Capital Taxation and Rent Seeking

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Abstract. We find the optimal capital income tax rate in an imperfectly competitive economy, where some part of recourses is devoted to rent-seeking activity. Optimal tax offsets the difference between marginal social and marginal private return to capital, which is a result of rent seeking, and the difference between the before tax interest rate and the marginal productivity of capital, which arises from imperfect competition. Optimal capital income tax rate depends neither on other tax rates nor on overall tax burden. Numerically it is close to zero.

At first glance, the nature of optimal fiscal policy in an imperfectly competitive economy or under decreased returns to scale is clear: pure profit should be intensively taxed, and subsidies should offset market distortions, which arise from imperfect competition; in all other respects the policy should be the same as under perfect competition and constant returns to scale. However, this recommendation is not applicable in practice, because fiscal authorities cannot distinguish pure profit from factor remuneration. Thus, it is not possible to tax pure profit without taxing wages or capital income.

To make the problem of taxation of imperfectly competitive economy more realistic, it is usually supposed that the pure profit tax is either zero or exogenously given (see Judd 1997, Guo & Lansing 1999, Auerbach & Hynes 2001). In this case, if higher stock of capital leads to higher economic profit, then the marginal productivity of capital should be higher than after-tax interest rate. Besides, subsidies should offset distortions between marginal productivity of capital and before tax interest rate, which arise from market power. The optimal capital income tax rate in such an economy is determined by many factors (consumption and pure profit taxes, marginal excess tax burden, and others), and its numerical value considerably varies when we slightly change the model structure or its calibration. For example, Guo and Lansing found that for US economy the optimal capital income tax rate is somewhere between −10% and +22%.

We take into consideration the fact that once pure profit has been produced, rent-seeking agents will spend their resources in order to seize it. This hypothesis makes the analysis more realistic: just as in the real world, we cannot distinguish pure profit from factor remuneration, and cannot tax them at different rates. Thus, we substitute the traditional hypothesis that pure profit enters into household budget constraint through a special channel by one that pure profit turns into private factor remuneration as a result of rent seeking.

We get general and intuitively clear results. Rent seeking distorts factor allocation. If an economy accumulates additional \$1 of capital, some part of it, say ξ_K, will be used for production of final goods, and the rest, $1-\xi_K$, for rent seeking. If the marginal productivity of capital used to produce final good is F_K, then the marginal productivity of capital, accumulated in the whole economy is $\xi_K F_K$. Therefore, the marginal social and marginal private returns to capital differ, and optimal policy offsets this distortion. Private and social returns to capital may differ also because capital accumulation may impact the division of labour between production and rent seeking. In addition, just as in previous researches, optimal policy offsets distortions, which arise from market power. Numerically, optimal capital income tax is close to zero.

It remains to note that this paper proceeds examination of hypotheses which contradict the Chamley (1986) and Judd (1985) result of zero long run optimal capital tax. In addition to the case of imperfect competition, the optimal capital tax in the long run is not zero under uncertainty (Zhu (1992), Chari and Kehoe (1994), Aiyagari (1995)), if some agents face with liquidity constraints (Hubbard and Judd (1986)), in the case of incomplete fiscal system (Correia (1896)), or under no-commitment (Benhabib and Rustichini (1997)).

The rest of the paper is organized as follows. In the first section we describe the economy with rent-seeking agents. Section 2 intuitively derives resource and implementability constraints, which we use to formulate the Ramsey policy problem; a formal proof may be found in the appendix. Section 3 poses the Ramsey problem of optimal policy, gives the first order conditions, and derives steady-state optimal capital income tax. Section 4 is devoted to numerical estimation of optimal capital tax rate, section 5 concludes.

1. Model description

A representative household solves the following problem:

$$\max_{C,L} \int_0^\infty e^{-\rho t} u(C,L)\,dt \tag{1}$$

$$\dot{A} = rA + wL - p_c C \tag{2}$$

where C is the consumption, L is the labor, ρ is the discount factor, A is the household's wealth, r, w and p_c are the after-tax capital income, wage and commodity price. Household's wealth consists of physical capital K and government bonds B, A_0 is given. The number of households is normalized to unity and the producer price of the final good is the numeraire. The first-order conditions are

$$U_C = p_c\gamma \tag{3a}$$

$$U_L = -w\gamma \tag{3b}$$

$$\dot{\gamma} = \gamma(\rho - r) \tag{3c}$$

where γ is the co-state variable.

There are two types of business activity: production and rent seeking. To produce final goods, firms use K_1 units of capital and L_1 units of labor:

$$Y = F\left(K_1, L_1\right) \tag{4}$$

Profit maximization requires:

$$\hat{r} + \delta = (1-\sigma) F_K \tag{5a}$$

$$\hat{w} = (1-\sigma) F_L \tag{5b}$$

Where $\hat{w}$ and $\hat{r}$ are the before tax wage and the interest rate, and δ is the depreciation rate. Parameter σ emerges as a result of imperfect competition on the final goods market and may be measured by the inverse of demand elasticity for one firm's output. σ may depend on the resource allocation in the economy.

The profit is given by

$$\pi = F\left(K_1, L_1\right) - (1-\sigma)\left[F_K K_1 + F_L L_1\right] \tag{6}$$

Case $\sigma = 0$ corresponds to the situation where all firms are price-takers. Note, that in this case the profit may still be positive if we assume decreasing returns to scale.

Rent-seekers compete with each other in order to seize the profit. Probability of success depends on amounts of capital and labor devoted to rent seeking. The seeker that achieves higher value of a function $Q(K, L)$ has a higher probability of success. To be precise, assume that the probability of success of agent i is given by

$$Prob_i = \frac{Q(K_i, L_i)}{\sum_j Q(K_j, L_j)}, \tag{7}$$

where K_i, L_i is the amount of capital and labor devoted to rent seeking by agent i.

For simplicity we assume that function Q exhibits constant returns to scale. The problem of agent i takes the following form:

$$\max_{K_i, L_i} \left[\frac{Q(K_i, L_i)}{\sum_j Q(K_j, L_j)} \pi - (\hat{r} + \delta) K_i - \hat{w} L_i\right] \tag{8}$$

For simplicity we assume that the depreciation rate in the rent-seeking sector is the same as in the production sector.

Firms assume all the risks. Rent-seekers' first-order conditions are:

$$Q_K(K_i, L_i) \frac{\sum_{j \neq i} Q(K_j, L_j)}{\left(\sum_j Q(K_j, L_j)\right)^2} \pi = (\hat{r} + \delta) \tag{9a}$$

$$Q_L(K_i, L_i) \frac{\sum_{j \neq i} Q(K_j, L_j)}{\left(\sum_j Q(K_j, L_j)\right)^2} \pi = \hat{w} \tag{9b}$$

Form these conditions we get:

(10) $$\frac{Q_K}{Q_L} = \frac{\hat{r}+\delta}{\hat{w}}$$

Free-entry assumption leads to the following market clearing condition:

(11) $$\hat{r}K_2 + \hat{w}L_2 = \pi$$

where K_2 and L_2 are the capital and the labor used to seek the rent in the economy.

Other market clearing conditions are

(12) $$Y = C + G + \dot{K} + \delta K$$

$$\begin{aligned} &(13a) & K &= K_1 + K_2 \\ &(13b) & L &= L_1 + L_2 \end{aligned}$$

The government collects taxes in order to finance an exogenously given amount of public goods G. Its budget constraint is

(14) $$\dot{B} = rB + G - (p_c - 1)\,C - [Y - rK - wL]$$

The tax rates are determined by ratios of consumer and producer prices. K_0, A_0, and B_0 are given.

The government solves the Ramsey problem. In other words, it chooses a tax system, which maximizes the utility of a representative household in decentralized economy. We confine ourselves to analysis of equilibrium policies, see Arefiev(2008).

2. Attainable allocation set

To derive the optimal policy we use primal approach to optimal taxation, developed by Ramsey (1927), Atkinson and Stiglitz (1980), Lucas and Stockey (1983), Chari and Kehoe (1998) and many others. The first step of this approach is to describe the set of allocations that can be decentralized without lump-sum taxes. The second one is to maximize utility of a representative agent on this set. The last one is to find tax rates, which decentralize the optimal allocation.

Attainable allocation set may be described by two constraints: resource and implementability ones. The resource constraint ensures that the considered allocation is consistent with firms' optimization: if for a given allocation the resource constraint is satisfied then there exists a vector of producer prices such that this allocation satisfies firms' budget constraints and their first-order conditions [1]. In the same sense the implementability constraint ensures that an allocation is consistent with the households optimization. If an allocation satisfies the both constraints then consumers and producers under some prices choose this allocation, and the

[1] Note, that this definition of the resource constraint diverges from the obvious one. The obvious resource constraint for the problem at hand is $\dot{K} = F(K, L) - C - G - \delta K$, and not the constraint we use (20). However, the production possibility frontier, which is given by the obvious constraint, is not attainable in our problem, because of non-productive use of K_2 and L_2. Thus, we augment the resource constraint by the conditions of firms' optimization. This means that the Diamond-Mirrlees principle of production efficiency is not satisfied in our framework.

government budget constraint will be satisfied by Walras law. The tax rates that decentralize the considered allocation are determined by ratios of consumer and producer prices.

In this section we intuitively derive both constraints from equilibrium conditions, and in the appendix A we prove that these constraints exactly describe the attainable allocation set.

To get the implementability constraint, consider the value of the household's wealth measured in units of utility:

$$a = \gamma A \tag{15}$$

Take a derivative of (15) with respect to time and substitute first-order conditions (3a, 3b, 3c) and household budget constraint (2) into obtained equation:

$$\dot{a} = \rho a - U_C C - U_L L \tag{16}$$

The condition of equilibrium policy gives the value of a_0, see Arefiev (2008) for details.

To get the resource constraint we need to determine how K_1 and L_1 depend on K and L. Consider the Cobb-Douglas example: $Y = K_1^{\alpha} L_1^{\beta}$, $Q = K_2^{\phi} L_2^{1-\phi}$. To get K_1/K ratio, divide the share of K_1 income in Y by the share of $K = K_1 + K_2$ income in Y. The share of K_1 income in Y is $(1-\sigma)\alpha$, and the share of K_2 income in Y is equal to the share of K_2 income in profit, ϕ, times the share of profit in Y, which is $[1-(1-\sigma)(\alpha+\beta)]$. We get:

$$\frac{K_1}{K} = \frac{\alpha(1-\sigma)}{\alpha(1-\sigma) + \phi[1-(1-\sigma)(\alpha+\beta)]} \tag{17}$$

In a similar way:

$$\frac{L_1}{L} = \frac{\beta(1-\sigma)}{\beta(1-\sigma) + (1-\phi)\left[1-(1-\sigma)(\alpha+\beta)\right]} \tag{18}$$

Thus, in the Cobb-Douglas case, the ratios K_1/K and L_1/L are constants. In a more general case, α, β and σ may depend on K_1 and L_1, and ϕ may depend on K_2 and L_2. In this case we get a system of two equations, which implicitly gives K_1 and L_1 as functions of K and L. Let's define

$$K_1 = \xi(K, L) \tag{19a}$$

$$L_1 = \eta(K, L) \tag{19b}$$

Substitution of (19a), (19b) and (4) into (12) gives us the resource constraint:

$$\dot{K} = F\left(\xi(K, L), \eta(K, L)\right) - C - G - \delta K \tag{20}$$

3. Optimal capital income taxation

The government maximizes utility of a representative household on the set of allocations, attainable in a decentralized economy:

$$\max_{C,L} \int_0^\infty e^{-\rho t} u(C,L)\, dt \tag{21a}$$

$$\dot{a} = \rho a - U_C C - U_L L \tag{21b}$$

$$\dot{K} = F(\xi(K,L), \eta(K,L)) - C - G - \delta K \tag{21c}$$

$$a(0) = a_0 \tag{21d}$$

$$K(0) = K_0 \tag{21e}$$

Let λ and μ be co-state variables for a and K. First order conditions are

$$U_C[1 - \lambda(1 + H_C)] = \mu \tag{22a}$$

$$U_L[1 - \lambda(1 + H_L)] = -\mu(F_K \xi_L + F_L \eta_L) \tag{22b}$$

$$\dot{\lambda} = 0 \tag{22c}$$

$$\dot{\mu} = \mu\rho - \mu[F_K \xi_K + F_L \eta_K - \delta] \tag{22d}$$

where

$$H_i = \frac{U_{Ci} C + U_{Li} L}{U_i} \qquad i = C, L \tag{23}$$

Let $p_c^{pv}(t)$ be the present value of the consumer price of the final good, and $\hat{p}_c^{pv}(t)$ be the present value of the producer price of the final good.

$$p_c^{pv}(t) = p_c(t) e^{-\int_0^t r(z)dz} \tag{24a}$$

$$\hat{p}_c^{pv}(t) = e^{-\int_0^t \hat{r}(z)dz} \tag{24b}$$

Let us we introduce the following cumulative tax rate:

$$T_{\frac{C(t)}{C(0)}} = \frac{p_c^{pv}(t)/p_c^{pv}(0)}{\hat{p}_c^{pv}(t)/\hat{p}_c^{pv}(0)} = \frac{p_c(t)}{p_c(0)} e^{\int_0^t [\hat{r}(z) - r(z)]dz} \tag{25}$$

In order to find the optimal value of $T_{C(t)/C(0)}$, first, divide (22a) by (22a), and substitute $\gamma_t = \gamma_0 e^{\rho t - \int_0^t r(z)dz}$ and $\mu_t = \mu_0 e^{\rho t - \int_0^t [F_K \xi_K + F_L \eta_K - \delta]dz}$, which solve respectively (3c) and (22d). We get the equation, which is analogous to the famous Judd (1999) multiplier of cumulative capital taxation:

$$p_c(t)[1 - \lambda(1 + H_C(t))] = \frac{\mu_0}{\gamma_0} e^{\int_0^t (r - [F_K \xi_K + F_L \eta_K - \delta])dz} \tag{26}$$

Then, derive $\frac{p_c(t)}{p_c(0)} e^{-\int_0^t r(z)dz}$ from (26), and substitute it to (25). Taking into account (5a), we get the optimal value of $T_{\frac{C(t)}{C(0)}}$:

$$T^*_{\frac{C(t)}{C(0)}} = \frac{1 - \lambda(1 + H_C(0))}{1 - \lambda(1 + H_C(t))} e^{\int_0^t ((1-\sigma)F_K - [F_K \xi_K + F_L \eta_K])dz} \tag{27}$$

There is an infinite number of policies, which satisfy (27), and decentralize the optimal allocation. In order to get the only policy we should normalize the fiscal system in a particular manner. Assume that the consumption taxation meets the Ramsey principles of optimal taxation at the micro level, i.e.

$$\frac{p_c(t)}{p_c(0)} \times \frac{1-\lambda(1+H_C(t))}{1-\lambda(1+H_C(0))} = 1 \tag{28}$$

We assume that the value of $p_c(0)$ is chosen to satisfy the condition of equilibrium policy at $t = 0$. Then from (25), (27) and (28) we get the following equation, which implies the optimal capital income tax:

$$r - \hat{r} = (1-\sigma)F_K - [F_K \xi_K + F_L \eta_K] \tag{29}$$

Thus, optimal capital tax offsets the difference between private and social marginal productivity of capital, which is determined by ξ_K and η_K, and the difference between before tax interest rate and marginal productivity of capital, which is given by σ. To be exact, if capital tax τ_K is defined by $(r+\delta) = (1-\tau_K)(\hat{r}+\delta)$, the optimal value of τ_K on a balanced growth path is given by

$$\tau_K = 1 - \frac{\left(\xi_K + \dfrac{F_L}{F_K}\eta_K\right)}{1-\sigma} \tag{30}$$

If $\sigma = \eta_K = 0$, and $\xi_K = 1$, we get the celebrated Chamley-Judd result of zero capital income taxation.

4. Value of optimal capital income tax

Let's suppose that the shares of K_1, K_2, L_1 and L_2 income in Y, and also the share of profit in Y are constants. From equation (17) we see that in this case

$$\xi_K = \frac{\alpha(1-\sigma)}{\alpha(1-\sigma) + \phi[1-(1-\sigma)(\alpha+\beta)]} \tag{31}$$

$$\eta_K = 0 \tag{32}$$

and the optimal capital income tax is given by

$$\tau_K = 1 - \left[\left(1 - \frac{\phi}{\alpha}(\alpha+\beta)\right)(1-\sigma) + \frac{\phi}{\alpha}\right]^{-1} \tag{33}$$

If an economy exhibits constant returns to scale ($\alpha + \beta = 1$), and the share of K_1 income in Y equals the share of K_2 income in profit ($\alpha = \phi$), then the optimal capital income tax is zero. To get more general results we need an estimation of returns to scale and the share of profit in GDP. Guo and Lansing (1999) used the estimations of Basu and Fernald (1997) and got that the optimal capital income tax rate is somewhere between -10% and +22%. We take the estimation of returns to scale in the typical US industry from the same source (Basu, Fernald, 1997), and hence assume the degree of homogeneity of the production function to be equal to 1,01, and the profit ratio of the typical US industry of about 3%. Let the gross share of capital income in GDP (the denominator in 31) be equal to 35%. Consequently, in our framework the optimal capital income tax is somewhere between -4,1% ($\phi = 0$) and 4,6% ($\phi = 1$).

When no capital is involved in rent-seeking, the capital is subsidized in order to offset distortions arising from imperfect competition (represented by σ). When all capital is involved in rent seeking, the effect of discouraging unproductive activity dominates and the tax rate is positive.

5. Conclusion

Taking account of unproductive use of resources in rent-seeking has allowed us to compactly pose the Ramsey problem, and to get intuitively clear and interpretable results. In particular, we found that the optimal capital tax offsets the difference between private and social marginal productivity of capital (given by ξ_K and η_K) and the difference between before tax return on capital and its marginal productivity (given by σ). The sign of the optimal tax in the long run is ambiguous. On one extreme, when all capital is tied up in rent seeking, the tax is positive, so that it distimulates capital accumulation. On the other extreme, when all capital is used in production, there arises a subsidy, which eliminates the distortions of imperfect competition. The bounds within which the tax varies are narrower than in previous works.

Appendix A. Attainable allocation set

Theorem 1. *(i) The implementability (21b) and the resource (21c)constraints, with the initial and transversality conditions (21d), (21e) and* $\lim\limits_{t\to\infty} e^{-\rho t} a(t) = 0$ *, are satisfied for any equilibrium allocation* $[C(t), L(t)]_{t\in[0,\infty)}$.

(ii) If the implementability (21b) and the resource (21c) constraints with the initial and transversality conditions are satisfied for a given allocation $[C(t), L(t)]_{t\in[0,\infty)}$, *then for given dynamics of any tax* (τ_K, τ_L, τ_C) *there exists the dynamics of the other two taxes such that this allocation will be implemented in the decentralized economy.*

Proof. (i) We get the implementability and the resource constraints from conditions, which are satisfied for any equilibrium allocation. Consequently, they are also satisfied for any equilibrium allocation.

The initial condition for the implementability constraint is given by the initial condition for the market clearing condition (12). The initial condition for a follows from the equilibrium policy condition, see Arefiev (2008). The transversality condition for a we derive from the transversality condition for A, $\lim_{t\to\infty} e^{\int_0^t -r(z)dz} A(t) = 0$, the definition of a, equation (15), and equation $\gamma = \gamma_0 \exp\left(\rho t - \int_0^t r(z)dz\right)$, which solves (3c). These equations give $\lim_{t\to\infty} e^{-\rho t} a(t) = 0$.

(ii) Suppose that the dynamics of p_c is exogenously given (we assume that $p_c > 0\ \forall t$, p_c is continuous, and $p_c(0)$ verifies the condition of equilibrium policy at $t = 0$ for the given value of $A(0)$). Chose γ, w, and r in such a way that (3a), (3b), and (3c) are satisfied. Substitution of (3a), (3b), and (3c) into (16) gives (2), consequently, the household's budget constraint is also satisfied. The transversality condition $\lim_{t\to\infty} e^{-\rho t} a(t) = 0$ with $\gamma = \gamma_0 \exp\left(\rho t - \int_0^t r(z)dz\right)$ implies $\lim_{t\to\infty} e^{\int_0^t -r(z)dz} A(t) = 0$.

For a given allocation $[C(t), L(t)]_{t\in(o,\infty)}$ and initial value of $K(0) = K_0$, the resource constraint (20) is a first-order differential equation with respect to $K(t)$, which has a unique backward-looking solution. For given trajectories of K and L, the production function (4) gives the dynamics of Y. Thus, we get the dynamics of K, and Y, which verifies (4) and (12).

Take the values of $\hat{r}$ and $\hat{w}$, which verify (5). Equation (6) gives the value of profit, which corresponds to the allocation under consideration. The functions $\xi(K, L)$ and $\eta(K, L)$ solve the system of equations (17) and (18), consequently, these equations are also satisfied. Substitution of the shares α, β, and ϕ into these equations gives:

$$\frac{Q_K}{Q_K K_2 + Q_L L_2} = \frac{\hat{r} + \delta}{\pi} \tag{34a}$$

$$\frac{Q_L}{Q_K K_2 + Q_L L_2} = \frac{\hat{w}}{\pi} \tag{34b}$$

If we divide (34a) by (34b), we get the rent-seekers' optimality conditions (10), consequently, they are satisfied. Multiply (34a) by K_2, (34b) by L_2 and, sum up the two term. We get (11), consequently, the free-entry condition is also satisfied.

The government budget constraint is satisfied by Walras low. The fiscal system, which decentralize the allocation, is impled by respective ratios of consumer to producer prices.

□

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