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ABSTRACT

Presented here is a simplified mathematical model describing a supply side crisis caused by the coronavirus pandemic (COVID – 19). Model of a single-product economy is presented where the supply shock has a constant acceleration. If amount of the supply shock has a modest positive acceleration the product earnings are positive and increasing with the passage of time. We observe an economic growth. If amount of the supply shock has a large positive acceleration the product earnings are negative and decreasing with the passage of time. We observe an economic decline. If amount of the supply shock has a negative acceleration the product earnings are negative and decreasing with the passage of time. We observe an economic decline. Economic mechanism of the supply side crisis is conceptually close to a mechanism of economic growth caused by investment. Moreover, economic system is able to overcome a modest supply-side shock and provide economic growth there. Further, the system with the passage of time produces and delivers enough amount of product to both satisfy the demand and compensate for the supply-side shock.

JEL Classification Numbers: C02, E32, O11

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1 Introduction

The work is devoted to theoretical and mathematical economics where the economic processes are analyzed by building appropriate simplified mathematical models.

In (Krouglov, 2006) there was described a framework of dynamic mathematical models where economic quantities of supply, demand and price were balanced through the system of ordinary differential equations. In particular, the framework stipulates that in equilibrium the quantities of supply and demand are equal. When the equality of supply and demand is broken then third quantity, the price, is changed in order to bring the situation back into equilibrium by affecting the quantities of both supply and demand.

In (Krouglov, 2017) the framework was applied to the concept of an economic growth. Particularly, it was investigated the phenomenon of financial crises and presented a simplified mathematical model to describe some features of the financial crises. Economic growth was viewed via the situation when some amount of supply (intended for investment purposes) was removed from the market. As result, the equality of supply and demand on market was broken, the price was increasing and demand was shrinking. An economic growth was analyzed via the product earnings (defined as a rate of demand multiplied by the product price). Increment of the product earnings was associated with an economic growth, and contraction of the product earnings was associated with an economic decline.

In 2020 the coronavirus pandemic (COVID – 19) caused an economic crisis identified by the supply-side shock. A simple mathematical model was developed to investigate a crisis of the supply-side shock. The supply-side shock was analyzed for a situation when the fractional amount of supply was permanently removed from market.

2 Outline of a Simple Mathematical Model

I deploy a simple mathematical model of the market with single-product economy, which produces an explicit description of interactions among economic variables. Economic forces acting on the market reflect inherent forces of demand and supply. The forces of demand and supply are complemented with the supply-side shock. The market actions are expressed through the system of ordinary differential equations.

When there are no disturbing economic forces, the market is in equilibrium position, i.e., the supply of and demand for product are equal, the quantities of supply and demand are developing with a constant rate and a price of product is fixed.

I assume the market had been in an equilibrium until time $t = t_0$, volumes of the product supply $V_S(t)$ and demand $V_D(t)$ on market were equal, and they both were developing with a constant rate r_D^0. The product price $P(t)$ at that time was fixed,

$$V_D(t) = r_D^0 (t - t_0) + V_D^0 \tag{1}$$

$$V_S(t) = V_D(t) \tag{2}$$

$$P(t) = P^0 \tag{3}$$

where $V_D(t_0) = V_D^0$.

When the balance between the volumes of product supply and demand is broken, market is experiencing economic forces, which act to bring the market to a new equilibrium position.

I will use a simple model of the single-product economy where the supply shock is developing with a constant acceleration.

According to the scenario, I assume the amount of supply shock $S_{SS}(t)$ on the market develops since time $t = t_0$ according to following formula,

$$S_{SS}(t) = \begin{cases} 0, & t < t_0 \\ \delta_{SS}(t - t_0) + \dfrac{\varepsilon_{SS}}{2}(t - t_0)^2, & t \ge t_0 \end{cases} \tag{4}$$

where $S_{SS}(t) = 0$ for $t < t_0$.

Economic forces trying to bring the market into a new equilibrium position are described by the following ordinary differential equations with regard to the volumes of product supply $V_S(t)$, demand $V_D(t)$, and price $P(t)$ given the accumulated amount of supply side shock $S_I(t)$ on the market,

$$\frac{dP(t)}{dt} = -\lambda_P \left(V_S(t) - V_D(t) - S_{SS}(t)\right) \tag{5}$$

$$\frac{d^2 V_S(t)}{dt^2} = \lambda_S \frac{dP(t)}{dt} \tag{6}$$

$$\frac{d^2 V_D(t)}{dt^2} = -\lambda_D \frac{d^2 P(t)}{dt^2} \tag{7}$$

In Eqs. (5) – (7) above the values $\lambda_P, \lambda_S, \lambda_D \ge 0$ are constants and they reflect the price inertness, supply inducement, and demand amortization correspondingly.

Let me use a new variable $D(t) \equiv \left(V_S(t) - V_D(t) - S_{SS}(t)\right)$ representing the volume of product surplus (or shortage) on the market due to a supply side shock. The behavior of $D(t)$ is described by the following equation for $t > t_0$,

$$\frac{d^2 D(t)}{dt^2} + \lambda_P \lambda_D \frac{dD(t)}{dt} + \lambda_P \lambda_S D(t) + \varepsilon_{SS} = 0 \tag{8}$$

with the initial conditions, $D(t_0) = 0$, $\dfrac{dD(t_0)}{dt} = -\delta_{SS}$.

If one introduces another variable $D_1(t) \equiv D(t) + \dfrac{\varepsilon_{SS}}{\lambda_P \lambda_S}$, then Eq. (8) becomes,

$$\frac{d^2 D_1(t)}{dt^2} + \lambda_P \lambda_D \frac{dD_1(t)}{dt} + \lambda_P \lambda_S D_1(t) = 0 \tag{9}$$

with the initial conditions, $D_1(t_0) = \frac{\varepsilon_{SS}}{\lambda_P \lambda_S}$, $\frac{dD_1(t_0)}{dt} = -\delta_{SS}$.

Similar to Eq. (8), the product price $P(t)$ is described by the following equation for $t > t_0$,

$$\frac{d^2 P(t)}{dt^2} + \lambda_P \lambda_D \frac{dP(t)}{dt} + \lambda_P \lambda_S \left(P(t) - P^0 - \frac{\delta_{SS}}{\lambda_S} - \frac{\varepsilon_{SS}}{\lambda_S}(t - t_0) \right) = 0 \tag{10}$$

with the initial conditions, $P(t_0) = P^0$, $\frac{dP(t_0)}{dt} = 0$.

Let me use variable $P_1(t) \equiv P(t) - P^0 - \frac{\delta_{SS}}{\lambda_S} - \frac{\varepsilon_{SS}}{\lambda_S}(t - t_0) + \frac{\lambda_D}{\lambda_S^2}\varepsilon_{SS}$ to simplify analysis of the product price behavior. The behavior of variable $P_1(t)$ is described by following equation for $t > t_0$,

$$\frac{d^2 P_1(t)}{dt^2} + \lambda_P \lambda_D \frac{dP_1(t)}{dt} + \lambda_P \lambda_S P_1(t) = 0 \tag{11}$$

with the initial conditions, $P_1(t_0) = -\frac{\delta_{SS}}{\lambda_S} + \frac{\lambda_D}{\lambda_S^2}\varepsilon_{SS}$, $\frac{dP_1(t_0)}{dt} = -\frac{\varepsilon_{SS}}{\lambda_S}$.

The behavior of solutions for $D_1(t)$ and $P_1(t)$ described by Eqs. (9) and (11) depends on the roots of the corresponding characteristic equations (Piskunov, 1965; Petrovski, 1966). Also Eqs. (9) and (11) have the same characteristic equations.

When the roots of characteristic equation are complex-valued (i.e., $\frac{\lambda_P^2 \lambda_D^2}{4} < \lambda_P \lambda_S$) both variables $D_1(t)$ and $P_1(t)$ experience damped oscillations for time $t \geq t_0$. If the roots of characteristic equation are real and different (i.e., $\frac{\lambda_P^2 \lambda_D^2}{4} > \lambda_P \lambda_S$) both variables $D_1(t)$ and $P_1(t)$ don't oscillate for time $t \geq t_0$. If the roots of characteristic equation are real and equal (i.e., $\frac{\lambda_P^2 \lambda_D^2}{4} = \lambda_P \lambda_S$) both variables $D_1(t)$ and $P_1(t)$ don't oscillate for time $t \geq t_0$ as well.

It takes place $D_1(t) \to 0$ and $P_1(t) \to 0$ for $t \to +\infty$ if roots of characteristic equations are complex-valued ($\frac{\lambda_P^2 \lambda_D^2}{4} < \lambda_P \lambda_S$), real and different ($\frac{\lambda_P^2 \lambda_D^2}{4} > \lambda_P \lambda_S$), or real and equal ($\frac{\lambda_P^2 \lambda_D^2}{4} = \lambda_P \lambda_S$).

We can observe for the product surplus (shortage) $D(t)$, for the product price $P(t)$, for the product demand $V_D(t)$, for the product supply $V_S(t)$, for the amount of supply side shock $S_{SS}(t)$ if $t \to +\infty$,

$$D(t) \to -\frac{\varepsilon_{SS}}{\lambda_P \lambda_S} \tag{12}$$

$$P(t) \to \frac{\varepsilon_{SS}}{\lambda_S}(t-t_0) + P^0 + \frac{\delta_{SS}}{\lambda_S} - \frac{\lambda_D}{\lambda_S^2}\varepsilon_{SS} \tag{13}$$

$$V_D(t) \to \left(r_D^0 - \frac{\lambda_D}{\lambda_S}\varepsilon_{SS} \right)(t-t_0) + V_D^0 - \frac{\lambda_D}{\lambda_S}\delta_{SS} + \frac{\lambda_D^2}{\lambda_S^2}\varepsilon_{SS} \tag{14}$$

$$V_S(t) \to \left(r_D^0 + \delta_{SS} - \frac{\lambda_D}{\lambda_S}\varepsilon_{SS} \right)(t-t_0) + \frac{\varepsilon_{SS}}{2}(t-t_0)^2 + V_D^0 - \frac{\lambda_D}{\lambda_S}\delta_{SS} - \frac{\varepsilon_{SS}}{\lambda_P \lambda_S} + \frac{\lambda_D^2}{\lambda_S^2}\varepsilon_{SS} \tag{15}$$

$$S_{SS}(t) = \delta_{SS}(t-t_0) + \frac{\varepsilon_{SS}}{2}(t-t_0)^2 \tag{16}$$

To analyze an economic growth I use the variable $E_D(t) \equiv P(t) \times r_D(t)$ where $r_D(t) \equiv \frac{dV_D(t)}{dt}$, i.e., a rate of the monetary demand for product describing the product earnings on the market.

The variable $E_D(t)$, a rate of the monetary demand adjusted by the amount of supply side shock $S_{SS}(t)$, converges toward for $t \to +\infty$

$$E_D(t) \to \left(\frac{\varepsilon_{SS}}{\lambda_S}(t - t_0) + P^0 + \frac{\delta_{SS}}{\lambda_S} - \frac{\lambda_D}{\lambda_S^2}\varepsilon_{SS} \right)\left(r_D^0 - \frac{\lambda_D}{\lambda_S}\varepsilon_{SS} \right) \tag{17}$$

while the variable $\tilde{E}_D(t)$, an original rate of the monetary demand, converges toward $\tilde{E}_D(t) \to P^0 \, r_D^0$ for $t \to +\infty$.

If $0 < \varepsilon_{SS} < \frac{\lambda_S}{\lambda_D} r_D^0$ then $\varepsilon_{SS} > 0$ and $r_D^0 - \frac{\lambda_D}{\lambda_S}\varepsilon_{SS} > 0$. The product price is positive and increases; the rate of demand is positive and fixed. Therefore, it brings an unrestricted increase of earnings $E_D(t)$ with the passage of time, i.e., $E_D(t) \to +\infty$ for $t \to +\infty$.

If $\frac{\lambda_S}{\lambda_D} r_D^0 < \varepsilon_{SS} < +\infty$ then $\varepsilon_{SS} > 0$ and $r_D^0 - \frac{\lambda_D}{\lambda_S}\varepsilon_{SS} < 0$. The product price is positive and increases; the rate of demand is negative and is fixed. Therefore, it brings an unrestricted decrease of earnings $E_D(t)$ with the passage of time, i.e., $E_D(t) \to -\infty$ for $t \to +\infty$.

If $\varepsilon_{SS} < 0$, then $r_D^0 - \frac{\lambda_D}{\lambda_S}\varepsilon_{SS} > 0$. The product price is negative and decreases; the rate of demand is positive and fixed. Therefore, it brings an unrestricted decrease of earnings $E_D(t)$ with the passage of time, i.e., $E_D(t) \to -\infty$ for $t \to +\infty$.

The last case is not economically plausible since economic models don't ordinarily deal with negative prices.

Additionally to the rate $r_D(t)$ of demand, we can also introduce a rate $r_S(t) \equiv \frac{dV_S(t)}{dt}$ of supply and a rate $r_{SS}(t) \equiv \frac{dS_{SS}(t)}{dt}$ of supply-side shock. Then, we may notice from Eqs. (14) – (16) for $t \to +\infty$,

$$r_S(t) \to r_D(t) + r_{SS}(t) \tag{18}$$

3 Economic Discussion

To build and investigate a mathematical model of the supply-side shock crisis I have employed a simple model of market with single-product economy. Economic forces acting on a market reflect the forces of demand and supply. Besides the forces of demand and supply, I want to describe economic forces that create a supply-side shock.

When there are no disturbing economic forces (lack of forces creating the supply shock), the market is in an equilibrium position, i.e., the supply of and demand for product is equal, and a price of product is fixed. When the balance between the quantities of product supply and demand is broken, market is experiencing the economic forces, which act to bring the market to a new equilibrium position.

The supply-side shock in this model represents a quantity of product that is not supplied to market relative to the equilibrium position. Thus, I remove some quantity of product from the supply side to imitate the supply side shock. Here I use a simple scenario where the supply shock is developing with a constant acceleration.

Economic forces trying to bring the market into a new equilibrium position are described via the system of ordinary differential equations with regard to the volumes of product supply, demand, and price given the accumulated amount of product's removal from market.

To investigate the economic growth I analyze the product earnings on a market, i.e., a rate of the demand for the product multiplied by the product price.

If an amount of the supply shock is developing with a low-value positive acceleration the product price is positive and increasing while the rate of demand is positive and fixed. Therefore, the product earnings are positive and increasing with the passage of time. Here, we can observe an economic growth.

If an amount of the supply shock is developing with a high-value positive acceleration the product price is positive and increasing while the rate of demand is negative and fixed. Therefore, the product earnings are negative and decreasing with the passage of time. Here, we can observe an economic decline.

If an amount of the supply shock is developing with a negative acceleration the product price is negative and decreasing while the rate of demand is positive and fixed. Therefore, the product earnings are negative and decreasing with the passage of time. Here, we can observe an economic decline. Though, this situation is not very significant since economic models rarely deal with negative prices.

Furthermore, we may notice that with the passage of time a rate of the demand is approaching to a rate of the supply decreased by a rate of the supply-side shock. It means that the rate of supply has to produce enough quantities to both satisfy the rate of demand and compensate for the rate of the supply-side shock.

If an amount of the supply shock is developing with a low-value positive acceleration the rate of demand is positive and fixed. If an amount of the supply shock is developing with a high-value positive acceleration the rate of demand is negative and fixed. If an amount of the supply shock is developing with a negative acceleration the rate of demand is positive and fixed.

If an amount of the supply shock is developing with a positive acceleration the rate of supply is positive and increasing with the passage of time. If an amount of the supply shock is developing with a negative acceleration the rate of supply is negative and decreasing with the passage of time.

If an amount of the supply shock is developing with a positive acceleration the rate of the supply-side shock is positive and increasing with the passage of time. If an amount of the supply shock is developing with a negative acceleration the rate of the supply-side shock is negative and decreasing with the passage of time.

4 Conclusion

Presented here is a simplified model of the supply-side shock crisis.

The model describes the supply shock whose development was caused by the permanent removal of high-quantity amount of supply from the market. If an amount of the supply shock has a high quantity the product price is positive and increasing and the rate of demand is negative and fixed. Hence, the product earnings are negative and decreasing with the passage of time. We can observe an economic decline caused by the supply-side shock.

We may notice that economic mechanism here is conceptually close to an economic growth caused by the investment. There also it takes place a permanent removal of some amount of supply from the market. That amount of supply is not wasted but intentionally reinvested into the production process in order to improve the product quality.

Another interesting fact is that a modest-size supply shock can be recuperated. If an amount of the supply shock has a low quantity the product price is positive and increasing and the rate of demand is positive and fixed. Hence, the product earnings are positive and increasing with the passage of time.

We can observe an economic growth here since the economic system is able to overcome crisis caused by the modest supply-side shock.

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