is weak). In the general case with N asymmetric countries, the presence of trade costs introduces an additional trade-off: while risk-minimizing entrepreneurs would like to export to countries with a lower covariance, they also want to sell to countries with lower trade costs.

To capture this trade-off, I compute for each country the average covariance with the other markets, weighted by the calibrated trade barriers in 2005.[65] Figure 10 plots the average change in the Sharpe Ratio against the weighted average of the covariances. The results illustrate that countries, like for instance Ireland, Singapore and Germany, that have cheaper access to destinations that are a good hedge against risk (i.e. destinations with a lower covariance with domestic demand), are able to significantly improve their risk-return profile upon a trade liberalization. On the other hand, for countries like Canada and Czech Republic, the stronger competition coming from foreign firms lowers the average profits by more than the reduction in the volatility of profits coming from international trade. In general, an improvement of the average risk-return profile is typically reflected into higher entrepreneurs gains, as shown in the right graph of Figure 10.

5.1.1 Robustness

In this section I investigate the robustness of the counterfactual predictions of the model along several dimensions. First, Panel A in Figure 11 plots the welfare gains from trade computed using means and demand covariances estimated using the alternative methodologies described in Section 3.1.4. For all these alternative measures of risk, the resulting welfare gains are very similar to the baseline.

Second, Panel B in Figure 11 plots the median welfare gains with different values of the entrepreneurs' risk aversion. Within a reasonable range for γ, gains from trade are not substantially different from the baseline, but they are a hump-shaped function of γ. Intuitively, going from an economy with risk neutrality to one with risk aversion implies that firms use more intensively international trade to diversify their demand risk. This leads to more trade competition among firms and lower prices. However, as the risk aversion becomes too high, entrepreneurs optimally choose to be less exposed to foreign

[65] In particular, I compute $\bar{Cov}_j = \sum_s \frac{\tau_{js}}{\sum_k \tau_{jk}} Cov(\alpha_j, \alpha_s)$, where τ_{js} are the trade costs calibrated in 2005.

risk, which leads to lower competitive pressure, and smaller welfare gains.[66]

Lastly, I repeat the counterfactual exercise using, instead of the constant risk aversion utility of the baseline, a CRRA utility function, which features a decreasing absolute risk aversion (see Appendix 8.1.15). I set the risk aversion parameter to 1, as estimated in Allen and Atkin (2016). Panel C in Figure 11 documents that welfare gains with CRRA are highly correlated with, but on average higher than, the ones predicted by the baseline model.

5.2 China Shock

The second counterfactual exercise I consider is an exogenous increase in productivity. Starting from the calibrated economy in 2005, I simulate an increase in the productivity of China by 30%, which is the observed increase in Chinese TFP from 2005 to 2015.[67] In the model, this corresponds to a 30% decrease in trade costs from China to all countries (including the domestic market).

As shown in Figure 12, the China shock generates in all countries an increase in the real wage, due to cheaper goods, that is partially offset by a reduction in entrepreneurs gains, with a net effect for the median country of only 0.2%.[68] For instance, countries such as Israel and Australia experience an increase in workers welfare due to cheaper consumption goods from China, but at the same time they have welfare losses for entrepreneurs because of reduced profits.[69] In addition, the productivity boom of Chinese firms decreases the export activity of the firms from other countries, which have a harder time to diversify domestic risk with international trade. Therefore, the China shock has a negative effect not only on average profits, but also on their volatility, as reflected in lower average Sharpe Ratios, as shown in Figure 13.

6 Conclusions

In this paper I characterize the link between demand risk, firms' exporting decisions,

[66]Note that in the limit case of an infinite risk aversion, firms optimally decide to not produce at all.

[67]I use the TFP estimates from Penn World Tables 9.1 (see Feenstra et al. (2015)). Ideally, I could calibrate the economy before the China's accession to WTO, and then simulate the observed increase in productivity and the decrease in trade costs. However, in such case I would only be using 5 years to estimate the moments of the demand shocks (since the dataset only starts in 1995), implying less precise estimates.

[68]Note that, while quite small, this number is remarkably consistent with the counterfactual result found for US, in a very different framework, by Caliendo et al. (2015).

[69]Interestingly, countries geographically closer to China, like South Korea and Japan, are able to reap higher real profits after the China shock, due to increased exports toward the richer Chinese consumers.

and welfare gains from trade. The proposed framework is sufficiently tractable to deliver testable implications and to be calibrated using firm-level data. Theoretically, I stress the importance of the cross-country multilateral covariance of demand in amplifying the impact of a change in trade costs through a novel "pro-competitive" effect. Empirically, I show that the Diversification Index, the country-level measure of demand risk, significantly affects trade patterns in a gravity framework. Quantitatively, an important message emerges from the analysis: a trade liberalization affects the risk-return trade-off that firms face on global markets, implying general equilibrium effects that may increase welfare gains from trade relative to standard trade models with risk neutrality.

Several avenues for future research may emerge from my study. For example, it would be interesting to introduce the possibility of product diversification as a tool to reduce profits volatility, as opposed to, or together with, geographical diversification, which has been the focus of this paper. In addition, one could enrich the model with dynamic learning, for example allowing firms to invest to reduce the degree of uncertainty over time. Lastly, the Diversification Index summarizes the entire pattern of spatial covariance of demand across countries, and therefore could be used in any empirical analysis to control for demand risk.

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7 Tables and Figures

Table 1: Moments: summary statistics

Statistic	**Covariance**	**Variance**	**Mean**	**SE**
Min	-1.73	1.01	1.27	0.007
Median	-0.04	3.35	1.55	0.0161
Max	2.49	24.85	2.35	1.643
Standard dev.	0.30	4.48	0.22	0.072

Notes: The first column reports some summary statistics for the estimated covariances, the second column for the variances, the third column for the means $\bar{\alpha}_i$, the fourth column for the bootstrapped standard errors associated with the covariance matrix Σ.

Table 2: Demand covariances and geography

Dep. var.: demand covariance	(1)	(2)	(3)
Bilateral log distance	-0.024***	-0.069***	-0.063***
	(0.009)	(0.015)	(0.017)
Observations	1,190	1,190	1,190
R-squared	0.006	0.038	0.041
Country FE	N	Y	Y
Gravity variables	N	N	Y

Notes: Column 1 reports the estimate of a regression of the bilateral demand covariances on the bilateral log distance between the countries, Column 2 adds country fixed effects, Column 3 adds dummies for contiguity, common language, common legal origins, colonial relationship.

Table 3: Diversification Index

Country	**Number of firms selling in 2005**	**Diversification Index**
Australia	266	0.99 (0.046)
Austria	367	0.68 (0.041)
Belgium-Lux.	949	0.87 (0.046)
Brazil	302	0.46 (0.033)
Canada	533	0.64 (0.045)
Chile	74	1.13 (0.045)
China	184	0.19 (0.016)
Czech Republic	211	0.57 (0.029)
Denmark	572	0.55 (0.036)
Finland	366	0.93 (0.055)
France	1,971	1.04 (0.056)
Germany	1,283	0.84 (0.049)
Greece	386	0.94 (0.049)
Hungary	189	0.82 (0.030)
Ireland	436	0.83 (0.052)
Israel	213	0.70 (0.039)
Italy	897	0.53 (0.036)
Japan	300	0.60 (0.034)
Rep. of Korea	112	0.47 (0.027)
Malaysia	55	0.21 (0.013)
Mexico	187	0.52 (0.031)
Morocco	286	1.12 (0.053)
Netherlands	954	0.77 (0.045)
Norway	370	0.83 (0.053)
Poland	241	0.55 (0.030)
Portugal	4,821	0.76 (0.054)
Romania	167	0.46 (0.033)
Russia	164	0.49 (0.037)
Singapore	100	0.52 (0.029)
South Africa	195	0.68 (0.045)
Spain	2,420	0.84 (0.049)
Sweden	597	0.76 (0.045)
Turkey	221	0.38 (0.022)
United Kingdom	1,294	0.91 (0.053)
United States	931	1.06 (0.061)
Total	4,821	

Notes: The second column reports, for each country, the number of Portuguese firms that were exporting there in 2005. The third column reports the estimated Diversification Indices, with bootstrapped standard errors in parenthesis.

Table 4: Diversification Index by Sector

Statistic	Baseline	Food/Tobacco	Textile	Wood	Minerals	Metals	Machinery	Others
Min	0.19	-0.54	0.00	-1.63	-0.88	-3.64	-0.56	-0.28
Median	0.70	0.62	0.59	0.76	0.40	0.57	0.64	0.68
Max	1.13	16.28	3.16	7.53	3.00	17.50	11.77	11.60
Average SE	0.04	0.48	0.04	0.41	0.21	0.21	0.09	0.07

Notes: The first row reports the minimum of the Diversification Indices for the baseline and for each sector, the second row reports the median, the third row reports the maximum, the fourth row reports the average bootstrapped standard error.

Table 5: Risk aversion

Dep. Var.: Average Marginal Profits	(1)	(2)	(3)
Variance of Marginal Profits	0.415***	0.282***	0.413***
	(0.106)	(0.052)	(0.065)
Destination Fixed Effect	Y	Y	Y
Observations	16,017	12,546	11,570
R^2	0.413	0.467	0.489

Notes: The table regresses the average marginal profits of Portuguese firms on the variance of marginal profits. Both statistics are computed using yearly data from 1995 to 2004. Column 1 uses the baseline sample, Column 2 excludes Portugal, Column 3 includes only established exporters. All regressions are weighted by firms' total sales in 2005, standard errors are clustered at the destination level to adjust for heteroskedasticity and within-country correlation across firms. Robust standard errors are shown in parenthesis (*** p<0.01, ** p<0.05, * p<0.1).

Table 6: Extensive margin and risk

Dep. var.: $Pr\left(x_{sj} > 0\right)$	(1)	(2)	(3)	(4)
$ln(D_j)$	0.119***	0.350**	0.119***	
	(0.044)	(0.136)	(0.001)	
Sharpe Ratio				0.477***
				(0.002)
Average covariance				-0.484***
				(0.009)
Log of GDP	0.100***	0.388***	0.100***	0.104***
	(0.019)	(0.066)	(0.000)	(0.000)
Log of Distance	-0.317***	-0.335***	-0.317***	-0.318***
	(0.111)	(0.120)	(0.002)	(0.002)
Firm F.E.	YES	NO	YES	YES
Obs.	86,823	86,823	86,823	86,823

Notes: The dependent variable is an indicator equal to 1 if Portuguese firm s enters market j in 2005, and equal 0 otherwise. Column 1 uses OLS, column 2 uses a Probit model, columns 3-5 report the results from the second stage of the two-step methodology proposed by Head and Mayer (2013) and Baker and Fortin (2001), using FGLS. Additional not reported controls are: remoteness, trade openness, log of population, average tariff rate, dummies for common language, common legal origins, contiguity, common currency, WTO membership, regional trade agreement, and a dummy for European countries. Regressions are weighted by firm's total sales. All data are for 2005. Clustered standard errors are shown in parenthesis (*** p<0.01, ** p<0.05, * p<0.1).

Table 7: Intensive margin and risk, firm level

	(1)	(2)	(3)	(4)	(5)
$ln(D_j)$	0.994***	0.869***	0.862***	1.031***	
	(0.264)	(0.194)	(0.103)	(0.013)	
Sharpe Ratio					2.99***
					(0.038)
Average covariance					-5.711***
					(0.196)
Log of GDP	0.674***	0.492***	0.394***	0.678***	0.745***
	(0.149)	(0.131)	(0.089)	(0.009)	(0.008)
Log of Distance	-1.151	-1.328*	0.017	-0.785***	-0.995***
	(0.936)	(0.685)	(0.328)	(0.049)	(0.048)
Firm F.E.	YES	NO	NO	YES	YES
Obs.	13,236	13,236	86,823	12,486	12,486

Notes: In Columns 1,3,4,5,6 the dependent variable is the log of exports of a Portuguese firm to country j in 2005, while Column 2 uses the level of exports. Column 1 uses OLS, column 2 uses PPML, column 3 reports the results from the second stage of a Heckman two-steps methodology, columns 4-6 report the results from the second stage of the two-step methodology proposed by Head and Mayer (2013) and Baker and Fortin (2001), using FGLS. Additional not reported controls are: remoteness, trade openness, log of population, average tariff rate, dummies for common language, common legal origins, contiguity, common currency, WTO membership, regional trade agreement, and a dummy for European countries. Regressions are weighted by firm's total sales. All data are for 2005. Clustered standard errors are shown in parenthesis (*** p<0.01, ** p<0.05, * p<0.1).

Table 8: Intensive margin and risk, country level

	(1)	(2)	(3)	(4)	(5)	(6)
$ln(D_j)$	0.162**	0.283***	0.143***			
	(0.072)	(0.084)	(0.024)			
$ln(D_{jk})$					0.109***	
					(0.022)	
Sharpe Ratio				0.409***		0.404***
				(0.065)		(0.060)
Average covariance				-3.314***		-0.828***
				(0.264)		(0.195)
Log of GDP	1.069***	1.014***	1.052***	1.103***		
	(0.032)	(0.046)	(0.011)	(0.012)		
Log of Bilateral Distance	-0.993***	-0.663***			-1.240***	-1.231***
	(0.045)	(0.047)			(0.056)	(0.054)
Origin F.E.	YES	YES	YES	YES	YES	YES
Destination F.E.	NO	NO	NO	NO	YES	YES
Obs.	1,122	1,122	1,122	1,122	7,060	7,060

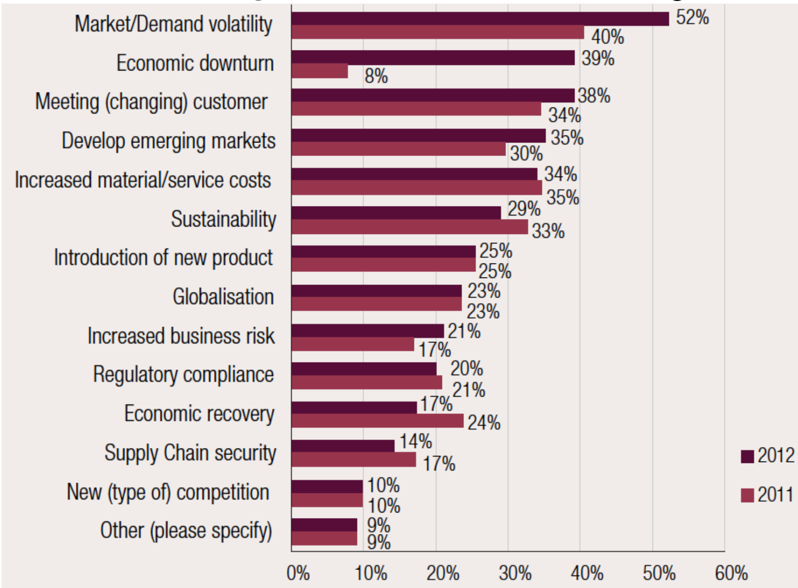
Notes: In Columns 1,3,4,5,6 the dependent variable is the log of bilateral exports between all pairs of countries $i \neq j$ in the sample, while Column 2 uses the level of exports. Columns 1,5,6 use OLS, column 2 uses PPML, columns 3,4 report the results from the second stage of the two-step methodology proposed by Head and Mayer (2013) and Baker and Fortin (2001), using FGLS. Additional not reported controls are: remoteness, trade openness, log of population, log of bilateral tariffs, dummies for common language, common legal origins, contiguity, common currency, WTO membership, regional trade agreement, and a dummy for European countries. Columns 3,4 additionally control for $\bar{\mathbf{X}}_j \equiv \frac{1}{N}\sum_i \mathbf{X}_{ij}$, the average across exporters of each bilateral variable (that is why the bilateral distance does not appear). All data are for 2005. Clustered standard errors are shown in parenthesis (*** p<0.01, ** p<0.05, * p<0.1).

Table 9: Sorting of exporters

Destination	Rank	Fraction sorted, data	Fraction sorted, model
France	2	77%	94%
UK	3	66%	77%
Germany	4	50%	95%
Netherland	5	46%	61%
Belgium	6	35%	60%
USA	7	23%	76%
Total		54%	76%

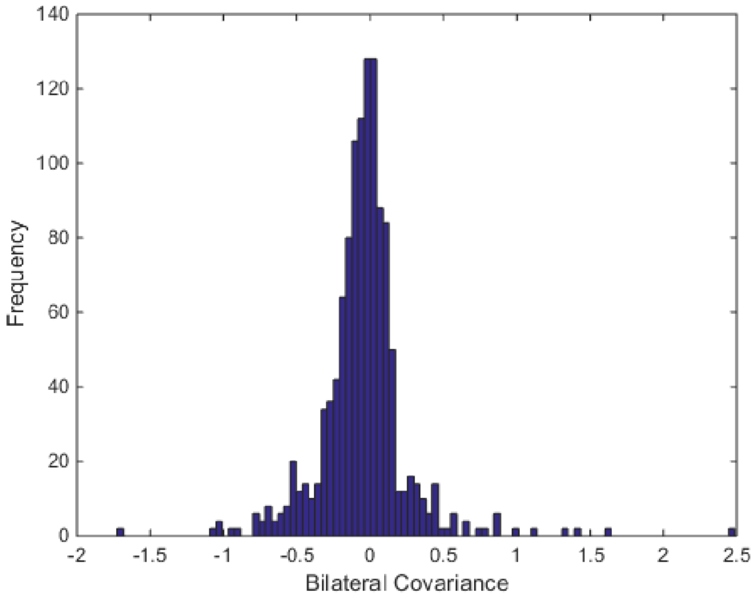
Notes: For each destination ranked k-th in terms of popularity, I consider all Portuguese firms exporting to that destination, and compute the fraction of exporters that sells to all markets with rank lower than k.

Figure 1: Main business challenges



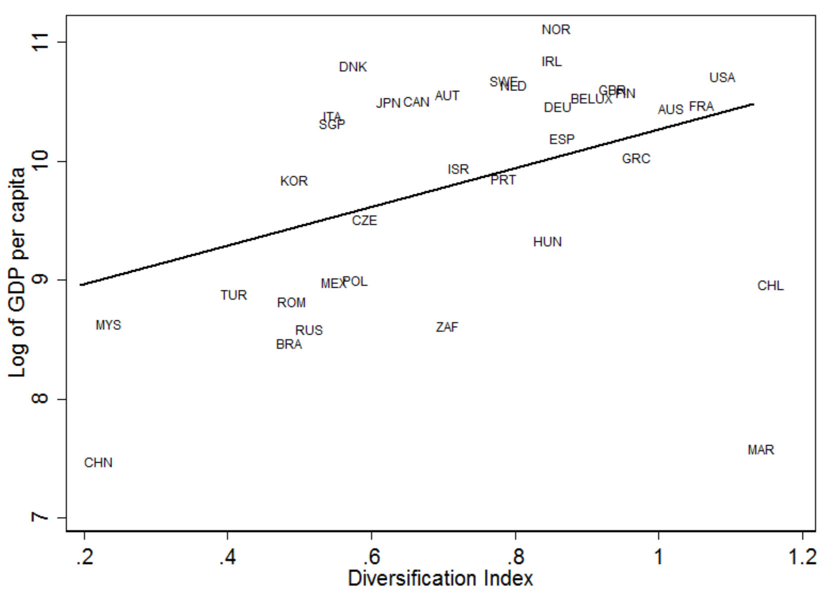
Notes: The figure shows the results from a survey conducted in 2011 and 2012 by the consulting firm Capgemini among 350 leading companies around the world. The question asked to the firms was: "What is the most important challenge for your firm?". The graph shows that 52% of the respondents thought demand volatility was the main challenge (or business driver) in 2012, up from 40% in 2011. Source: https://www.capgemini.com/wp-content/uploads/2017/07/The_2012_Global_Supply_Chain_Agenda.pdf

Figure 2: Distribution of Covariances



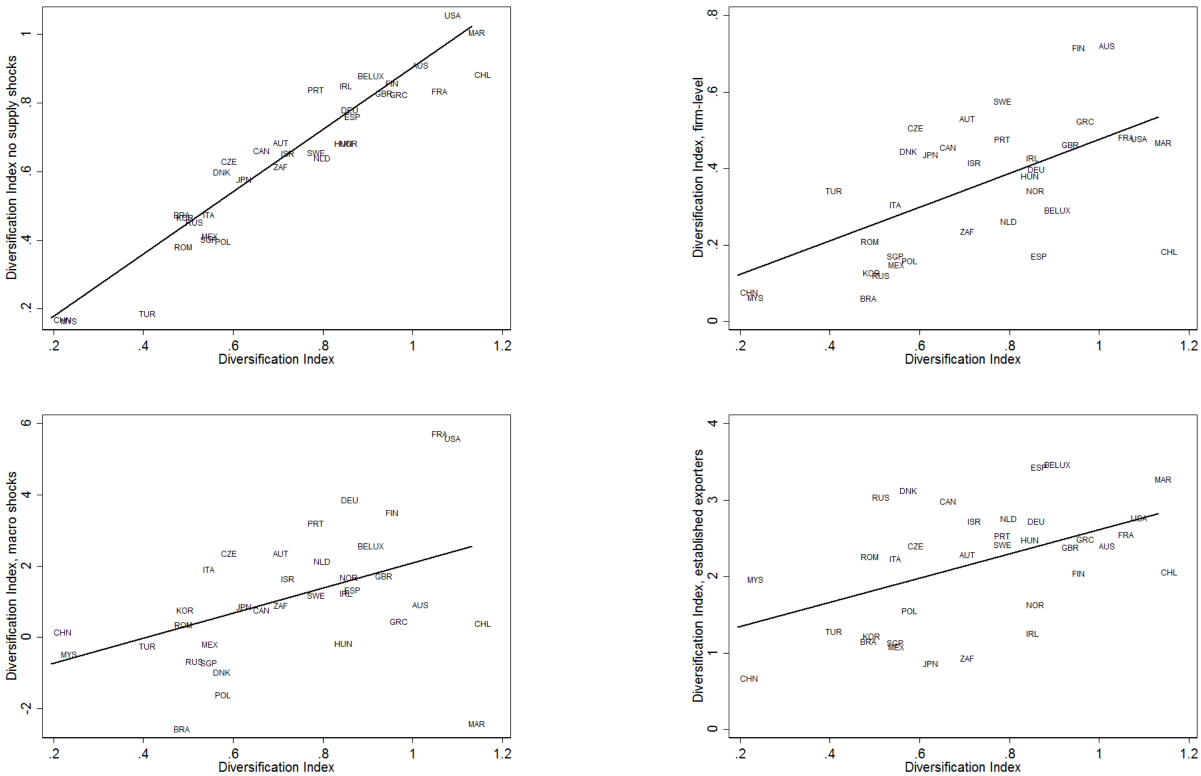
Notes: The figure plots the distribution of the estimated bilateral covariances of demand.

Figure 3: Diversification Index and income per capita



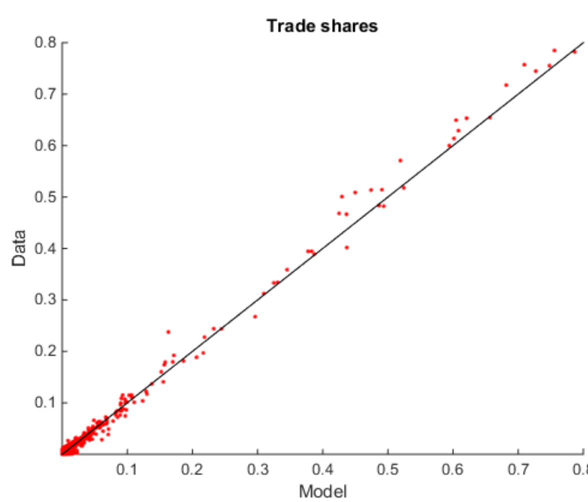
Notes: The figure plots the log of GDP per capita in 2005 against the estimated Diversification Index of the countries in the sample. The displayed relationship is 5% significant.

Figure 4: Diversification Index: robustness



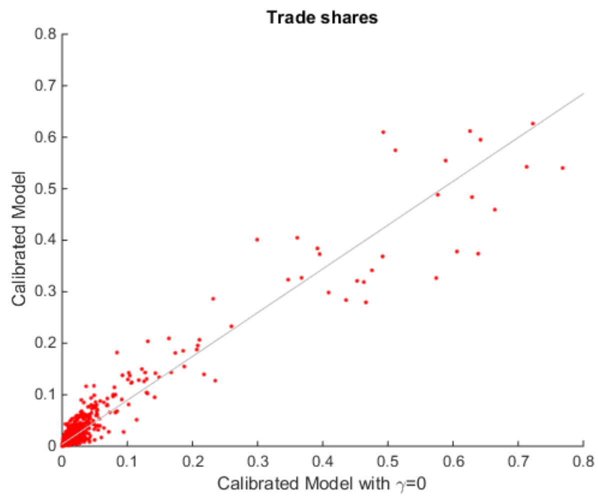
Notes: The figures plot the baseline Diversification Index against, respectively: i) the Diversification Index computed without controlling for firm-destination supply shocks; ii) the Diversification Index computed taking an average across each firm's covariances; iii) the Diversification Index computed taking into account for macro shocks; iv) the Diversification Index computed with only established firm-destination pairs. The displayed relationships are all 1% significant.

Figure 5: Calibration of trade shares



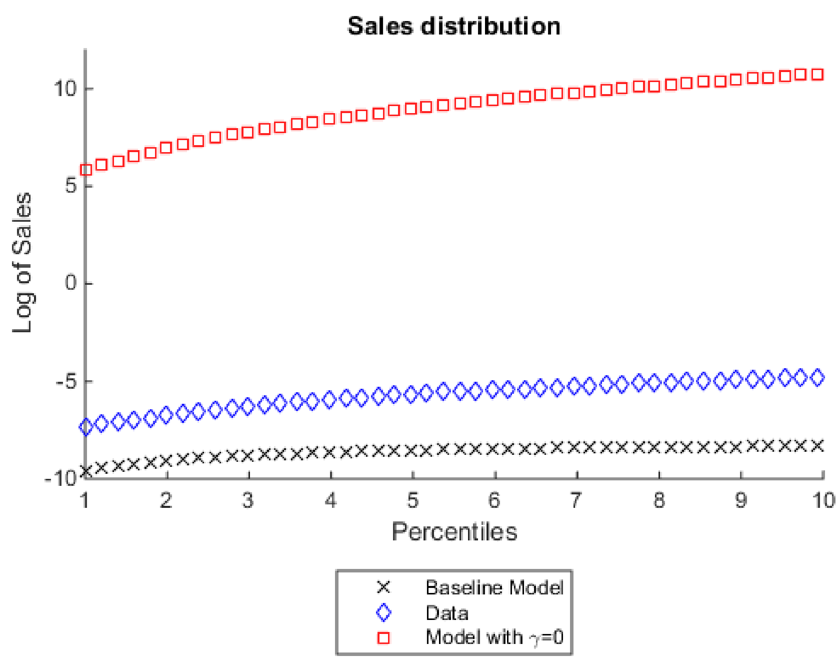
Notes: The figure plots the simulated and the actual values of bilateral trade shares in 2005. The correlation is 0.98.

Figure 6: Fraction of trade explained by risk



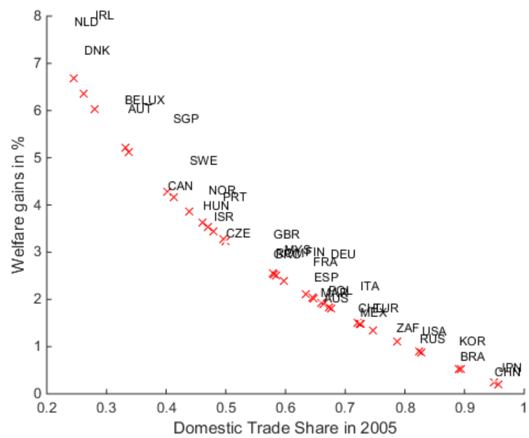
Notes: The figure plots the trade shares of the calibrated model vs the corresponding shares of the calibrated model with the risk aversion parameter set to 0. The figure also displays the best fit line, which has an intercept of 0.0048, a slope of 0.8310, and an R^2 of 85%. The remaining variation, 15%, is the fraction of trade shares that is not explained by the risk neutral model.

Figure 7: Sales distribution



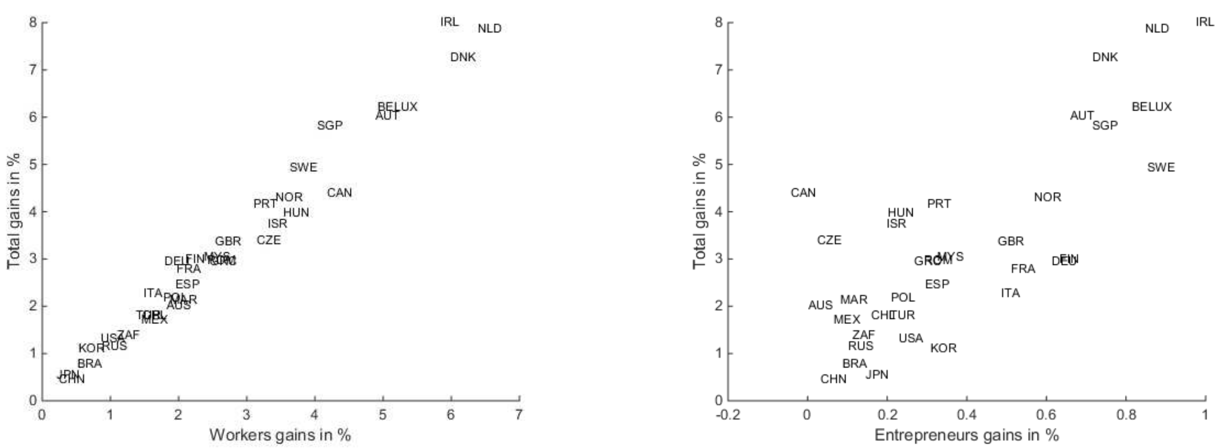
Notes: The figure plots the left tail of the distribution of log sales in the calibrated model and in the data. I first compute the percentiles for each destination and then I take an average across destinations for each percentile.

Figure 8: Welfare gains from trade



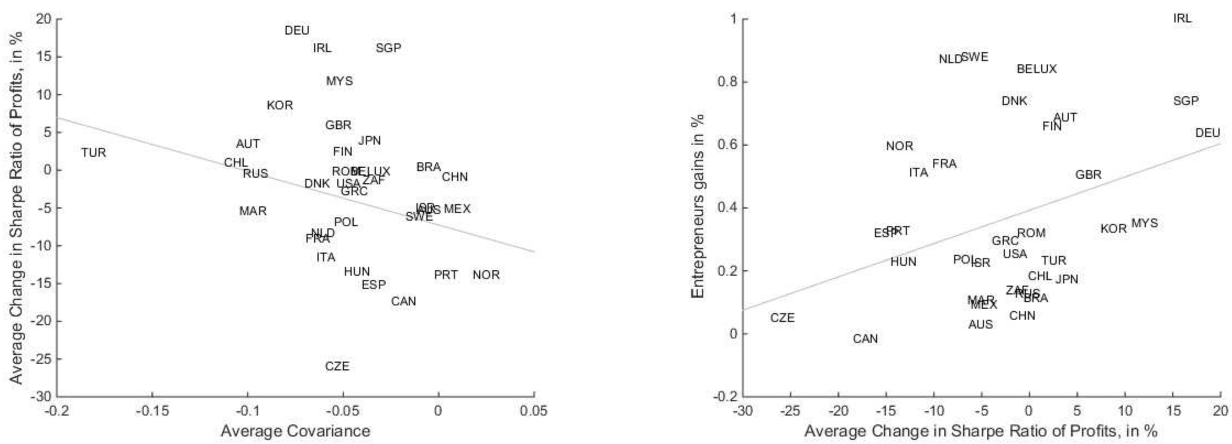
Notes: The figure plots the percentage change in welfare after moving from autarky to the calibrated equilibrium. The red crosses are the gains predicted by a model with risk neutrality, while the dots with country names are the gains predicted by the model with risk aversion.

Figure 9: Decomposition of Welfare Gains from Trade



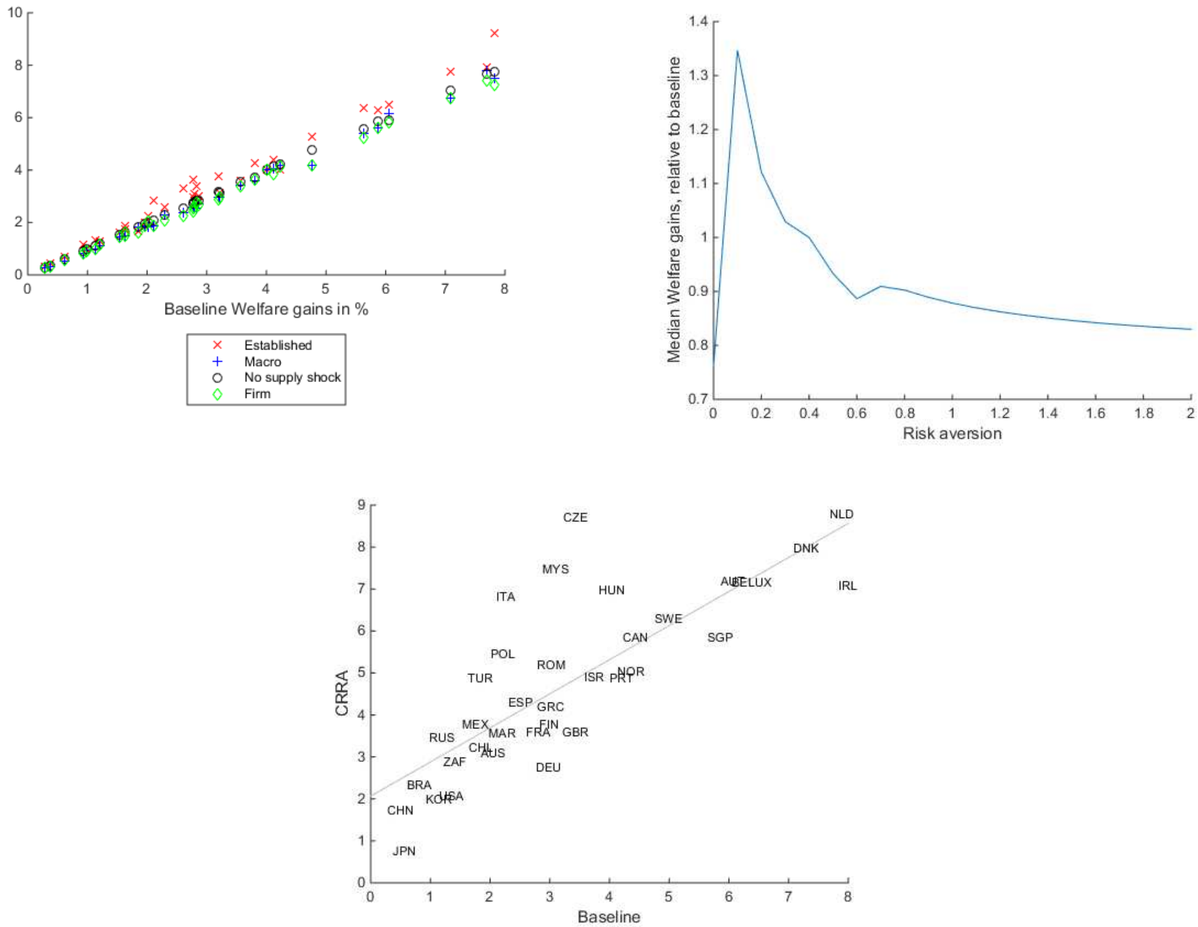
Notes: The figure on the left plots the percentage change in workers' welfare after moving from autarky to the calibrated equilibrium, against the total welfare gains. The figure on the right plots instead the percentage change in entrepreneurs' welfare against the total welfare gains.

Figure 10: Risk Diversification and Entrepreneurs gains



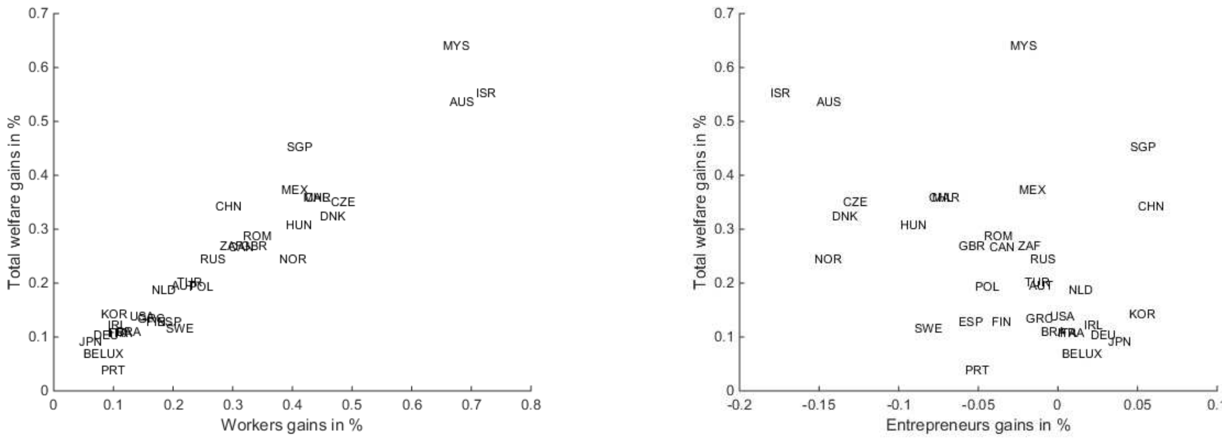
Notes: The figure on the left plots the average change in $SR(z)$ across firms after a trade liberalization against the average covariance with other countries, weighted by the bilateral trade costs. The OLS coefficient is -.712 and the p-value is 0.01.The figure on the right plots the entrepreneurs gains against the average change in $SR(z)$. The OLS coefficient is 1.06 and the p-value is 0.01.

Figure 11: Welfare Gains from Trade - Robustness



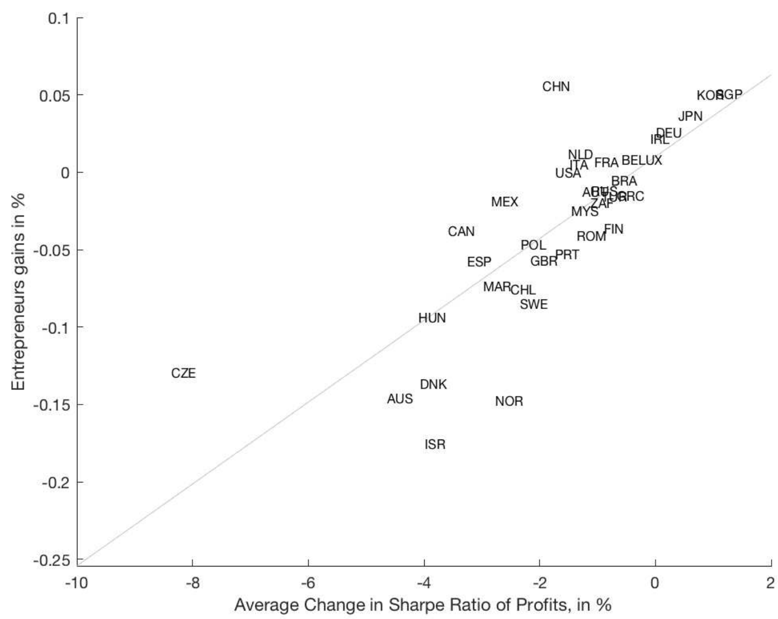
Notes: The figure on the top left plots the baseline welfare gains from trade against the gains computed using means and demand covariances estimated with the alternative methodologies described in section 3.1.4. These are the moments computed: i) using only established firm-destination pairs; ii) taking into account for macro shocks; iii) without controlling for firm-destination supply shocks; iv) taking an average across each firm's covariances. The figure on the top right reports the welfare gains from trade for different values for the risk aversion, relative to the baseline welfare gains (which assumes a risk aversion of 0.41). The figure at the bottom plots the baseline welfare changes against the corresponding changes computed with a CRRA utility function with risk aversion parameter $\rho = 1$, as in Allen and Atkin (2016).

Figure 12: Welfare gains after China Shock



Notes: The figure on the left plots the counterfactual percentage change in total welfare against the workers' welfare gains after the China shock. The figure on the right plots instead total welfare gains against entrepreneurs gains. See equation (52).

Figure 13: China Shock and Risk Diversification



Notes: The figure plots the entrepreneurs gains after the counterfactual increase in Chinese productivity against the average, across firms, change in the Sharpe Ratio of real profits.

8 For Online Publication: Appendix

8.1 Analytical appendix

8.1.1 Proof of Proposition 1

Since the firm sets the optimal price after the realization of the shock, in the first stage it chooses the optimal fraction of consumers to reach in each market based on the expectation of what the price will be in the second stage. I solve the optimal problem of the firm by backward induction, starting from the second stage. At this stage, there is no uncertainty and thus the firm chooses the optimal pricing policy that maximizes profits, *given* the optimal $n_{ij}(z, E[p_{ij}(z)])$ chosen in the first stage:

$$\max_{\{p_{ij}\}} \sum_j \alpha_j(z) \frac{p_{ij}(z)^{-\sigma}}{P_j^{1-\sigma}} n_{ij}(z, E[p_{ij}(z)]) Y_j \left(p_{ij}(z) - \frac{\tau_{ij} w_i}{z} \right).$$

noting that the firm has already paid the marketing costs in the first stage. It is easy to see that this leads to the standard constant markup over marginal cost:

$$p_{ij}(z) = \frac{\sigma}{\sigma - 1} \frac{\tau_{ij} w_i}{z}. \tag{53}$$

Notice that, given the linearity of profits in $n_{ij}(z, E[p_{ij}(z)])$ and $\alpha_j(z)$, due to the assumptions of CES demand and constant returns to scale in labor, the optimal price does not depend on neither $n_{ij}(z, E[p_{ij}(z)])$ nor α_j. The optimal quantity produced is:

$$q_{ij}(z) = \alpha_j(z) \left(\frac{\sigma}{\sigma - 1} \frac{\tau_{ij} w_i}{z} \right)^{-\sigma} \frac{n_{ij}(z, E[p_{ij}(z)]) Y_j}{P_j^{1-\sigma}}. \tag{54}$$

I now solve the firm problem in the first stage, when there is uncertainty on the realization of the shocks. By backward induction, in the first stage the firm takes as given the pricing rule in (53) and the quantity produced in (54). The maximization problem of firm z is:

$$\max_{\{n_{ij}\}} \sum_j \bar{\alpha}_j n_{ij}(z) r_{ij}(z) - \frac{\gamma}{2} \sum_j \sum_s n_{ij}(z) r_{ij}(z) n_{is}(z) r_{is}(z) Cov(\alpha_j, \alpha_s) - \sum_j w_j n_{ij}(z) f_j L_j$$

$$\text{s. to } 1 \geq n_{ij}(z) \geq 0$$

where $r_{ij}(z) \equiv \frac{1}{P_i}\frac{p_{ij}(z)^{-\sigma}Y_j}{P_j^{1-\sigma}}\left(p_{ij}(z) - \frac{\tau_{ij}w_i}{z}\right)$. Given the optimal price in (53), this simplifies to:

$$r_{ij}(z) = \frac{1}{P_i}\left(\frac{\sigma}{\sigma-1}\frac{\tau_{ij}w_i}{z}\right)^{1-\sigma}\frac{Y_j}{P_j^{1-\sigma}\sigma}$$

The Lagrangian is, omitting the z for simplicity:

$$L = \sum_j \bar{\alpha}_j n_{ij} r_{ij} - \frac{\gamma}{2}\sum_j\sum_s n_{ij} r_{ij} n_{is} r_{is} Cov(\alpha_j, \alpha_s) - \sum_j w_j n_{ij}(z) f_j L_j / P_i - \sum_j \mu_{ij} g(n_{ij})$$

where $g(n_{ij}) = n_{ij} - 1$. The necessary Karush–Kuhn–Tucker conditions are:

$$\frac{\partial L}{\partial n_{ij}} = \frac{\partial U}{\partial n_{ij}} - \mu_{ij}\frac{\partial g(n_{ij})}{\partial n_{ij}} \leq 0 \quad \frac{\partial L}{\partial n_{ij}} n_{ij} = 0$$
$$\frac{\partial L}{\partial \mu_{ij}} \geq 0 \quad \frac{\partial L}{\partial \mu_j}\mu_{ij} = 0$$

A more compact way of writing the above conditions is to introduce the auxiliary variable λ_{ij}, which is such that

$$\frac{\partial U}{\partial n_{ij}} - \mu_{ij}\frac{\partial g(n_{ij})}{\partial n_{ij}} + \lambda_{ij} = 0$$

and thus $\lambda_{ij} = 0$ if $n_{ij} > 0$, while $\lambda_{ij} > 0$ if $n_{ij} = 0$. Then the first order condition for n_{ij} becomes:

$$\bar{\alpha}_j r_{ij} - \gamma \sum_s r_{ij} n_{is} r_{is} Cov(\alpha_j, \alpha_s) - w_j f_j L_j / P_i - \mu_{ij} + \lambda_{ij} = 0$$

I can write the solution for $n_{ij}(z)$ in matrix form as:

$$\mathbf{n}_i = \frac{1}{\gamma}\left(\tilde{\Sigma}_i\right)^{-1}\mathbf{r}_i, \tag{55}$$

where each element of the $N-$dimensional vector $\mathbf{r}_i$ equals:

$$\mathbf{r}_i^j \equiv r_{ij}\bar{\alpha}_j - w_j f_j L_j / P_i - \mu_{ij} + \lambda_{ij}, \tag{56}$$

and $\tilde{\Sigma_i}$ is a NxN covariance matrix, whose k, j element is, from equation (13):

$$\tilde{\Sigma}_{i,kj} = r_{ij} r_{ik}(z) Cov(\alpha_j, \alpha_k).$$

The inverse of $\tilde{\Sigma}_i$ is, by the Cramer's rule:

$$\left(\tilde{\Sigma}_i\right)^{-1} = r_i \frac{1}{det(\Sigma)} C_i r_i, \tag{57}$$

where r_i is the inverse of a diagonal matrix whose $j-th$ element is r_{ij}, and C_i is the (symmetric) matrix of cofactors of Σ.[70] Since $r_{ij} > 0$ for all i and j, then

$$det(\Sigma) \neq 0$$

is a sufficient condition to have invertibility of $\tilde{\sum}_i$. This is Assumption 2 in the main text. Replacing equations (57) and (56) into (55), the optimal n_{ij} is:

$$n_{ij} = \frac{\sum_k \frac{C_{jk}}{r_{ik}} \left(r_{ik}\bar{\alpha}_k - w_i^{\beta} w_k^{1-\beta} f_k L_k / P_i - \mu_{ik} + \lambda_{ik} \right)}{\gamma r_{ij}},$$

where C_{jk} is the j,k cofactor of Σ, rescaled by $det(\Sigma)$. Finally, the solution above is a global maximum if i) the constraints are quasi convex and ii) the objective function is concave. The constraints are obviously quasi convex since their are linear. The Hessian matrix of the objective function is:

$$H(z) = \begin{bmatrix} \frac{\partial^2 U}{\partial^2 n_{ij}} & & \frac{\partial^2 U}{\partial n_{ij} \partial n_{iN}} \\ . & . & \\ . & . & \\ \frac{\partial^2 U}{\partial n_{iN} \partial n_{ij}} & & \frac{\partial^2 U}{\partial^2 n_{iN}} \end{bmatrix},$$

where, for all pairs j,k:

$$\frac{\partial^2 U}{\partial n_{ij} \partial n_{ik}} = \frac{\partial^2 U}{\partial n_{ik} \partial n_{ij}} = -\gamma \delta_{ij} \delta_{ik} Cov(\alpha_j, \alpha_k) < 0$$

Given that $\frac{\partial^2 U}{\partial^2 n_{ij}} < 0$, the Hessian is negative semi-definite if and only if its determinant is positive. It is easy to see that the determinant of the Hessian can be written as:

$$det(H) = \prod_{j=1}^{N} \gamma \delta_{ij}(z)^2 det(\Sigma),$$

[70]The cofactor is defined as $C_{kj} \equiv (-1)^{k+j} M_{kj}$, where M_{kj} is the (k,j) minor of Σ. The minor of a matrix is the determinant of the sub-matrix formed by deleting the k-th row and j-th column.

which is always positive if

$$det(\Sigma) > 0,$$

which always holds by Assumption 2 and since Σ is a covariance matrix. Therefore the function is concave and the solution is a global maximum, given the price index P, income Y and wage w.■

8.1.2 Alternative production structure

In this section I solve the problem of the firm under the assumption that the firm makes also the production decision under uncertainty, i.e. it pre-commits to production in the first stage. In such case, in the first stage the firm not only pays the marketing costs, but also pays workers to produce. After the demand shocks are realized, then the firm adjusts the price in each location such that the amount of goods already produced for each location exactly equal the realized demand. Specifically, there are three stages:

i. The firm maximizes the expected utility by choosing $n_{ij}(E[p_{ij}(z)], z)$, where $E[p_{ij}(z)]$ is the expectation of the price it will charge after the shock is realized. This stage is solved exactly in the same way as in Proposition 1.

ii. Given the set of destinations and the number of consumers chosen in the first stage, $n_{ij}(E[p_{ij}(z)], z)$, the firm hires workers to produce the corresponding quantity, still under uncertainty. However, since the firm has already decided the locations in which to sell to, the firm simply has to maximize expected gross profits in each destination separately:

$$\max_{\{p_{ij}\}} \sum_j E[\alpha_j] \frac{p_{ij}(z)^{-\sigma}}{P_j^{1-\sigma}} n_{ij}(E[p_{ij}(z)], z) Y_j \left(p_{ij}(z) - \frac{\tau_{ij} w_i}{z} \right)$$

which leads to

$$\tilde{p}_{ij}(z) = \frac{\sigma}{\sigma - 1} \frac{\tau_{ij} w_i}{z} \tag{58}$$

Thus the quantity produced at this stage is:

$$q_{ij}^{produced} = E[\alpha_j] \frac{\tilde{p}_{ij}(z)^{-\sigma}}{P_j^{1-\sigma}} n_{ij}(E[p_{ij}(z)], z) Y_j \tag{59}$$

iii. The firm learns the demand shocks α_j in the countries it entered, and adjusts the price so that

$$q_{ij}^{produced} = q_{ij}^{realized} \tag{60}$$

in each destination j. Thus, the firm cannot reallocate resources across locations after the shocks, because it has already produced and paid workers, but it will only change the price.

Given equation (59), the above becomes

$$E[\alpha_j]\frac{\tilde{p}_{ij}(z)^{-\sigma}}{P_j^{1-\sigma}}n_{ij}(E[p_{ij}(z)],z)Y_j = \alpha_j(z)\frac{p_{ij}(z)^{-\sigma}}{P_j^{1-\sigma}}n_{ij}(E[p_{ij}(z)],z)Y_j$$

where $n_{ij}(E[p_{ij}(z)],z)$ is chosen in the first stage and cannot be changed. Then we have:

$$\bar{\alpha}_j\tilde{p}_{ij}(z)^{-\sigma} = \alpha_j(z)p_{ij}(z)^{-\sigma}$$

where $\bar{\alpha}_j \equiv E[\alpha_j]$. This implies the following final price

$$p_{ij}(z) = \frac{\sigma}{\sigma-1}\frac{\tau_{ij}w_i}{z}\left(\frac{\alpha_j(z)}{\bar{\alpha}_j}\right)^{\frac{1}{\sigma}} \tag{61}$$

Intuitively, the higher the realized demand shock relative to the expected shock, the higher is the effective price charged, in order to extract more revenues from the higher demand.

By backward induction, in the first stage the firm takes the expectation over the price it will charge in the third stage, which is:

$$E[p_{ij}(z)] = \frac{\sigma}{\sigma-1}\frac{\tau_{ij}w_i}{z}\tilde{\alpha}_j$$

where $\tilde{\alpha}_j \equiv E\left[\left(\alpha_j(z)\right)^{\frac{1}{\sigma}}\right]\left(\bar{\alpha}_j\right)^{-\frac{1}{\sigma}}$. I assume that $E\left[\left(\alpha_j(z)\right)^{\frac{1}{\sigma}}\right]$ is a finite moment. Therefore we have that:

$$r_{ij}(z) = \frac{1}{P_i}\left(\frac{\sigma}{\sigma-1}\frac{\tau_{ij}w_i}{z}\right)^{1-\sigma}\tilde{\alpha}_j\frac{Y_j}{P_j^{1-\sigma}\sigma}$$

and $n_{ij}(z)$ is still given by:

$$n_{ij} = \frac{\sum_k \frac{C_{jk}}{r_{ik}}\left(r_{ik}\bar{\alpha}_k - w_i^{\beta}w_k^{1-\beta}f_kL_k/P_i - \mu_{ik} + \lambda_{ik}\right)}{\gamma r_{ij}}.$$

Under this alternative formulation, the price index is

$$P_i^{1-\sigma} = \sum_j M_j \int_{\underline{z}}^{\infty} \int_0^{\infty} \alpha_i(z) n_{ji}(z) p_{ji}(z)^{1-\sigma} g(z) h(\alpha) d\alpha dz =$$

$$= \tilde{\alpha}_i \bar{\alpha}_i \sum_j M_j \int_{\underline{z}}^{\infty} n_{ji}(z) \left(\frac{\sigma}{\sigma-1} \frac{\tau_{ji} w_j}{z} \right)^{1-\sigma} g(z) dz$$

which differs from the baseline price index only by the presence of $\tilde{\alpha}_i$. The profits are

$$\Pi_i = M_i \sum_j \left(\frac{1}{\sigma} \tilde{\alpha}_j \bar{\alpha}_j \int_{\underline{z}}^{\infty} \frac{n_{ij}(z) Y_j}{P_j^{1-\sigma}} \left(\frac{\sigma}{\sigma-1} \frac{\tau_{ij} w_i}{z} \right)^{1-\sigma} g(z) dz - \int_{\underline{z}}^{\infty} f_{ij}(z) g(z) dz \right)$$

which also differ only by the presence of $\tilde{\alpha}_i$. Finally, the labor clearing condition becomes:

$$M_i \sum_j \bar{\alpha}_j \int_{\underline{z}}^{\infty} \frac{\tau_{ij}}{z} \frac{1}{P_j^{1-\sigma}} \left(\frac{\sigma}{\sigma-1} \frac{\tau_{ij} w_i}{z} \right)^{-\sigma} n_{ij}(z) Y_j g(z) dz + M_i \sum_j \int_{\underline{z}}^{\infty} f_j n_{ij}(z) L_j g(z) dz = \tilde{L}_i$$

as in the baseline model.

8.1.3 Proof of Proposition 2

From Proposition 1, the optimal solution can be written as (omitting the z to simplify notation):

$$n_{ij} = \frac{\sum_k \frac{C_{jk}}{r_{ik}} \left(r_{ik} \bar{\alpha}_k - w_i^{\beta} w_k^{1-\beta} f_k L_k / P_i - \mu_{ik} + \lambda_{ik} \right)}{\gamma r_{ij}} =$$

$$= \frac{D_j}{\gamma r_{ij}} - \frac{\sum_k \frac{C_{jk}}{r_{ik}} \left(w_i^{\beta} w_k^{1-\beta} f_k L_k / P_i \right)}{\gamma r_{ij}} + \frac{\sum_k \frac{C_{jk}}{r_{ik}} \left(\lambda_{ik} - \mu_{ik} \right)}{\gamma r_{ij}} \tag{62}$$

where $D_j = \sum_k C_{jk} \bar{\alpha}_k$ is the Diversification Index of destination j. In the case of an interior solution, we have that:

$$n_{ij}(z) = \frac{D_j}{\gamma r_{ij}} - \frac{\sum_k \frac{C_{jk}}{r_{ik}} \left(w_i^{\beta} w_k^{1-\beta} f_k L_k / P_i \right)}{\gamma r_{ij}} \tag{63}$$

and therefore both the probability of entering j (i.e. the probability that $n_{ij}(z) > 0$) and the level of exports to j,

$$x_{ij}(z) = \alpha_j(z)\left(\frac{\sigma}{\sigma-1}\frac{\tau_{ij}w_i}{z}\right)^{1-\sigma}\frac{Y_j}{P_j^{1-\sigma}}n_{ij}(z) \tag{64}$$

are increasing in D_j. When instead there is at least one binding constraint (either the firm sets $n_{ik}(z) = 0$ or $n_{ik}(z) = 1$ for at least one k), then the corresponding Lagrange multiplier will be positive. Therefore:

$$\frac{\partial n_{ij}(z)}{\partial D_j} = \underbrace{\frac{1}{\gamma r_{ij}}}_{\text{direct effect}} + \underbrace{\frac{1}{\gamma r_{ij}}\left[\sum_{k\neq j}\frac{C_{jk}}{r_{ik}}\frac{\partial \lambda_{ik}}{\partial D_j} - \sum_{k\neq j}\frac{C_{jk}}{r_{ik}}\frac{\partial \mu_{ik}}{\partial D_j}\right]}_{\text{indirect effect}} \tag{65}$$

Note that λ_{ik} is zero if $n_{ik}(z) > 0$, otherwise it equals:

$$\lambda_{ik} = -\bar{\alpha}_k r_{ik} + \gamma r_{ik}\sum_{s\neq j} n_{is}r_{is}Cov(\alpha_k,\alpha_s) + w_i^{\beta}w_k^{1-\beta}f_k L_k/P_i$$

and therefore

$$\frac{\partial \lambda_{ik}}{\partial D_j} = \gamma r_{ik}\sum_{s\neq j}\frac{\partial n_{is}(z)}{\partial D_j}r_{is}Cov(\alpha_k,\alpha_s) \tag{66}$$

Similarly for the other Lagrange multiplier:

$$\mu_{ik} = \bar{\alpha}_k r_{ik} - \gamma r_{ik}\sum_{s\neq j} n_{is}r_{is}Cov(\alpha_k,\alpha_s) - \gamma r_{ik}^2 Var(\alpha_k) - w_i^{\beta}w_k^{1-\beta}f_k L_k/P_i$$

and thus:

$$\frac{\partial \mu_{ik}}{\partial D_j} = -\gamma r_{ik}\sum_{s\neq j}\frac{\partial n_{is}(z)}{\partial D_j}r_{is}Cov(\alpha_k,\alpha_s) = -\frac{\partial \lambda_{ik}}{\partial D_j} \tag{67}$$

Now notice that either $\mu_{ik} > 0$ and $\lambda_{ik} = 0$, or $\lambda_{ik} > 0$ and $\mu_{ik} = 0$. Combining this fact with equations 66 and 67, equation 65 becomes:

$$\frac{\partial n_{ij}(z)}{\partial D_j} = \frac{1}{\gamma r_{ij}}\left[1 + \gamma\sum_{k\neq j}C_{jk}\sum_{s\neq j}\frac{\partial n_{is}(z)}{\partial D_j}r_{is}Cov(\alpha_k,\alpha_s)\right]$$

Define $x_j \equiv \frac{\partial n_{ij}(z)}{\partial D_j}\gamma r_{ij}$. Since $\gamma r_{ij} > 0$, the sign of x_j equals the sign of the derivative of interest, $\frac{\partial n_{ij}(z)}{\partial D_j}$. Then the above can be written as:

$$x_j = 1 + \sum_{k \neq j} C_{jk} \sum_{s \neq j} x_s Cov(\alpha_k, \alpha_s)$$

This is a linear system of N equations in N unknowns, x_j. We can rewrite it as $AX = B$, where A is a NxN matrix:

$$A = \begin{bmatrix} 1 & -\sum_{k\neq 1} C_{1k}Cov(\alpha_k, \alpha_2) & \dots & -\sum_{k\neq 1} C_{1k}Cov(\alpha_k, \alpha_N) \\ -\sum_{k\neq 2} C_{2k}Cov(\alpha_k, \alpha_1) & 1 & \dots & -\sum_{k\neq 2} C_{2k}Cov(\alpha_k, \alpha_N) \\ \dots & \dots & \dots & \dots \\ -\sum_{k\neq N} C_{Nk}Cov(\alpha_k, \alpha_1) & -\sum_{k\neq N} C_{2k}Cov(\alpha_k, \alpha_2) & \dots & 1 \end{bmatrix},$$

that is

$$A_{ij} = \begin{cases} -\sum_{k\neq i} C_{ik}Cov(\alpha_k, \alpha_j)\,, & i \neq j \\ 1 \qquad\qquad , & i = j \end{cases}.$$

and B is a $Nx1$ vector of ones. It follows that

$$X = A^{-1}B.$$

Since B is a positive vector, in order to have X positive, it is sufficient that A^{-1} is a non-negative matrix. By Theorem 2.3. in chapter 6 of Berman and Plemmons (1994) (see also Pena (1995)), a necessary and sufficient condition for A^{-1} to be non-negative is A being a M-matrix, i.e. all off-diagonal elements are negative. Finally, it is easy to verify that A is a M-matrix whenever some, but not all, demand correlations are negative.[71] ■

8.1.4 Model with risk neutrality

With risk neutrality, the objective function is:

[71] For example, this can be seen for the case $N = 4$, where a typical element of the matrix A looks like:

$$A_{21} = \rho_{12}\sigma_1^3\sigma_2\sigma_3^2\sigma_4^2(1 - \rho_{13}^2 - \rho_{14}^2 - \rho_{34}^2 + 2\rho_{13}\rho_{14}\rho_{34}).$$

$$\max_{\{n_{ij}\}} \sum_j \bar{\alpha}_j n_{ij}(z) r_{ij}(z) - \sum_j w_j n_{ij}(z) f_j L_j / P_i$$

Notice that the above is simply linear in $n_{ij}(z)$, and therefore it is always optimal, upon entry, to set $n_{ij}(z) = 1$. Therefore the firm's problem boils down to a standard entry decision, as in Melitz (2003), which implies that the firm enters a market j only if expected profits are positive. This in turn implies the existence of an entry cutoff, given by:

$$\left(\bar{z}_{ij}\right)^{\sigma-1} = \frac{w_j f_j L_j P_j^{1-\sigma} \sigma}{\bar{\alpha}_j \left(\frac{\sigma}{\sigma-1} \tau_{ij} w_i\right)^{1-\sigma} Y_j} \tag{68}$$

To find the welfare gains from trade in the case of $\gamma = 0$, I first write the equation for trade shares

$$\lambda_{ij} = \frac{M_i \int_{\bar{z}_{ij}}^{\infty} \bar{\alpha}_j p_{ij}(z) q_{ij}(z) g_i(z) dz}{w_j L_j} = \frac{M_i \int_{\bar{z}_{ij}}^{\infty} \bar{\alpha}_j p_{ij}\left(z\right)^{1-\sigma} g_i(z) dz}{P_j^{1-\sigma}} \tag{69}$$

Inverting the above:

$$\frac{M_i \phi (\tau_{ij} w_i)^{1-\sigma} \left(\bar{z}_{ij}\right)^{\sigma-\theta-1}}{\lambda_{ij}} = P_j^{1-\sigma}. \tag{70}$$

where ϕ is a constant. Substituting for the cutoff, and using the fact that when $\gamma = 0$ profits are a constant share of total income (see ACR), I can write the real wage as a function of trade shares:

$$\left(\frac{w_j}{P_j}\right) = \vartheta \lambda_{jj}^{-\frac{1}{\theta}}, \tag{71}$$

where ϑ is a constant. Since the risk aversion is zero, and profits are a constant share of total income, the percentage change in welfare is simply:

$$d\ln W_j = -d\ln P_j \tag{72}$$

where I have also set the wage as the numeraire. Substituting 71 into 72, we get:

$$d\ln W_j = -\frac{1}{\theta} d\ln \lambda_{jj}$$

Lastly, from the equation for trade shares and using the assumption of Pareto distributed productivities, it is easy to verify that $-\theta$ equals the trade elasticity.

8.1.5 Model with autarky

Lemma 1. *Assume that* $\gamma > \left(\chi\tilde{L}\right)^{\frac{\theta+1-\sigma}{(1-\sigma)\theta}} \left(\bar{\alpha} M D_A \sigma \left(\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1}\frac{\sigma f}{\bar{\alpha}}\right)^{\frac{\theta}{1-\sigma}}\left(\frac{\sigma-1}{\theta+\sigma-1}\right)\right)^{-\frac{1}{\theta}}\left(\frac{D_A\bar{\alpha}}{4f}\right)^{\frac{1+\theta}{\theta}}$.
Then the optimal solution is:
- $n(z) = 0$ *if* $z \leq z^*$
- $0 < n(z) < 1$ *if* $z > z^*$, *where:*

$$n(z) = \frac{D_A}{\gamma}\frac{\left(1-\left(\frac{z^*}{z}\right)^{\sigma-1}\right)}{r(z)}$$

where

$$z^* = \left(\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1}\frac{fP^{1-\sigma}\sigma}{\bar{\alpha}Y}\right)^{\frac{1}{\sigma-1}}$$

and

$$r(z) = \frac{1}{P}\left(\frac{\sigma}{\sigma-1}\frac{1}{z}\right)^{1-\sigma}\frac{Y}{P^{1-\sigma}\sigma}.$$

Proof. As in Proposition 1, the optimal price is a constant markup over marginal cost:

$$p = \frac{\sigma}{\sigma-1}\frac{1}{z}$$

and thus real gross profits are:

$$r(z) = \frac{1}{P}\left(\frac{\sigma}{\sigma-1}\frac{1}{z}\right)^{1-\sigma}\frac{Y}{P^{1-\sigma}\sigma}$$

The Lagrangian is:

$$\mathcal{L}_i(z) = \bar{\alpha}n(z)r(z) - \frac{\gamma}{2}Var(\alpha)n^2(z)r^2(z) - n(z)f + \lambda n(z) + \mu(1-n(z))$$

and the FOCs are:

$$\bar{\alpha}r(z) - f/P - \gamma n(z)r^2(z)Var(\alpha) + \lambda - \mu = 0$$

Thus $n(z)$ becomes:

$$n(z) = \frac{\bar{\alpha}r(z) - f/P + \lambda - \mu}{r^2(z)Var(\alpha)\gamma}$$

To get rid of the upper bound multiplier μ, I now find a restriction on parameters such that it is always optimal to choose $n(z) < 1$. When the optimal solution is $n = 0$, then this holds trivially. If instead $n > 0$, and thus $\lambda = 0$, then it must hold that:

$$n(z) = \frac{\bar{\alpha}r(z) - f/P}{r^2(z)Var(\alpha)\gamma} < 1$$

Rearranging:

$$\gamma > \frac{\bar{\alpha}r(z) - f/P}{r^2(z)Var(\alpha)} \tag{73}$$

The RHS of the above inequality is a function of the productivity z. For the inequality to hold for any z, it suffices to hold for the productivity z that maximizes the RHS. It is easy to verify that such z is:

$$z_{max} = \left(\frac{2f}{\bar{\alpha}\tilde{u}}\right)^{\frac{1}{\sigma-1}} \tag{74}$$

where $\tilde{u} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \frac{Y}{P^{1-\sigma}\sigma}$. Therefore a sufficient condition to have 73 is:

$$\gamma > \frac{\bar{\alpha}\frac{\tilde{u}}{P}\frac{2f}{\bar{\alpha}\tilde{u}} - f/P}{\left(\frac{\tilde{u}}{P}\frac{2f}{\bar{\alpha}\tilde{u}}\right)^2 Var(\alpha)} = P\frac{\bar{\alpha}^2}{f4Var(\alpha)} \tag{75}$$

In what follows (see equation (81)), I show that if the above inequality holds, the optimal price index is given by:

$$P = (\chi\tilde{L})^{\frac{\theta+1-\sigma}{(1-\sigma)(1+\theta)}} (\kappa_2)^{-\frac{1}{\theta+1}} \tag{76}$$

where χ depends only on σ and θ, and where $\kappa_2 \equiv \bar{\alpha}M\frac{D_A\sigma}{\gamma}(x)^{\frac{\theta}{1-\sigma}}\left(\frac{\sigma-1}{\theta+\sigma-1}\right)$ and $x \equiv \left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1}\frac{\sigma f}{\bar{\alpha}}$. Plugging equation (81) into the above inequality implies that:

$$\gamma > (\chi\tilde{L})^{\frac{\theta+1-\sigma}{(1-\sigma)(1+\theta)}} \left(\bar{\alpha}MD_A\sigma\frac{1}{\gamma}(x)^{\frac{\theta}{1-\sigma}}\left(\frac{\sigma-1}{\theta+\sigma-1}\right)\right)^{-\frac{1}{\theta+1}} \frac{D_A\bar{\alpha}}{f4}$$

Rearranging:

$$\gamma > (\chi\tilde{L})^{\frac{\theta+1-\sigma}{(1-\sigma)\theta}} \left(\bar{\alpha}MD_A\sigma(x)^{\frac{\theta}{1-\sigma}}\left(\frac{\sigma-1}{\theta+\sigma-1}\right)\right)^{-\frac{1}{\theta}} \left(\frac{D_A\bar{\alpha}}{f4}\right)^{\frac{1+\theta}{\theta}} \tag{77}$$

If (77) holds, then any firm will always choose to set $n_{ij}(z) < 1$. Then, the FOC becomes:

$$\bar{\alpha}r(z) - f/P - \gamma n(z) r^2(z) Var(\alpha) + \lambda = 0$$

I now guess and verify that the optimal $n(z)$ is such that: if $z > z^*$ then $n(z) > 0$, otherwise $n(z) = 0$. First I find such cutoff by solving $n(z^*) = 0$:

$$z^* = \left(\left(\frac{\sigma}{\sigma - 1} \right)^{\sigma-1} \frac{f P^{1-\sigma} \sigma}{\bar{\alpha} Y} \right)^{\frac{1}{\sigma-1}}$$

and the corresponding optimal $n(z)$ is:

$$n(z) = \frac{1}{\gamma} \frac{\bar{\alpha}}{Var(\alpha)} \frac{\left(1 - \left(\frac{z^*}{z}\right)^{\sigma-1}\right)}{r(z)}$$

If the guess is correct, then it must be that, when $z < z^*$, the FOC is satisfied with a positive λ and thus $n(z) = 0$. Indeed, notice that setting $n(z) = 0$ gives:

$$\bar{\alpha}r(z) - f + \lambda = 0$$

and so the multiplier is:

$$\lambda = f - \bar{\alpha}r(z)$$

which is positive only if $f > \bar{\alpha}r(z)$, that is, when $z < z^*$. Therefore the guess is verified. Lastly, the optimal solution can be written more compactly as:

$$n(z) = \frac{D_A}{\gamma} \frac{\left(1 - \left(\frac{z^*}{z}\right)^{\sigma-1}\right)}{r(z)}$$

where $D_A \equiv \frac{\bar{\alpha}}{Var(\alpha)}$ is the Diversification Index.■

Equilibrium. Assuming that $\theta > \sigma - 1 > 0$, and normalizing the wage to 1, current account balance implies that total income is:

$$Y_A = w_i \tilde{L}_i + \Pi_i = \tilde{L} + \kappa_1 P^{1+\theta} Y_A^{\frac{\theta}{\sigma-1}} \tag{78}$$

where $\kappa_1 \equiv \frac{MD_A}{\gamma} (x)^{\frac{\theta}{1-\sigma}} \bar{\alpha} \left[\frac{\sigma-1-\theta}{\theta+\sigma-1} + \frac{\theta}{\theta+2\sigma-2} \right]$ and where $x \equiv \left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1} \frac{\sigma f}{\bar{\alpha}}$.
The price index equation is:

$$P_i^{1-\sigma} = \bar{\alpha} M \int_{z^*}^{\infty} n_{ji}(z) p_{ji}(z)^{1-\sigma} \theta z^{-\theta-1} dz =$$

$$= Y_A^{\frac{-\theta-1+\sigma}{1-\sigma}} P^{2-\sigma+\theta} \kappa_2$$

where $\kappa_2 \equiv \bar{\alpha} M \frac{D_A \sigma}{\gamma} (x)^{\frac{\theta}{1-\sigma}} \left(\frac{\sigma-1}{\theta+\sigma-1}\right)$. Rearranging:

$$Y_A^{\frac{\theta+1-\sigma}{1-\sigma}} / \kappa_2 = P^{1+\theta} \tag{79}$$

Plug equation 79 into equation 78:

$$Y_A = \tilde{L} + \kappa_1 P^{1+\theta} Y_A^{\frac{\theta}{\sigma-1}} =$$

$$= \tilde{L} + \frac{\kappa_1}{\kappa_2} Y_A^{\frac{\theta+1-\sigma}{1-\sigma}} Y_A^{\frac{\theta}{\sigma-1}} = \tilde{L} + \frac{\kappa_1}{\kappa_2} Y_A$$

and therefore total income is:

$$Y_A = \chi \tilde{L} \tag{80}$$

where $\chi \equiv \frac{\sigma\left(\frac{\sigma-1}{\theta+\sigma-1}\right)}{\sigma\left(\frac{\sigma-1}{\theta+\sigma-1}\right) - \left[\frac{\sigma-1-\theta}{\theta+\sigma-1} + \frac{\theta}{\theta+2\sigma-2}\right]}$, and the price index is:

$$P_A = \left(\chi \tilde{L}\right)^{\frac{\theta+1-\sigma}{(1-\sigma)(1+\theta)}} (\kappa_2)^{-\frac{1}{\theta+1}} \tag{81}$$

8.1.6 Model with two symmetric countries and free trade

Lemma 2. *Assume countries are perfectly symmetric and there is free trade. Assume that* $\gamma > \left(\chi \tilde{L}\right)^{\frac{\theta+1-\sigma}{(1-\sigma)\theta}} \left(\bar{\alpha} 2 M D_{FT} \sigma \left(\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1} \frac{\sigma f}{\bar{\alpha}}\right)^{\frac{\theta}{1-\sigma}} \left(\frac{\sigma-1}{\theta+\sigma-1}\right)\right)^{-\frac{1}{\theta}} \left(\frac{D\bar{\alpha}}{4f}\right)^{\frac{\theta+1}{\theta}}$. *Then the optimal solution is:*

- $n_{ij} = 0$ *if* $z \leq z^*$

- $0 < n(z) < 1$ *if* $z > z^*$*, where:*

$$n(z) = \frac{D_{FT}}{\gamma} \frac{\left(1 - \left(\frac{z^*}{z}\right)^{\sigma-1}\right)}{r(z)}$$

where

$$z^* = \left(\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1} \frac{f P^{1-\sigma} \sigma}{\bar{\alpha} Y}\right)^{\frac{1}{\sigma-1}}$$

and

$$r(z) = \frac{1}{P} \left(\frac{\sigma}{\sigma-1} \frac{1}{z}\right)^{1-\sigma} \frac{Y}{P^{1-\sigma} \sigma}.$$

Proof: As in Proposition 1, the optimal price is a constant markup over marginal cost:

$$p = \frac{\sigma}{\sigma - 1}\frac{1}{z}$$

and thus real gross profits are:

$$r_{ij}(z) = \frac{1}{P}\left(\frac{\sigma}{\sigma - 1}\frac{1}{z}\right)^{1-\sigma}\frac{Y_j}{P_j^{1-\sigma}\sigma}$$

In the first stage, the FOCs are:

$$\bar{\alpha}r_{ih}(z) - f/P - \gamma\left(n_{ih}r_{ih}^2(z)Var(\alpha_h) + r_{ih}(z)n_{if}(z)r_{if}(z)Cov(\alpha_h, \alpha_f)\right) + \lambda_h - \mu_h = 0$$

$$\bar{\alpha}r_{if}(z) - f/P - \gamma\left(n_{if}r_{if}^2(z)Var(\alpha_f) + r_{if}(z)n_{ih}(z)r_{ih}(z)Cov(\alpha_h, \alpha_f)\right) + \lambda_f - \mu_f = 0$$

From the above we have that:

$$n_{ih} = \frac{d_{ih}r_{if}(z) - d_{if}r_{ih}(z)\rho + r_{if}(z)\left(\lambda_h - \mu_h\right) - r_{ih}(z)\rho\left(\lambda_f - \mu_f\right)}{\gamma Var(\alpha)r_{ih}^2(z)r_{if}(z)\left(1 - \rho^2\right)}$$

$$n_{if} = \frac{d_{if}r_{ih}(z) - d_{ih}r_{if}(z)\rho + r_{ih}(z)\left(\lambda_f - \mu_f\right) - r_{if}(z)\rho\left(\lambda_h - \mu_h\right)}{\gamma Var(\alpha)r_{if}^2(z)r_{ih}(z)\left(1 - \rho^2\right)}$$

where

$$d_{ij} \equiv \bar{\alpha}r_{ij}(z) - f/P$$

To get rid of the upper bound multipliers μ_h and μ_f, I now find a restriction on parameters such that it is always optimal to choose $n_{ij}(z) < 1$. When the optimal solution is $n_{ij} = 0$, then this holds trivially. If instead $n_{ij} > 0$, and thus $\lambda_j = 0$, then it must hold that:

$$n_{ij} = \frac{d_{ij}r_{ik}(z) - d_{ik}r_{ij}(z)\rho}{\gamma Var(\alpha)r_{ij}^2(z)r_{ik}(z)\left(1 - \rho^2\right)} < 1$$

for all j, where $k \neq j$. For the home country, this becomes:

$$\left(\bar{\alpha}r_{ih}(z) - f/P\right)r_{if}(z) - \left(\bar{\alpha}r_{if}(z) - f/P\right)r_{ih}(z)\rho < \gamma Var(\alpha)r_{ih}^2(z)r_{if}(z)\left(1 - \rho^2\right)$$

Invoking symmetry:

$$\left(\bar{\alpha}uz^{\sigma-1} - f/P\right)uz^{\sigma-1} - \left(\bar{\alpha}uz^{\sigma-1} - f/P\right)uz^{\sigma-1}\rho < \gamma Var(\alpha)u^2z^{2(\sigma-1)}uz^{\sigma-1}\left(1 - \rho^2\right)$$

$$\left(\bar{\alpha}uz^{\sigma-1}-f/P\right)(1-\rho)<\gamma Var(\alpha)u^2z^{2(\sigma-1)}\left(1-\rho^2\right)$$

$$\left(\bar{\alpha}uz^{\sigma-1}-f/P\right)<\gamma Var(\alpha)u^2z^{2(\sigma-1)}(1+\rho)$$

where $u=\frac{1}{P}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\frac{Y}{P^{1-\sigma}\sigma}$. Rearranging:

$$\gamma>\frac{1}{Var(\alpha)uz^{\sigma-1}(1+\rho)}\left(\bar{\alpha}-\frac{f/P}{z^{\sigma-1}u}\right) \tag{82}$$

The RHS of the above inequality is a function of the productivity z. For the inequality to hold for any z, it suffices to hold for the productivity z that maximizes the RHS. It is easy to verify that such z is:

$$z_{max}=\left(\frac{2f}{\bar{\alpha}\tilde{u}}\right)^{\frac{1}{\sigma-1}} \tag{83}$$

where $\tilde{u}=\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\frac{Y}{P^{1-\sigma}\sigma}$. Therefore a sufficient condition to have 82 is:

$$\gamma>\frac{1}{Var(\alpha)u\frac{2f}{\bar{\alpha}\tilde{u}}(1+\rho)}\left(\bar{\alpha}-\frac{f}{\frac{2f}{\bar{\alpha}\tilde{u}}\tilde{u}}\right)=P\frac{\bar{\alpha}^2}{Var(\alpha)4f(1+\rho)} \tag{84}$$

In what follows, I show that if the above inequality holds, the optimal price index is given by:

$$P=\left(\chi\tilde{L}\right)^{\frac{\theta+1-\sigma}{(1-\sigma)(1+\theta)}}(\kappa_4)^{-\frac{1}{\theta+1}} \tag{85}$$

where χ depends only on σ and θ, and $\kappa_4\equiv\bar{\alpha}2M\frac{D\sigma}{\gamma}\left(\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1}\frac{\sigma f}{\bar{\alpha}}\right)^{\frac{\theta}{1-\sigma}}\left(\frac{\sigma-1}{\theta+\sigma-1}\right)$. Therefore the risk aversion has to satisfy:

$$\gamma>\left(\chi\tilde{L}\right)^{\frac{\theta+1-\sigma}{(1-\sigma)(1+\theta)}}(\kappa_4)^{-\frac{1}{\theta+1}}\frac{\bar{\alpha}^2}{Var(\alpha)4f(1+\rho)}$$

Rearranging:

$$\gamma>\left(\chi\tilde{L}\right)^{\frac{\theta+1-\sigma}{(1-\sigma)\theta}}\left(\bar{\alpha}2MD_{FT}\sigma\left(\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1}\frac{\sigma f}{\bar{\alpha}}\right)^{\frac{\theta}{1-\sigma}}\left(\frac{\sigma-1}{\theta+\sigma-1}\right)\right)^{-\frac{1}{\theta}}\left(\frac{D_{FT}\bar{\alpha}}{4f}\right)^{\frac{\theta+1}{\theta}} \tag{86}$$

where the right hand side is only function of parameters.

If (86) holds, then any firm will always choose to set $n_{ij}(z)<1$. Then, given the symmetry of the economy, each firm will either sell to both the domestic and foreign market, or to none. This implies that the FOC becomes:

$$\bar{\alpha}r(z) - f/P - \gamma n_{ih}(z)r^2(z)Var(\alpha_h)\,(1+\rho) + \lambda_h = 0$$

I now guess and verify that the optimal $n_{ih}(z)$ is such that: if $z > z^*$ then $n_{ih}(z) > 0$, otherwise $n_{ih}(z) = 0$. First I find such cutoff by solving $n_{ih}(z^*) = 0$:

$$z^* = \left(\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1}\frac{fP^{1-\sigma}\sigma}{\bar{\alpha}Y}\right)^{\frac{1}{\sigma-1}}$$

and the corresponding optimal $n(z)$ is:

$$n(z) = \frac{1}{\gamma}\frac{\bar{\alpha}}{Var(\alpha)\,(1+\rho)}\frac{\left(1-\left(\frac{z^*}{z}\right)^{\sigma-1}\right)}{r(z)}$$

If the guess is correct, then it must be that, when $z < z^*$, the FOC is satisfied with a positive λ_h and thus $n(z) = 0$. Indeed, notice that setting $n(z) = 0$ gives:

$$\bar{\alpha}r(z) - f + \lambda_h = 0$$

and so the multiplier is:

$$\lambda_h = f - \bar{\alpha}r(z)$$

which is positive only if $f > \bar{\alpha}r(z)$, that is, when $z < z^*$. Therefore the guess is verified. Lastly, the optimal solution can be written as:

$$n(z) = \frac{D_{FT}}{\gamma}\frac{\left(1-\left(\frac{z^*}{z}\right)^{\sigma-1}\right)}{r(z)}$$

where $D_{FT} \equiv \frac{\bar{\alpha}}{Var(\alpha)(1+\rho)}$ is the Diversification Index.■

Equilibrium with free trade. Assuming as before that $\theta > \sigma - 1$, and normalizing the wage to 1, current account balance implies that total income is:

$$Y_{FT} = w_i\tilde{L}_i + \Pi_i = \tilde{L} + \kappa_3 P_{FT}^{1+\theta}Y^{\frac{\theta}{\sigma-1}} \tag{87}$$

where $\kappa_3 \equiv \frac{2MD_{FT}}{\gamma}\,(x)^{\frac{\theta}{1-\sigma}}\,\bar{\alpha}\left[\frac{\sigma-1-\theta}{\theta+\sigma-1} + \frac{\theta}{\theta+2\sigma-2}\right]$ and where $x \equiv \left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1}\frac{\sigma f}{\bar{\alpha}}$.

The price index equation is:

$$P_{FT}^{1-\sigma} = \bar{\alpha}2M\int_{z^*}^{\infty} n_{ji}(z)p_{ji}(z)^{1-\sigma}\theta z^{-\theta-1}dz =$$

$$= Y_{FT}^{\frac{-\theta-1+\sigma}{1-\sigma}} P_{FT}^{2-\sigma+\theta}\kappa_4$$

where $\kappa_4 \equiv \bar{\alpha}2M\frac{D_{FT}\sigma}{\gamma}(x)^{\frac{\theta}{1-\sigma}}\left(\frac{\sigma-1}{\theta+\sigma-1}\right)$. Rearranging:

$$Y_{FT}^{\frac{\theta+1-\sigma}{1-\sigma}}/\kappa_4 = P_{FT}^{1+\theta} \tag{88}$$

Plug equation (88) into equation (87):

$$Y_{FT} = \tilde{L} + \kappa_3 P_{FT}^{1+\theta} Y_{FT}^{\frac{\theta}{\sigma-1}} =$$

$$= \tilde{L} + \frac{\kappa_3}{\kappa_4} Y_{FT}^{\frac{\theta+1-\sigma}{1-\sigma}} Y_{FT}^{\frac{\theta}{\sigma-1}} = \tilde{L} + \frac{\kappa_3}{\kappa_4} Y_{FT}$$

and therefore total income is:

$$Y_{FT} = \chi\tilde{L} \tag{89}$$

where $\chi \equiv \frac{\sigma\left(\frac{\sigma-1}{\theta+\sigma-1}\right)}{\sigma\left(\frac{\sigma-1}{\theta+\sigma-1}\right)-\left[\frac{\sigma-1-\theta}{\theta+\sigma-1}+\frac{\theta}{\theta+2\sigma-2}\right]}$, and the price index is:

$$P_{FT} = \left(\chi\tilde{L}\right)^{\frac{\theta+1-\sigma}{(1-\sigma)(1+\theta)}}(\kappa_4)^{-\frac{1}{\theta+1}} \tag{90}$$

8.1.7 Proof of Proposition 3

In Lemma 1, under autarky we impose that

$$\gamma > \tilde{\gamma}_A \equiv \left(\chi\tilde{L}\right)^{\frac{\theta+1-\sigma}{(1-\sigma)\theta}}\left(\bar{\alpha}MD_A\sigma\left(\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1}\frac{\sigma f}{\bar{\alpha}}\right)^{\frac{\theta}{1-\sigma}}\left(\frac{\sigma-1}{\theta+\sigma-1}\right)\right)^{-\frac{1}{\theta}}\left(\frac{D_A\bar{\alpha}}{4f}\right)^{\frac{1+\theta}{\theta}}$$

In Lemma 2, under free trade we impose that

$$\gamma > \tilde{\gamma}_{FT} \equiv \left(\chi\tilde{L}\right)^{\frac{\theta+1-\sigma}{(1-\sigma)\theta}}\left(\bar{\alpha}2MD_{FT}\sigma\left(\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1}\frac{\sigma f}{\bar{\alpha}}\right)^{\frac{\theta}{1-\sigma}}\left(\frac{\sigma-1}{\theta+\sigma-1}\right)\right)^{-\frac{1}{\theta}}\left(\frac{D_{FT}\bar{\alpha}}{4f}\right)^{\frac{\theta+1}{\theta}} =$$

$$= \tilde{\gamma}_A \frac{1}{1+\rho}$$

Thus we can combine Lemma 1 and 2 and state that if

$$\gamma > (\chi \tilde{L})^{\frac{\theta+1-\sigma}{(1-\sigma)\theta}} \left(\bar{\alpha} M \sigma \left(\left(\frac{\sigma}{\sigma-1} \right)^{\sigma-1} \frac{\sigma f}{\bar{\alpha}} \right)^{\frac{\theta}{1-\sigma}} \left(\frac{\sigma-1}{\theta+\sigma-1} \right) \right)^{-\frac{1}{\theta}} \left(\frac{\bar{\alpha}}{4f} \right)^{\frac{1+\theta}{\theta}} max \left\{ \frac{1}{1+\rho}, 1 \right\},$$

then

$$n(z) = 0 \qquad \text{if } z \leq z^*$$
$$n(z) = \frac{D}{\gamma} \frac{\left(1 - \left(\frac{z^*}{z}\right)^{\sigma-1}\right)}{r(z)} \text{ if } z > z^*$$

8.1.8 Proof of Proposition 4

The Sharpe Ratio is defined as

$$SR(z) = \frac{E\left[\frac{\pi^{FT}(z)}{P^{FT}}\right]}{Std\left[\frac{\pi^{FT}(z)}{P^{FT}}\right]}.$$

Under free trade expected real profits are

$$E\left[\frac{\pi^{FT}(z)}{P^{FT}}\right] = \frac{1}{P} \left(2\bar{\alpha} n(z) \left(\frac{\sigma}{\sigma-1} \frac{1}{z} \right)^{1-\sigma} \frac{Y}{P^{1-\sigma}\sigma} - 2fn(z) \right) =$$

$$= 2\bar{\alpha} \frac{D}{\gamma} \left(1 - \left(\frac{z^*}{z} \right)^{\sigma-1} \right) \left(1 - \frac{fP^{1-\sigma}\sigma}{\left(\frac{\sigma}{\sigma-1}\frac{1}{z}\right)^{1-\sigma} Y\bar{\alpha}} \right) =$$

$$= 2\bar{\alpha} \frac{D}{\gamma} \left(1 - \left(\frac{z^*}{z} \right)^{\sigma-1} \right)^2.$$

Instead, under free trade the standard deviation of real profits is

$$Std\left[\frac{\pi^{FT}(z)}{P^{FT}}\right] = \sqrt{Var(\alpha)\left(\frac{\pi(z)}{P}\right)^2 + Var(\alpha)\left(\frac{\pi(z)}{P}\right)^2 + 2\left(\frac{\pi(z)}{P}\right)^2 Cov\left(\alpha_H, \alpha_F\right)} =$$

$$= \frac{\pi(z)}{P} Std(\alpha)\sqrt{2(1+\rho)} =$$

$$= \frac{D}{\gamma}\left(1-\left(\frac{z^{*FT}}{z}\right)^{\sigma-1}\right) Std(\alpha)\sqrt{2(1+\rho)}.$$

Therefore the firm-level Sharpe Ratio under free trade is

$$SR^{FT}(z) = \frac{2\bar{\alpha}\frac{D}{\gamma}\left(1-\left(\frac{z^{*FT}}{z}\right)^{\sigma-1}\right)^2}{\frac{D}{\gamma}\left(1-\left(\frac{z^{*FT}}{z}\right)^{\sigma-1}\right) Std(\alpha)\sqrt{2(1+\rho)}} =$$

$$= \sqrt{\frac{2}{1+\rho}}\frac{\bar{\alpha}}{Std(\alpha)}\left(1-\left(\frac{z^{*FT}}{z}\right)^{\sigma-1}\right).$$

Similarly, under autarky the Sharpe Ratio equals

$$SR^{A}(z) = \frac{\bar{\alpha}}{Std(\alpha)}\left(1-\left(\frac{z^{*A}}{z}\right)^{\sigma-1}\right).$$

Therefore the Sharpe Ratio changes by

$$\hat{SR}(z) = \frac{SR^{FT}(z)}{SR^{A}(z)} = \sqrt{\frac{2}{1+\rho}}\frac{\left(1-\left(\frac{z^{*FT}}{z}\right)^{\sigma-1}\right)}{\left(1-\left(\frac{z^{*A}}{z}\right)^{\sigma-1}\right)}$$

Substituting the entry cutoffs from Lemma 1 and 2, and then substituting the equilibrium solutions for P and Y from equations (80)-(81) and (89)-(90), we obtain

$$= \sqrt{\frac{2}{1+\rho}}\frac{\left(1-(1+\rho)^{-\frac{\sigma-1}{\theta+1}}(2)^{\frac{\sigma-1}{\theta+1}}\psi\,(z)^{1-\sigma}\right)}{\left(1-(z)^{1-\sigma}\,\psi\right)}$$

where

$$\psi \equiv \gamma^{\frac{1-\sigma}{\theta+1}} M^{\frac{\sigma-1}{\theta+1}}\left(\tilde{L}\right)^{\frac{-\sigma}{(1+\theta)}} f^{1+\theta}\tilde{\sigma}. \tag{91}$$

and $\tilde{\sigma} \equiv \left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1}\left(\frac{\sigma\left(\frac{\sigma-1}{\theta+\sigma-1}\right)}{\sigma\left(\frac{\sigma-1}{\theta+\sigma-1}\right)-\left[\frac{\sigma-1-\theta}{\theta+\sigma-1}+\frac{\theta}{\theta+2\sigma-2}\right]}\right)^{\frac{-\sigma}{(1+\theta)}}\left(\left(\frac{\sigma}{\sigma-1}\right)^{-\theta}\sigma^{\frac{\theta+1-\sigma}{1-\sigma}}\left(\frac{\sigma-1}{\theta+\sigma-1}\right)\right)^{\frac{\sigma-1}{\theta+1}}\sigma.$

The change in the Sharpe Ratio is increasing in firm productivity:

$$\frac{\partial \hat{SR}(z)}{\partial z} = \sqrt{\frac{2}{1+\rho}} \frac{\left((\sigma-1)(1+\rho)^{-\frac{\sigma-1}{\theta+1}}(2)^{\frac{\sigma-1}{\theta+1}}(z)^{-\sigma}\psi\right)\left(1-(z)^{1-\sigma}\psi\right)-\left((\sigma-1)(z)^{-\sigma}\psi\right)\left(1-(1+\rho)^{-\frac{\sigma-1}{\theta+1}}(2)^{\frac{\sigma-1}{\theta+1}}(z)^{1-\sigma}\psi\right)}{\left(1-(z)^{1-\sigma}\psi\right)^2}$$

this is always positive since

$$(1+\rho)^{-\frac{\sigma-1}{\theta+1}}(2)^{\frac{\sigma-1}{\theta+1}}\left(1-(z)^{1-\sigma}\psi\right) > 1-(1+\rho)^{-\frac{\sigma-1}{\theta+1}}(2)^{\frac{\sigma-1}{\theta+1}}(z)^{1-\sigma}\psi$$

$$(1+\rho)^{-\frac{\sigma-1}{\theta+1}}(2)^{\frac{\sigma-1}{\theta+1}} > 1$$

$$2 > (1+\rho)$$

$$1 > \rho$$

The change in the Sharpe Ratio is decreasing in the correlation as long as

$$\frac{\partial \hat{SR}(z)}{\partial \rho} = \frac{1}{\left(1-(z)^{1-\sigma}k\right)}\sqrt{2}\left[-\frac{1}{2}(1+\rho)^{-\frac{3}{2}}+\left(\frac{\sigma-1}{\theta+1}+\frac{1}{2}\right)(1+\rho)^{-\frac{\sigma-1}{\theta+1}-\frac{3}{2}}(2)^{\frac{\sigma-1}{\theta+1}}\psi(z)^{1-\sigma}\right] < 0$$

$$\left(\frac{\sigma-1}{\theta+1}+\frac{1}{2}\right)(2)^{\frac{\sigma+\theta}{\theta+1}}\psi(z)^{1-\sigma} < (1+\rho)^{\frac{\sigma-1}{\theta+1}}$$

8.1.9 Proof of Proposition 5

Welfare under autarky is:

$$W_A = \frac{Y_A}{P_A} - M\int_{z^*}\frac{\gamma}{2}Var\left(\frac{\pi(z)}{P_A}\right)\theta z^{-\theta-1}dz =$$

$$= \frac{Y_A}{P_A} - M\int_{z^*}\frac{\gamma}{2}Var(\alpha)n^2(z)r^2(z)\theta z^{-\theta-1}dz$$

since marketing costs are non-stochastic. Then

$$W_A = \frac{Y_A}{P_A} - \frac{M}{2}Var(\alpha)\frac{D^2}{\gamma}\int_{z^*}\left(1-\left(\frac{z^*}{z}\right)^{\sigma-1}\right)^2\theta z^{-\theta-1}dz =$$

$$= \frac{Y_A}{P_A} - \frac{M}{2}Var(\alpha)\frac{D^2}{\gamma}\int_{z^*}\left(1+\left(\frac{z^*}{z}\right)^{2(\sigma-1)}-2\left(\frac{z^*}{z}\right)^{\sigma-1}\right)\theta z^{-\theta-1}dz =$$

$$= \frac{Y_A}{P_A} - \frac{M}{2}Var(\alpha)\frac{D^2}{\gamma}(z^*)^{-\theta}\left(\frac{\sigma-1-\theta}{\theta+\sigma-1}+\frac{\theta}{\theta+2-2\sigma}\right) =$$

$$= \frac{Y_A}{P_A} - P_A^{\theta} Y_A^{\frac{\theta}{\sigma-1}} \frac{M}{2\gamma} D\bar{\alpha} x^{\frac{\theta}{1-\sigma}} \left(\frac{\theta}{\theta - 2 + 2\sigma} + \frac{\sigma - 1 - \theta}{\theta + \sigma - 1} \right) =$$

$$= \frac{Y_A}{P_A} - \kappa_5 P_A^{\theta} Y_A^{\frac{\theta}{\sigma-1}} \tag{92}$$

where $\kappa_5 = M \frac{D_A \bar{\alpha}}{2\gamma} (x)^{\frac{\theta}{1-\sigma}} \left[\frac{\sigma-1-\theta}{\theta+\sigma-1} + \frac{\theta}{\theta+2\sigma-2} \right]$. Let's further simplify the above:

$$W_A = \left(\chi\tilde{L}\right)^{\frac{\sigma\theta}{(\sigma-1)(1+\theta)}} (\kappa_2)^{\frac{1}{\theta+1}} - \kappa_5 \left(\chi\tilde{L}\right)^{\frac{\theta\sigma}{(\sigma-1)(1+\theta)}} (\kappa_2)^{-\frac{\theta}{\theta+1}} =$$

$$= \left(\chi\tilde{L}\right)^{\frac{\sigma\theta}{(\sigma-1)(1+\theta)}} \left[(\kappa_2)^{\frac{1}{\theta+1}} - \kappa_5 (\kappa_2)^{-\frac{\theta}{\theta+1}} \right] \tag{93}$$

Welfare under free trade is instead:

$$W_{FT} = \frac{Y}{P} - M \int_{z^*}^{\infty} \frac{\gamma}{2} Var\left(\frac{\pi(z)}{P}\right) \theta z^{-\theta-1} dz =$$

$$= \frac{Y}{P} - M \int_{z^*}^{\infty} \frac{\gamma}{2} \left(Var(\alpha) \left(\frac{\pi_{HH}(z)}{P}\right)^2 + Var(\alpha) \left(\frac{\pi_{HF}(z)}{P}\right)^2 + 2\frac{\pi_{HF}(z)}{P}\frac{\pi_{HH}(z)}{P} Cov\left(\alpha_H, \alpha_F\right) \right) \theta z^{-\theta-1} dz$$

where π_{ij} are gross profits (since marketing costs are non-stochastic). By symmetry (and by absence of trade costs):

$$W_{FT} = \frac{Y}{P} - M \int_{z^*}^{\infty} \frac{\gamma}{2} \left(Var(\alpha) \left(\frac{\pi(z)}{P}\right)^2 + Var(\alpha) \left(\frac{\pi(z)}{P}\right)^2 + 2\left(\frac{\pi(z)}{P}\right)^2 Cov\left(\alpha_H, \alpha_F\right) \right) \theta z^{-\theta-1} dz =$$

$$= \frac{Y}{P} - M\gamma Var(\alpha) (1+\rho) \int_{z^*}^{\infty} \left(\frac{\pi(z)}{P}\right)^2 \theta z^{-\theta-1} dz =$$

$$= \frac{Y}{P} - MVar(\alpha) (1+\rho) \frac{D^2}{\gamma} \int_{z^*}^{\infty} \left(1 - \left(\frac{z^*}{z}\right)^{\sigma-1}\right)^2 \theta z^{-\theta-1} dz =$$

$$= \frac{Y}{P} - MVar(\alpha) (1+\rho) \frac{D^2}{\gamma} (z^*)^{-\theta} \left(\frac{\sigma-1-\theta}{\theta+\sigma-1} + \frac{\theta}{\theta+2-2\sigma} \right) =$$

$$= \frac{Y}{P} - MVar(\alpha) (1+\rho) \frac{D^2}{\gamma} \left(\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1} \frac{fP^{1-\sigma}\sigma}{\bar{\alpha}Y} \right)^{-\frac{\theta}{\sigma-1}} \left(\frac{\sigma-1-\theta}{\theta+\sigma-1} + \frac{\theta}{\theta+2-2\sigma} \right) =$$

$$= \frac{Y}{P} - \kappa_6 P^{\theta} Y^{\frac{\theta}{\sigma-1}} \tag{94}$$

where $\kappa_6 = M\frac{1}{\gamma}\bar{\alpha}D_{FT}\left(x\right)^{\frac{\theta}{1-\sigma}}\left[\frac{\sigma-1-\theta}{\theta+\sigma-1}+\frac{\theta}{\theta+2\sigma-2}\right]$. Further simplify:

$$W_{FT} = \left(\chi\tilde{L}\right)^{1-\frac{\theta+1-\sigma}{(1-\sigma)(1+\theta)}}\left(\kappa_4\right)^{\frac{1}{\theta+1}} - \kappa_6 P^{\theta} Y^{\frac{\theta}{\sigma-1}} =$$

$$= \left(\chi\tilde{L}\right)^{\frac{\sigma\theta}{(\sigma-1)(1+\theta)}}\left[\left(\kappa_4\right)^{\frac{1}{\theta+1}} - \kappa_6\left(\kappa_4\right)^{-\frac{\theta}{\theta+1}}\right] \tag{95}$$

Using equations (93) and (95), welfare gains are:

$$\hat{W} = \frac{W_{FT}}{W_A} =$$

$$= \frac{\left(\chi\tilde{L}\right)^{\frac{\sigma\theta}{(\sigma-1)(1+\theta)}}\left[\left(\kappa_4\right)^{\frac{1}{\theta+1}} - \kappa_6\left(\kappa_4\right)^{-\frac{\theta}{\theta+1}}\right]}{\left(\chi\tilde{L}\right)^{\frac{\sigma\theta}{(\sigma-1)(1+\theta)}}\left[\left(\kappa_2\right)^{\frac{1}{\theta+1}} - \kappa_5\left(\kappa_2\right)^{-\frac{\theta}{\theta+1}}\right]} =$$

$$= \frac{\left(\kappa_4\right)^{\frac{1}{\theta+1}} - \kappa_6\left(\kappa_4\right)^{-\frac{\theta}{\theta+1}}}{\left(\kappa_4\frac{1+\rho}{2}\right)^{\frac{1}{\theta+1}} - \kappa_5\left(\kappa_4\frac{1+\rho}{2}\right)^{-\frac{\theta}{\theta+1}}} =$$

$$= (2)^{\frac{1}{\theta+1}}\left(1+\rho\right)^{-\frac{1}{\theta+1}} \tag{96}$$

since I set $Var(\alpha) = \bar{\alpha} = 1$. Given the symmetry, with free trade $\lambda_{jj} = \frac{1}{2}$ in both ACR and in my model. In autarky instead, $\lambda_{jj} = 1$. Therefore the change in trade shares is the same across models, and we can use the ACR formula to compare welfare gains:

$$\hat{W}_{ACR} = \left(\lambda_{jj}\right)^{-\frac{1}{\theta}} = \left(\frac{1}{2}\right)^{-\frac{1}{\theta}} \tag{97}$$

Therefore I can write welfare gains in my model as:

$$\hat{W} = \left(\frac{1}{2}\right)^{-\frac{1}{\theta+1}}\left(1+\rho\right)^{-\frac{1}{\theta+1}} =$$

$$= \left(\frac{1}{2}\right)^{-\frac{1}{\theta}}\left(\frac{1}{2}\right)^{\frac{1}{\theta(\theta+1)}}\left(1+\rho\right)^{-\frac{1}{\theta+1}}$$

Taking logs

$$ln\hat{W} = -\frac{1}{\theta}ln\left(\frac{1}{2}\right) + \frac{1}{\theta(\theta+1)}ln\left(\frac{1}{2}\right) - \frac{1}{\theta+1}ln\left(1+\rho\right) =$$

$$= \hat{W}_{ACR} - \hat{W}_{ACR}\frac{1}{(\theta+1)} - \frac{1}{\theta+1}ln\left(1+\rho\right)$$

$$= \ln\hat{W}_{ACR}\cdot\frac{\theta}{\theta+1} - \underbrace{\frac{1}{\theta+1}\ln\left(1+\rho\right)}_{\text{diversification effect}}$$

■

8.1.10 Model with free-entry

I assume that there is free-entry of firms. Specifically, there is an unbounded set of risk-averse entrepreneurs that decide whether to pay a fixed entry cost to draw a productivity level and start a business. The free-entry assumption implies that entrepreneurs enter up to the point in which the expected utility of running a firm equals the entry cost:

$$\int_{z^*} E\left(\frac{\pi_i(z)}{P_i}\right)\theta z^{-\theta-1}dz - \frac{\gamma}{2}\int_{z^*} Var\left(\frac{\pi_i(z)}{P_i}\right)\theta z^{-\theta-1}dz = f^e \tag{98}$$

Note that this is different from the usual free-entry condition in Melitz (2003), where firms enter until expected profits are equated to entry costs.

Free-trade. Equation (98) can be expanded as:

$$2\left(\bar{\alpha}\int_{z^*}^{\infty} n(z)r(z)\theta z^{-\theta-1}dz - \int_{z^*}^{\infty}\frac{f}{P}n(z)\theta z^{-\theta-1}dz\right) - \frac{\gamma}{2}\int_{z^*}\xi(z)\theta z^{-\theta-1}dz = f^e$$

where $\xi(z) \equiv \left(Var(\alpha)\left(\frac{\pi_{HH}(z)}{P}\right)^2 + Var(\alpha)\left(\frac{\pi_{HF}(z)}{P}\right)^2 + 2\frac{\pi_{HF}(z)}{P}\frac{\pi_{HH}(z)}{P}Cov\left(\alpha_H,\alpha_F\right)\right)$, which after some algebra becomes

$$P_{FT} = (\zeta_1)^{-\frac{1}{\theta}}\,Y_{FT}^{\frac{1}{1-\sigma}}$$

where $\zeta_1 \equiv \bar{\alpha}\frac{D_{FT}}{\gamma f^e}\left(x\right)^{\frac{-\theta}{\sigma-1}}\left(\frac{\sigma-1-\theta}{\theta+\sigma-1} + \frac{\theta}{\theta+2\sigma-2}\right)$. The price index equation is

$$Y_{FT}^{\frac{\theta+1-\sigma}{1-\sigma}}/\zeta_2 = MP_{FT}^{1+\theta}$$

where $\zeta_2 \equiv \bar{\alpha}2\frac{D_{FT}\sigma}{\gamma}\left(x\right)^{\frac{\theta}{1-\sigma}}\left(\frac{\sigma-1}{\theta+\sigma-1}\right)$. The current account balance is

$$Y_{FT} = \tilde{L} + M\zeta_3 P_{FT}^{1+\theta}Y_{FT}^{\frac{\theta}{\sigma-1}}$$

where $\zeta_3 \equiv \zeta_1 f^e$. Replace this into the price index equation:

$$Y_{FT} = \tilde{L} + Y_{FT}^{\frac{\theta}{\sigma-1}}\zeta_3 Y_{FT}^{\frac{\theta+1-\sigma}{1-\sigma}}/\zeta_2$$

which rearranged gives:

$$Y_{FT} = \chi \tilde{L}$$

where

$$\chi = \frac{\sigma(\sigma-1)}{(\sigma-1)^2 + \theta - \frac{\theta(\theta+\sigma-1)}{\theta+2\sigma-2}}$$

Note that Y_{FT} is exactly the same as in the model without free entry. Thus the price index is

$$P_{FT} = \left(\bar{\alpha}\frac{D_{FT}}{f^e\gamma}(x)^{\frac{-\theta}{\sigma-1}}\left(\frac{\sigma-1-\theta}{\theta+\sigma-1} + \frac{\theta}{\theta+2\sigma-2}\right)\right)^{-\frac{1}{\theta}}(\chi\tilde{L})^{\frac{1}{(1-\sigma)}}$$

The number of firms is

$$M_{FT} = (\chi\tilde{L})^{\frac{\sigma}{(\sigma-1)}}\left(\bar{\alpha}\frac{D_{FT}}{f^e\gamma}(x)^{\frac{-\theta}{\sigma-1}}\left(\frac{\sigma-1-\theta}{\theta+\sigma-1} + \frac{\theta}{\theta+2\sigma-2}\right)\right)^{\frac{1+\theta}{\theta}}/\zeta_2$$

Autarky. Under autarky the free entry equation becomes

$$P_A = (\zeta_5)^{-\frac{1}{\theta}} Y_A^{\frac{1}{(1-\sigma)}}$$

where $\zeta_5 \equiv \frac{1}{2}\frac{D_A}{f^e\gamma}(x)^{\frac{\theta}{1-\sigma}}\bar{\alpha}\left[\frac{\sigma-1-\theta}{\theta+\sigma-1} + \frac{\theta}{\theta+2\sigma-2}\right]$. The price index equation implies:

$$Y_A^{\frac{\theta+1-\sigma}{1-\sigma}}/\zeta_6 = M_A P_A^{1+\theta}$$

where $\zeta_6 \equiv \bar{\alpha}\frac{D_A\sigma}{\gamma}(x)^{\frac{\theta}{1-\sigma}}\left(\frac{\sigma-1}{\theta+\sigma-1}\right)$. The current account balance is:

$$Y_A = \tilde{L} + \zeta_5 M_A P_A^{1+\theta} Y_A^{\frac{\theta}{\sigma-1}}$$

Thus income under autarky and free entry is:

$$Y_A = \chi\tilde{L}$$

where $\chi = \frac{1}{1-\zeta_5/\zeta_6}$ is the same as before. The price index is

$$P_A = (\zeta_5)^{-\frac{1}{\theta}}(\chi\tilde{L})^{\frac{1}{(1-\sigma)}} =$$

$$= \left(\frac{D_A\bar{\alpha}}{2f^e\gamma}(x)^{\frac{\theta}{1-\sigma}}\left[\frac{\sigma-1-\theta}{\theta+\sigma-1} + \frac{\theta}{\theta+2\sigma-2}\right]\right)^{-\frac{1}{\theta}}(\chi\tilde{L})^{\frac{1}{(1-\sigma)}}$$

and the number of firms is

$$M_A = Y_A^{\frac{\theta+1-\sigma}{1-\sigma}} P_A^{-1-\theta}/\zeta_6 =$$

$$= (\zeta_5)^{\frac{1+\theta}{\theta}} \left(\chi \tilde{L}\right)^{\frac{\sigma}{\sigma-1}} / \zeta_6$$

It is easy to verify that the change of the number of firms between the two equilibria equals:

$$\frac{M_{FT}}{M_A} = (2D_{FT})^{\frac{1}{\theta}} \tag{99}$$

Welfare gains from trade. The welfare gains from trade with free-entry of firms are:

$$\hat{W} = \frac{W_{FT}}{W_A} = \frac{\frac{w}{P} + \frac{\Pi}{P} - R}{\frac{w}{P} + \frac{\Pi}{P} - R} = \frac{\frac{1}{P_{FT}} + M_{FT} f^e}{\frac{1}{P_A} + M_A f^e} =$$

$$= \frac{\left(\bar{\alpha} \frac{D_{FT}}{f^e \gamma} (x)^{\frac{-\theta}{\sigma-1}} \left(\frac{\sigma-1-\theta}{\theta+\sigma-1} + \frac{\theta}{\theta+2\sigma-2}\right)\right)^{\frac{1}{\theta}} \left(\chi \tilde{L}\right)^{\frac{1}{\sigma-1}} + M_{FT} f^e}{\left(\frac{D_A \bar{\alpha}}{2 f^e \gamma} (x)^{\frac{\theta}{1-\sigma}} \left[\frac{\sigma-1-\theta}{\theta+\sigma-1} + \frac{\theta}{\theta+2\sigma-2}\right]\right)^{\frac{1}{\theta}} \left(\chi \tilde{L}\right)^{\frac{1}{\sigma-1}} + M_A f^e} =$$

$$= \frac{\left(\bar{\alpha} \frac{D_{FT}}{f^e \gamma} (x)^{\frac{-\theta}{\sigma-1}} \left(\frac{\sigma-1-\theta}{\theta+\sigma-1} + \frac{\theta}{\theta+2\sigma-2}\right)\right)^{\frac{1}{\theta}} \left(\chi \tilde{L}\right)^{\frac{1}{\sigma-1}} + \left(\chi \tilde{L}\right)^{\frac{\sigma}{(\sigma-1)}} \left(\frac{1}{2}\zeta_1\right)^{\frac{1+\theta}{\theta}} / \zeta_2 f^e}{\left(\frac{D_A \bar{\alpha}}{2 f^e \gamma} (x)^{\frac{\theta}{1-\sigma}} \left[\frac{\sigma-1-\theta}{\theta+\sigma-1} + \frac{\theta}{\theta+2\sigma-2}\right]\right)^{\frac{1}{\theta}} \left(\chi \tilde{L}\right)^{\frac{1}{\sigma-1}} + \left(\chi \tilde{L}\right)^{\frac{\sigma}{\sigma-1}} \left(\frac{1}{2}\zeta_5\right)^{\frac{1+\theta}{\theta}} / \zeta_6 f^e} =$$

$$= (2D_{FT})^{\frac{1}{\theta}}$$

Therefore welfare gains can be written as

$$\ln \hat{W} = \ln \hat{W}_{ACR} - \frac{1}{\theta} \ln(1+\rho)$$

8.1.11 Covariance of log shocks

Starting from the covariance of the log-change of the shocks:

$$Cov\left(\Delta\tilde{\alpha}_j, \Delta\tilde{\alpha}_k\right) = E\left[\Delta\tilde{\alpha}_j \Delta\tilde{\alpha}_k\right] - E\left[\Delta\tilde{\alpha}_j\right] E\left[\Delta\tilde{\alpha}_k\right] =$$

$$= E\left[\left(\tilde{\alpha}_{j,t} - \tilde{\alpha}_{j,t-1}\right)\left(\tilde{\alpha}_{k,t} - \tilde{\alpha}_{k,t-1}\right)\right] - E\left[\tilde{\alpha}_{j,t} - \tilde{\alpha}_{j,t-1}\right] E\left[\tilde{\alpha}_{k,t} - \tilde{\alpha}_{k,t-1}\right] =$$

$$= Cov\left(\tilde{\alpha}_{j,t}, \tilde{\alpha}_{k,t}\right) - Cov\left(\tilde{\alpha}_{j,t}, \tilde{\alpha}_{k,t-1}\right) - Cov\left(\tilde{\alpha}_{j,t-1}, \tilde{\alpha}_{k,t}\right) + Cov\left(\tilde{\alpha}_{j,t-1}, \tilde{\alpha}_{k,t-1}\right) =$$

$$= 2Cov\left(\tilde{\alpha}_j, \tilde{\alpha}_k\right) - 2Cov\left(\tilde{\alpha}_{j,t-1}, \tilde{\alpha}_{k,t}\right) \tag{100}$$

where the last equality follows because the covariance is assumed constant over the estimation period, i.e. $Cov\left(\tilde{\alpha}_{j,t}, \tilde{\alpha}_{k,t}\right) = Cov\left(\tilde{\alpha}_{j,t-1}, \tilde{\alpha}_{k,t-1}\right) = Cov\left(\tilde{\alpha}_j, \tilde{\alpha}_k\right)$, and by symmetry of the covariance matrix. By Assumption 4c, $Cov\left(\tilde{\alpha}_{j,t-1}\tilde{\alpha}_{k,t}\right) = 0$ for all j and k. Therefore, the covariance of the log of the shocks is

$$Cov\left(\tilde{\alpha}_j, \tilde{\alpha}_k\right) = Cov\left(\Delta\tilde{\alpha}_j, \Delta\tilde{\alpha}_k\right)/2.$$

8.1.12 Equivalence in the moments estimation

In this section I prove that, if the shocks are i.i.d. over time, computing the moments of the shocks using variation across time for each year firm and taking the average across firms is equivalent to stacking together the observations for all years and computing the moments once.

To save notation, define $X \equiv \Delta\tilde{\alpha}_x$ and $Y \equiv \Delta\tilde{\alpha}_y$, where x and y are any two destinations. The covariance between X and Y, computed stacking together the structural residuals, is:

$$Cov(X,Y) = \frac{1}{T \cdot S} \sum_{k=1}^{T \cdot S} \left(y_k - \bar{y}\right)\left(x_k - \bar{x}\right) \tag{101}$$

where x_k (y_k) is the observed change in the log of the shock in destination $x(y)$ for k, where k is a pair of firm s and year t, S is the number of firms selling to both x and y, T is the number of years. Since $\bar{x} \equiv E[\Delta\tilde{\alpha}_x] = 0$ and $\bar{y} \equiv E[\Delta\tilde{\alpha}_p] = 0$, the above becomes:

$$Cov(X,Y) = \frac{1}{T \cdot S} \sum_{k=1}^{T \cdot S} y_k x_k \tag{102}$$

If instead I compute the covariance for each firm, it equals:

$$Cov(X,Y)_s = \frac{1}{T} \sum_{y=1}^{T} y_{ts} x_{ts} \tag{103}$$

where x_{ts} (y_{ts}) is the observed change in the log of the shock in destination x (y) in year t and for firm s. The average across firms of this covariance is simply:

$$\frac{1}{S} \sum_{s=1}^{S} Cov(X,Y)_s = \frac{1}{S} \sum_{s=1}^{S} \frac{1}{T} \sum_{t=1}^{T} y_{ts} x_{ts} =$$

$$= \frac{1}{T \cdot S} \sum_{s=1}^{S} \sum_{t=1}^{T} y_{ts} x_{ts} = \frac{1}{T \cdot S} \sum_{k=1}^{T \cdot S} y_k x_k \tag{104}$$

by the associative property. Therefore, equation (102) is equivalent to equation (104). A similar proof works for the expected value of the shocks.

8.1.13 Allowing for aggregate shocks

Assume that the log demand shocks can be decomposed into a macro component, common to all firms, and a firm-destination component:

$$\tilde{\alpha}_{js} = \tilde{\tilde{\xi}}_j + \tilde{\varsigma}_{js}$$

The macro shock represents any aggregate demand shock, for example a financial crisis, a monetary policy shock, or an exchange rate fluctuation, that affect all firms selling to that country. In the structural equation (35), the macro shock $\tilde{\tilde{\xi}}_j$ is absorbed by the destination fixed effect. In order to back it out, I use as before the assumption that the parameters stay constant between two consecutive years, and take the first difference of both the estimated destination fixed effect and the residual (controlling for supply shocks as in equation (38)):

$$\delta_{jt} - \delta_{j,t-1} + \varepsilon_{js,t} - \varepsilon_{js,t-1} = \Delta\tilde{\tilde{\xi}}_{j,t} + \Delta\tilde{\varsigma}_{js,t} = \Delta\tilde{\alpha}_{js,t}$$

Using $\Delta\tilde{\alpha}_{js,t}$, I compute the moments of the demand shocks in exactly the same way I do in the baseline, thus assuming that $\tilde{\alpha}_{js}$ is normally distributed.

8.1.14 Estimation of risk aversion

In equation (43), both the marginal expected profits and their marginal variance are not observable in the data. However, I do observe gross profits from each destination, since they are a share σ of observed revenues, due to the CES assumption. The ratio between expected marginal gross profits, $MEGP_{js}$, and observed gross profits, GP_{js}, for firm s and market j, is:

$$\frac{MEGP_{js}}{GP_{js}} = \frac{\bar{\alpha}_j r_{js}}{\alpha_{js} n_{js} r_{js}} = \frac{\bar{\alpha}_j}{\alpha_{js} n_{js}} \tag{105}$$

Taking logs and rearranging:

$$ln\left(MEGP_{js}\right) = ln\left(GP_{js}\right) + ln\left(\bar{\alpha}_j\right) - ln\left(\alpha_{js}\right) - ln\left(n_{js}\right) \tag{106}$$

Note that $ln\left(\alpha_{js}\right) - ln\left(n_{js}\right)$ is just the residual from the regression in (38), which I have already estimated for 2005 (and for all other years). I directly observe $ln\left(GP_{js}\right)$ in the data, and I have already estimated $ln\left(\bar{\alpha}_j\right)$ in section (3.1). Therefore, I can back out $MEGP_{js}$ for all firms and destinations in 2005.

To compute the marginal variance, note that the total covariance of gross profits in country j, i.e. the variance in j plus the covariances with all other countries, equals:

$$CGP_{js} = \sum_k Cov(GP_{js}, GP_{ks}) = \sum_k r_{js} r_{ks} n_{js} n_{ks} Cov(\alpha_j, \alpha_k) \tag{107}$$

I use data on observed gross profits from 1995 to 2004, for each firm-destination pair, to compute the left hand side of equation (107):

$$CGP_{js} = \sum_k \frac{1}{T} \sum_{t=1}^{T} \left(GP_{kst} - E[GP_{ks}]\right)\left(GP_{jst} - E[GP_{js}]\right) \tag{108}$$

Then I compute r_{js} in equation (107) as $\frac{MEGP_{js}}{\bar{\alpha}_j}$ for all s and j, and since I have already estimated $Cov(\alpha_j, \alpha_k)$ in section (3.1), I only need to solve for the vector n_{sj}. Thus, for each firm s selling to G markets, I solve a system of G equations (107) in G unknowns, i.e. $n_{gs}(z)$, for $g = 1, ...G$.

Finally, I compute the marginal variance of gross profits as

$$MVGP_{js} = \sum_k r_{js} n_{ks} r_{ks} Cov(\alpha_j, \alpha_k) \tag{109}$$

8.1.15 CRRA Utility

In the baseline model, I assume that entrepreneurs maximize an expected CARA utility in real income. One shortcoming of the CARA utility is that the absolute risk aversion is independent from wealth. This implies that large firms display the same risk aversion as small firms, which may be too restrictive. In this subsection, I consider an extension of the model where the entrepreneurs have a CRRA utility, and thus a decreasing absolute risk aversion. In particular, the owners now maximize the following utility:

$$\max E\left[\frac{1}{1-\rho}\left(\frac{y_i(z)}{P_i}\right)^{1-\rho}\right] \tag{110}$$

where the coefficient of absolute risk aversion is given by

$$ARA = \rho \left(\frac{y_i(z)}{P_i} \right)^{-1} \tag{111}$$

and therefore is decreasing in the size of the firm, and the coefficient of relative risk aversion is simply $\rho > 0$.

Define $z \equiv \frac{y_i(z)}{P_i}$. Take a second-order expansion of $E\left[\frac{1}{1-\rho}z^{1-\rho}\right]$ around $\bar{z} \equiv E\left(\frac{y_i(z)}{P_i}\right)$:

$$E\left[\frac{1}{1-\rho}z^{1-\rho}\right] \approx E\left[\frac{1}{1-\rho}\bar{z}^{1-\rho} + \bar{z}^{-\rho}\left(z-\bar{z}\right) + \frac{-\rho}{1-\rho}\frac{1}{2}\bar{z}^{-\rho-1}\left(z-\bar{z}\right)^2\right] =$$

$$= \frac{1}{1-\rho}\bar{z}^{1-\rho} + \bar{z}^{-\rho}E\left(z-\bar{z}\right) - \frac{\rho}{(1-\rho)2}\bar{z}^{-\rho-1}E\left(z-\bar{z}\right)^2 =$$

$$= \frac{1}{1-\rho}\bar{z}^{1-\rho} - \frac{\rho}{(1-\rho)2}\bar{z}^{-\rho-1}Var\left(z\right)$$

Multiplying the above expression by $1-\rho$, we obtain:

$$\max \left(E\left[\frac{y_i(z)}{P_i}\right]\right)^{1-\rho} - \frac{\rho}{2}\left(E\left[\frac{y_i(z)}{P_i}\right]\right)^{-1-\rho} Var\left(\frac{y_i(z)}{P_i}\right). \tag{112}$$

8.2 Data Appendix

Trade data. Statistics Portugal collects data on export and import transactions by firms that are located in Portugal on a monthly basis. These data include the value and quantity of internationally traded goods (i) between Portugal and other Member States of the EU (intra-EU trade) and (ii) by Portugal with non-EU countries (extra-EU trade). Data on extra-EU trade are collected from customs declarations, while data on intra-EU trade are collected through the Intrastat system, which, in 1993, replaced customs declarations as the source of trade statistics within the EU. The same information is used for official statistics and, besides small adjustments, the merchandise trade transactions in our dataset aggregate to the official total exports and imports of Portugal. Each transaction record includes, among other information, the firm's tax identifier, an eight-digit Combined Nomenclature product code, the destination/origin country, the value of the transaction in euros, the quantity (in kilos and, in some case, additional product-specific measuring units) of transacted goods, and the relevant international commercial term (FOB, CIF, FAS, etc.). I use data on export transactions only, aggregated at the firm-destination-year level.

Data on firm characteristics. The second main data source, Quadros de Pessoal, is a longitudinal dataset matching virtually all firms and workers based in Portugal. Currently, the dataset collects data on about 350,000 firms and 3 million employees. As for the trade data, I was able to gain access to information from 1995 to 2005. The data is made available by the Ministry of Employment, drawing on a compulsory annual census of all firms in Portugal that employ at least one worker. Each year, every firm with wage earners is legally obliged to fill in a standardized questionnaire. Reported data cover the firm itself, each of its plants, and each of its workers. Variables available in the dataset include the firm's location, industry (at 5 digits of NACE rev. 1), total employment, sales, ownership structure (equity breakdown among domestic private, public or foreign), and legal setting. Each firm entering the database is assigned a unique, time-invariant identifying number which I use to follow it over time.

The two datasets are merged by means of the firm identifier. As in Mion and Opromolla (2014) and Cardoso and Portugal (2005), I account for sectoral and geographical specificities of Portugal by restricting the sample to include only firms based in continental Portugal while excluding agriculture and fishery (Nace rev.1, 2-digit industries 1, 2, and 5) as well as minor service activities and extra-territorial activities (Nace rev.1, 2-digit industries 95, 96, 97, and 99). The analysis focuses on manufacturing firms only (Nace rev.1 codes 15 to 37) because of the closer relationship between the export of goods and the industrial activity of the firm. The location of the firm is measured according to the NUTS 3 regional disaggregation.

Data on $\tilde{L}_j$. $\tilde{L}_j$ is the total number of workers in the manufacturing sector in 2005, obtained from UNIDO. For some countries, I do not observe $\tilde{L}_j$, and thus I set it proportional to the population in country j. In particular, I compute $\tilde{L}_j = L_j/r$, where r is the average ratio of population over manufacturing workers in the other countries.

Data on M. From UNIDO, I also observe the number of *establishments* in the manufacturing sector. To compute the number of *firms*, M_j, I divide the number of establishments in each country by the ratio between number of firms and number of establishments in Portugal, which is 0.32. I obtain the number of manufacturing firms in Portugal, $M_P = 27,970$, from Quadros de Pessoal. For the countries for which I do not have data on number of establishments, I set $M_j = 0.021\tilde{L}_j$, where 0.021 is the median ratio of workers to firms in the other countries. Setting the number of firms to be proportional to the working population of a country has been shown to be a good approximation of the data (see Bento and Restuccia (2016) and Fernandes et al. (2016)).

Data on firms' profits. I obtain data on firms' net profits, investment rate, capital expenditures from Central de Balanços, a repository of yearly balance sheet data for non financial firms in Portugal.

List of countries. The countries in the sample are the top destinations of Portuguese exporters for which there is available data, from WIOD or UNIDO, to construct manufacturing trade shares. The final list of destinations is provided in Table 3.

8.3 Algorithm used for calibration

The calibration algorithm works as follows:

1) Guess a matrix of trade costs τ_{ij} (normalizing domestic trade costs to 1). Stack them into the vector Θ.

2) Solve the trade equilibrium using the system of equations (15), (23), (25) and (26). To solve the general equilibrium, I create a grid composed by 10,000 firms, each with a given productivity z, and compute the optimal $n_{ij}(z)$ for all firms and countries. By Walras' Law, one equation is redundant. Therefore, I normalize world GDP to a constant, as in Allen et al. (2014) and Caliendo and Parro (2014).

3) Compute aggregate trade shares, $\lambda_{ij} \equiv \frac{X_{ij}}{\sum_k X_{kj}}$, for $i \neq j$, where X_{ij} are total trade flows from i to j, as shown in equation (22). I stack these trade shares in a $N(N-1)$-element vector $\hat{m}(\Theta)$ and compute the analogous moment in the data, m^{data}, using manufacturing trade data in 2005.

4) Stack the differences between observed and simulated moments into a vector of length 1,190, $y(\Theta) \equiv m^{data} - \hat{m}(\Theta)$. I update Θ as follows:

$$\Theta^{new} = \Theta + \varepsilon y(\Theta)$$

where ε is arbitrarily small. The intuition is that, for example, if the trade shares predicted by the model using some guess Θ_i are lower than the corresponding shares observed in the data, i.e. $y_i(\Theta) < 0$, the new trade costs will have to be lower, $\Theta_i^{new} < \Theta_i$.

5) Iterate over 1-4 until $\max\{y(\Theta)\} < tol$, for tol sufficiently small.