Quantifying Goodwin Growth Cycles with Minimum Wage Shares

Sasaki, Hiroaki and Asada, Yasukuni

Kyoto University, The Norinchukin Bank

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Hiroaki Sasaki[†] **Yasukuni Asada**
Kyoto University **The Norinchuku Bank**

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Abstract
This study extends Goodwin's (1967) growth cycle model to consider two types of workers, low- and high-skilled workers. Using Japanese data from 1989 to 2018, we theoretically and empirically investigate how the introduction of the minimum wage share affects the wage shares and employment rates. Introducing the minimum wage share diminishes the amplitude of fluctuations of both the wage shares and the employment rates, and in this sense, it has a stabilizing effect. Reducing the wage gap between low- and high-skilled workers increases the amplitude of fluctuations of the wage shares and employment rates.

Keywords: growth cycles; low-skilled and high-skilled workers; minimum wage share
JEL Classification: E11; E24; E25; E32; J31

1. Introduction

This study presents a Goodwin's growth cycle model that considers low- and high-skilled workers, and theoretically and empirically investigates the effects of the minimum wage and a reduction in wage gap between the two types of workers on business cycles.

Many countries introduce minimum wages to protect workers, that is, to prevent workers from receiving an extremely low wage. On the other hand, minimum wages can have a negative effect on workers. First, even if unemployed workers want to work at a wage lower than the minimum wage, firms cannot hire these workers because it is a violation of the minimum wage law, which consequently leads to a higher unemployment rate. Second, an increase in labor costs due to high minimum wages depresses the management of firms and increases the probability of bankruptcy. At the same time, to decrease labor costs, firms decrease their employment. Using data for the entire U.S. economy, Brown (1988) demonstrates that an increase in minimum wages lowers employment rates. Card (1992) reveals that an increase in the minimum wage in New Jersey increases the employment rate. Card and Krueger (1994) extend Card's (1992) analysis to the entire U.S. and conclude that an increase in minimum wages has no definite effect on the employment rate.

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[†] Corresponding author. E-mail: sasaki@econ.kyoto-u.ac.jp

The Goodwin model (Goodwin 1967) is based on the Lotka-Volterra equation developed in the field of mathematical biology. In the Goodwin model, business cycles arise for the following reasons. Suppose that the unemployment rate falls for some reason. The bargaining power of workers increases, and the real wage rate increases through the reserve army affect, which increases the wage share. This lowers the profit rate and the capital accumulation rate therefore declines. If the capital accumulation rate declines below the natural growth rate, unemployment increases. The bargaining power of workers then decreases, and the real wage rate also decreases through the reserve army effect, which in turn decreases the wage share. This leads to an increase in the profit rate, and hence, capital accumulation increases. If the capital accumulation rate increases above the natural growth rate, the employment rate increases. This series of processes is repeated endlessly. The Goodwin model shows that the capitalist economy necessarily creates endogenous perpetual business cycles through changes in the bargaining position between workers and capitalists.[1]

The Goodwin model is a model of growth cycles in which the equilibrium output grows at a constant rate; that is, an output trend exists, and the actual output fluctuates around the trend. The Goodwin model is outstanding in that it can simultaneously explain business cycles and economic growth. In addition, the structure of the Goodwin model is relatively simple, and hence, it can easily be extended.

Figure 1 shows the business cycle of the Japanese economy during the period 1989–2018. The horizontal and vertical axes correspond to the employment rate and the wage share, respectively. The data for the employment rate are from the “Labour Force Survey” and those for the wage share are from “Financial Statements Statistics of Corporations by Industry.” The wage share is calculated by dividing labor costs by the sum of labor costs, operating surplus, and capital depreciation.

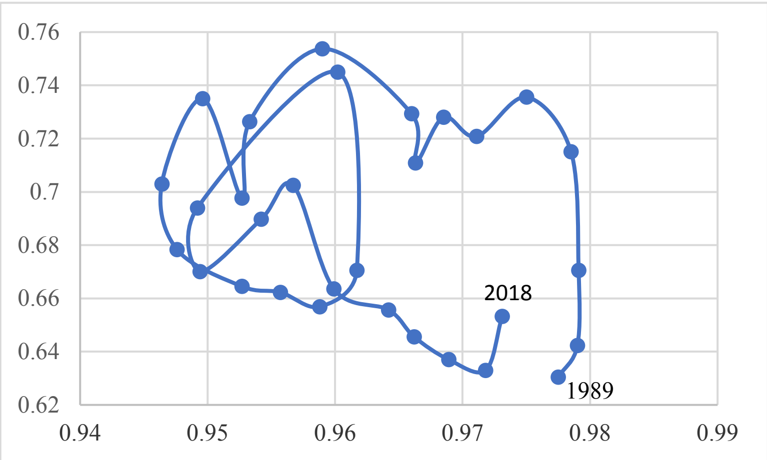

Figure 1: Fluctuations in the employment rate and wage share of the Japanese economy. Source: Labour Force Survey and Financial Statements Statistics of Corporations by Industry.

[1] This study uses a continuous time version of the Goodwin model. For discrete time versions of the Goodwin model, see Pohjola (1981) and Chapters 6 and 7 of Foley *et al*. (2019).

As Figure 1 shows, these variables rotate counterclockwise as the Goodwin model suggests. Such patterns of business cycles are observed in many developed countries. Using data for OECD countries, Zipperer and Skott (2011) show that the pair of the employment rate and the wage share rotates counterclockwise. We use the Goodwin model because it can explain the pattern of the business cycle.

Some theoretical studies that investigate the effect of the minimum wage on an economy using the Goodwin model. Flaschel and Greiner (2009) theoretically show that the introduction of the minimum wage can diminish the extent of the business cycle. Flaschel and Greiner (2011) introduce the minimum wage into an extended Goodwin model that considers dual labor markets and demonstrate that it can diminish the size of the business cycle. However, these studies are theoretical and hence, cannot quantitatively determine the degree to which the sizes of business cycles can be diminished.[2]

Some empirical studies are based on the Goodwin model.[3] Harvie (2000) conducts an empirical analysis using data for OECD countries during the period 1951–1994. The author estimates the growth rate of labor productivity, the growth rate of labor supply, the Phillips curve, and the capital-output ratio. The study then obtains the steady-state equilibrium values and fluctuations of the employment rate and wage share. The author concludes that a closed orbit around the steady state produced by the theoretical model cannot be obtained, but business cycles can be roughly captured. Using quarterly data for the United States during the period 1948–2004, Mohun and Veneziani (2008) investigate whether it is possible to observe the business cycle produced by the Goodwin model. They conclude that the Goodwin model cannot reproduce long-run business cycles, but it can reproduce short-run business cycles to some extent. Using quarterly data for the U.S. economy, Tarassow (2010) conducts an empirical analysis and concludes that the mechanism of the business cycle of the Goodwin model is appropriate. Grasselli and Maheshwari (2018) perform an econometric test on a modified Goodwin model in which the saving rate of capitalists, which is equal to unity in the original Goodwin model, is less than unity. In addition, they address the methodological and reporting issues in Harvie (2000), which leads to a remarkably better result. They conclude that despite its simplicity and obvious limitations, the performance of the modified Goodwin model can be used as a starting point for more sophisticated models for endogenous growth cycles.[4] Araujo *et al*. (2019) present an extended Goodwin model in which the capacity utilization rate as well as the employment rate and the wage share are endogenous variables, and theoretically show that limit cycles always occur.

[2] Flaschel *et al*. (2012) also investigate the effect of the minimum wage on an economy using a Goodwin model.

[3] Ryzhenkov (2009) conducts an empirical analysis of the Italian economy by applying a Goodwin model.

[4] Grasselli and Maheshwari (2017) highlight a mistake in Harvie (2000) and demonstrate that the correction of the mistake leads to significantly different results that support the empirical performance of the Goodwin model.

In addition, using quarterly data for the U.S. economy during the period 1948–2016, we estimate the parameters of the model, conduct numerical simulations, and show that different initial values converge to different limit cycles. This interesting result suggests path dependency. However, these empirical studies do not analyze the effect of minimum wages on business cycles.

Based on the above observation, the current study uses an extended Goodwin model to investigate the effect of the minimum wage on growth cycles both theoretically and empirically. The main contributions of this study are as follows:

First, we classify workers into two groups: low- and high-skilled workers, and assume that both types save their wages. When investigating the effect of the minimum wage, we should not abstract workers' savings because it prevents us from strictly analyzing the effect of minimum wage. Moreover, some workers are engaged in work around the minimum wage, while others are not. Hence, each group is affected differently by the minimum wage.[5]

Second, we conduct both theoretical and empirical analyses. As previously stated, existing studies that incorporate minimum wages into the Goodwin model theoretically show that the introduction of the minimum wage can diminish cyclical fluctuations, but they cannot clarify the extent to which its introduction can diminish cyclical fluctuations. Using Japanese annual data during the period 1989–2018, we estimate the parameters of our Goodwin model, conduct numerical simulations to reproduce business cycles, and quantitatively examine the extent to which business cycles are diminished. The minimum wage herein corresponds to the minimum wage share. In our model, labor productivity increases at a constant rate; hence, we must increase the minimum wage at the same rate as labor productivity to keep the minimum wage share at a target level. From this, a lower bound is set to the wage share level.

Third, our model includes an exogenously given wage gap between low- and high-skilled workers, and we quantitatively investigate the degree by which cyclical fluctuations are diminished when the wage gap is reduced.

We obtain the following results. First, the introduction of the minimum wage share policy diminishes cyclical fluctuations of the employment rates and the wage shares for the entire economy and both high- and low-skilled workers. In this sense, the business stabilizing effect of the minimum wage policy is large. Second, a reduction in the wage gap between the two types of workers increases the extent of cyclical fluctuations of the employment rates and the wage share for the entire economy and both high- and low-skilled workers. This implies that reducing the wage gap according to the concept of equal pay for equal work can lead to an unstable economy.

The effect of a reduction in the wage gap was also examined by Sasaki *et al.* (2013). They build a Kaleckian model that incorporates two types of workers, regular workers

[5] Pasinetti (1962) asserts that when workers save, not only capitalists but also workers own capital stock. For this issue, see the debate between Pasinetti (1962) and Samuelson and Modigliani (1966). For simplicity, we do not consider workers' capital accumulation. Van der Ploeg (1984) considers workers' capital accumulation in the Goodwin model. For workers' capital accumulation, see also Taylor *et al*. (2019).

and non-regular workers. Using the model, they theoretically demonstrate that the introduction of the minimum wage diminishes the extent of cyclical fluctuations and that a reduction in the wage gap between the two types of workers stabilizes the economy.[6] For the introduction of the minimum wage, we reach the same conclusion, but for the reduction in the wage gap, we reach the opposite conclusion. This is because their study is based on a demand-led growth model in which the principle of effective demand prevails, while our study is based on a supply-led growth model in which Say's law prevails. This difference suggests that policy implications differ depending on which model is used to consider policy effects, that is, a demand-led or a supply-led model.

The remainder of this paper is organized as follows. Section 2 presents an extended Goodwin model. Section 3 estimates the parameters of the model using Japanese data and analyzes the patterns of business cycles generated by our model. Section 4 investigates the effect of the introduction of the minimum wage on business cycles. Section 5 examines the effect of a reduction in the wage gap between the two types of workers. Finally, concluding remarks are presented in section 6.

2. Model

Suppose we have an economy in which low-skilled workers, high-skilled workers, and capitalists coexist. Firms produce a single good used for both consumption and investment using low- and high-skilled workers, and capital stock according to the following Leontief production function:

$$Y = \min\{aE_H, bE_L, \sigma K\}, \quad \sigma > 0 \tag{1}$$

$$a = \bar{a}e^{\phi t}, \ \bar{a} > 1, \ \phi > 0 \tag{2}$$

$$b = e^{\phi t} \tag{3}$$

where Y denotes output, E_H is employment of high-skilled workers, E_L is employment of low-skilled workers, K is capital stock, a is labor productivity of high-skilled workers, b is labor productivity of low-skilled workers, and σ is capital productivity. Labor productivity of both groups increases at the same constant rate ϕ, which is given exogenously.[7] The inequality $\bar{a} > 1$ means that the level of labor productivity of high-skilled workers always exceeds that of low-skilled workers.

Firms are assumed to adopt cost-minimizing behaviors, and accordingly, from equation (1), they operate at a point where $aE_H = bE_L = \sigma K$. Let L denote the supply of labor. Then, the employment rate of high-skilled workers, that of low-skilled workers, and that of the whole economy can respectively be given as follows:

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[6] Sasaki (2016) introduces profit sharing into Sasaki *et al.*'s (2013) model. Here, profit sharing is a rule such that a constant fraction of profits is redistributed to regular workers. Sonoda and Sasaki (2019) endogenize the wage gap between regular and non-regular workers, which is given exogenously in Sasaki *et al*. (2013).

[7] For Goodwin models with endogenous technical change, see Shah and Desai (1981), Van der Ploeg (1987), and Julius (2005). In the Goodwin model, endogenous technical change can stabilize business cycles. For micro-founded Goodwin models, see Tavani (2012, 2013).

$$x_H = \frac{\sigma K}{aL} \tag{4}$$

$$x_L = \frac{\sigma K}{bL} = \bar{a} x_H \tag{5}$$

$$x = \frac{E}{L} = \frac{E_H + E_L}{L} = (1 + \bar{a}) x_H \tag{6}$$

We assume that the labor supply L grows at a constant rate n, which is given exogenously. From equations (4)–(6), the dynamics and the values of x_L and x can be obtained from those of x_H. This property is used when conducting the numerical simulations introduced below.

Suppose that the growth rate of the real wage rate of high-skilled workers w_H is an increasing function of the employment rate of high-skilled workers x_H, which is specified as follows:

$$\frac{\dot{w}_H}{w_H} = -\alpha + \beta x_H, \alpha > 0, \beta > 0 \tag{7}$$

where α denotes the constant term of the real wage Phillips curve, and β is the response of the growth rate of the real wage rate to the employment rate of high-skilled workers. Skilled workers belong to labor unions, and through negotiations between them and managers, the real wage rate changes. When the employment rate is high, the bargaining power of labor unions is high, leading to a large change in the real wage rate. On the other hand, when the employment rate is low, the bargaining power of labor unions is low, leading to a small change in the real wage rate.

Suppose that the real wage rate of high-skilled workers is more than that of low-skilled workers by a constant factor.[8]

$$w_H = \gamma w_L, \gamma > 1 \tag{8}$$

where γ is a positive constant. From equation (8), the real wage rate of the entire economy is given by

$$w = \frac{E_H}{E} w_H + \frac{E_L}{E} w_L = \frac{\bar{a} + \gamma}{(1 + \bar{a})\gamma} w_H \tag{9}$$

Equation (9) shows that the real wage rate of the entire economy is proportional to that of high-skilled workers. From equations (8) and (9), the wage share of high-skilled workers, that of low-skilled workers, and that of the entire economy can respectively be given by

$$y_H = \frac{w_H E_H}{Y} = \frac{w_H}{a} \tag{10}$$

$$y_L = \frac{w_L E_L}{Y} = \frac{\bar{a}}{\gamma} y_H \tag{11}$$

$$y = \frac{w_H E_H + w_L E_L}{Y} = y_L + y_H = \frac{\bar{a} + \gamma}{\gamma} y_H \tag{12}$$

[8] A similar specification is also adopted by Lavoie (2009), Sasaki *et al*. (2013), and Sasaki (2016).

From equations (10)–(12), the dynamics and the values of y_L and y can be obtained. This property is used for the numerical simulations introduced later.

By definition, the profit rate is equal to capital productivity multiplied by profit share.

$$r = \sigma(1 - y) = \sigma\left(1 - \frac{\bar{a} + \gamma}{\gamma} y_H\right) \tag{13}$$

We now define saving. Suppose that low- and high-skilled workers save their wage incomes at constant rates s_w^L and s_w^H, respectively, and that capitalists save their profit incomes at a constant rate s_c. From this, the ratio of savings to capital stock is given by:

$$\frac{S}{K} = \frac{s_w^L w_L E_L + s_w^H w_H E_H + s_c r K}{K} = s_w^L \sigma \frac{\bar{a}}{\gamma} y_H + s_w^H \sigma y_H + s_c \sigma \left(1 - \frac{\bar{a} + \gamma}{\gamma} y_H\right) \tag{14}$$

where we assume that $s_w^L < s_w^H < s_c$. This assumption is reasonable based on the reality that capitalists' income is highest, whereas low-skilled workers' incomes are the lowest.

Let δ denote the capital depreciation rate. Net investment $\dot{K}$ is equal to gross investment I minus capital depreciation δK. Hence, the capital accumulation rate is given by:

$$\frac{\dot{K}}{K} = \frac{I}{K} - \delta, 0 < \delta < 1 \tag{15}$$

Goods market clearing is attained when total saving is equal to total investment, that is, $S = I$. Taking logarithms of equation (4), differentiating them with respect to time, and applying equations (14) and (15) to the resultant expressions, we obtain the rate of change in the employment rate of high-skilled workers as follows:

$$\frac{\dot{x}_H}{x_H} = \frac{I}{K} - \delta - \phi - n = s_w^L \sigma \frac{\bar{a}}{\gamma} y_H + s_w^H \sigma y_H + s_c \sigma \left(1 - \frac{\bar{a} + \gamma}{\gamma} y_H\right) - \delta - \phi - n \tag{16}$$

Using equations (7) and (10), we obtain the rate of change in the wage share of high-skilled workers as follows:

$$\frac{\dot{y}_H}{y_H} = \frac{\dot{w}_H}{w_H} - \frac{\dot{a}}{a} = -\alpha + \beta x_H - \phi \tag{17}$$

From equations (16) and (17), our model can be reduced to the following system of differential equations:

$$\dot{x}_H = \left[(s_c \sigma - \delta - \phi - n) - \sigma\left(\frac{\bar{a} + \gamma}{\gamma} s_c - s_w^H - \frac{\bar{a}}{\gamma} s_w^L\right) y_H\right] x_H \tag{18}$$

$$\dot{y}_H = -[(\alpha + \phi) - \beta x_H] y_H \tag{19}$$

The system composed of equations (18) and (19) has the same structure as that of Goodwin (1967), that is, the typical Lotka-Volterra equation. Therefore, the dynamics of the employment rate and the wage share of high-skilled workers show closed orbits as long as the steady-state equilibrium exists in the first quadrant of (x_H, y_H) the plane.

We define the steady state as a situation in which $\dot{x}_H = \dot{y}_H = 0$. Letting the right-hand sides of equations (18) and (19) be zero, we can obtain the steady state values of the employment rate and the wage share of high-skilled workers as follows:

$$x_H^* = \frac{\alpha + \phi}{\beta} \tag{20}$$

$$y_H^* = \frac{s_c\sigma - \delta - \phi - n}{\sigma\left(s_c\dfrac{\bar{a}+\gamma}{\gamma} - s_w^H - \dfrac{\bar{a}}{\gamma}s_w^L\right)} = \frac{\gamma(s_c\sigma - \delta - \phi - n)}{\sigma[(\bar{a}+\gamma)s_c - \bar{a}s_w^L - \gamma s_w^H]} \tag{21}$$

Hereafter, a variable with "*" denotes the steady-state value of the variable.

Both the employment rate and the wage share must be more than zero and less than unity. First, from equation (20), the condition in which the employment rate is less than unity is given by

$$\alpha + \phi < \beta \tag{22}$$

This means that the response coefficient of the real wage Phillips curve must be larger than the sum of the constant term of the curve and the growth rate of labor productivity. Second, for the wage share to be more than zero and less than unity, from equation (21), the following three conditions must be satisfied.[9]

$$s_c\sigma > \delta + \phi + n \tag{23}$$

$$s_c > \frac{\bar{a}}{\bar{a}+\gamma}s_w^L + \frac{\gamma}{\bar{a}+\gamma}s_w^H \tag{24}$$

$$s_c\sigma - \delta - \phi - n < \sigma\left(s_c\frac{\bar{a}+\gamma}{\gamma} - s_w^H - \frac{\bar{a}}{\gamma}s_w^L\right) \tag{25}$$

Equation (24) states that the saving rate of capitalists must be more than the weighted average of the saving rates of the two types of workers. This condition is necessarily satisfied as long as $s_w^L < s_w^H < s_c$. In the empirical analysis and numerical simulations introduced below, the conditions given by equations (22)–(25) are satisfied.

With the steady-state values of x_H^* and y_H^*, we can obtain the steady-state values of other endogenous variables as follows:

$$x_L^* = \frac{\bar{a}(\alpha+\phi)}{\beta} \tag{26}$$

$$x^* = (1+\bar{a})\frac{\alpha+\phi}{\beta} \tag{27}$$

$$y_L^* = \frac{\bar{a}(s_c - \delta - \phi - n)}{\sigma[(\bar{a}+\gamma)s_c - \bar{a}s_w^L - \gamma s_w^H]} \tag{28}$$

$$y^* = \frac{(\bar{a}+\gamma)(s_c\sigma - \delta - \phi - n)}{\sigma[(\bar{a}+\gamma)s_c - \bar{a}s_w^L - \gamma s_w^H]} \tag{29}$$

Based on the above results, we can draw the phase diagram of the employment rate and the wage share of high-skilled workers, which is shown in Figure 2. The horizontal axis and the vertical axis correspond to the employment rate of high-skilled workers and the wage share of high-skilled workers, respectively. Point E corresponds to the steady state. An economy starting from the initial value S_1 moves along the corresponding closed orbit and returns to point S_1. This process is repeated endlessly. In addition, an economy starting from the initial value S_2 moves along the corresponding closed orbit and returns to point S_2. This means that different closed

[9] In equation (18), for the rate of change in x_H to be positive when $y_H = 0$, we need $s_c\sigma > \delta + \phi + n$. From this, the numerator of y_H^* becomes positive, and hence, the denominator of y_H^* must be positive, which produces the condition given by equation (24).

orbits exist for different initial values. In this way, the employment rate and the wage share of high-skilled workers continue to fluctuate periodically.

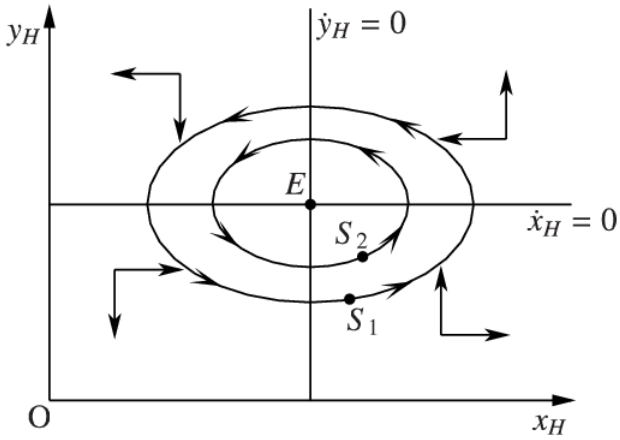

Figure 2: Closed orbits of employment rate and wage share of high-skilled workers

3. Quantifying the Goodwin growth cycles

In this section, we estimate the parameters of the model and reproduce business cycles using Japanese data. We need to estimate 11 parameters: three kinds of saving rates, capital productivity, the coefficient and the constant term of the real wage Phillips curve, the wage gap between low- and high-skilled workers, the growth rate of labor productivity, the initial value of the labor productivity of high-skilled workers, the capital depreciation rate, and the population growth rate.

First, we find the savings rates of the two types of workers and capitalists. We use data from the "Survey of Households' Financial Behavior" published by the Bank of Japan in 2018. This survey provides savings rates according to household income levels, which are classified into five income categories. We regard workers with income levels lower than 3 million yen as low-skilled workers and those with income levels between 3 million yen and 12 million yen as high-skilled workers, and then obtain each class's average saving rate. For the saving rate of capitalists, we use the saving rate of households with income levels higher than 12 million yen. From the data, we obtain $s_w^L = 0.06$, $s_w^H = 0.13$, $s_c = 0.19$. These savings rates satisfy the condition $s_w^L < s_w^H < s_c$.

Second, we obtain capital productivity. Capital productivity is the inverse of the capital coefficient, which is the ratio of real capital stock to real GDP. The data on capital coefficients were obtained from "National Accounts of Japan" in 2016. From this, we obtain $\sigma = 0.4$.

Third, we estimate the real wage Phillips curve.[10] We regard university graduates as high-skilled workers. From the data of "2018 Basic Survey on Wage Structure" by the Ministry of Health, Labour, and Welfare, we obtain nominal wages of university

[10] This study estimates the real wage Phillips curve. On the other hand, Flaschel *et al*. (2007) estimate the nominal wage Phillips curve and the price Phillips curve separately.

graduates and deflate them by the consumer price index in 2015 to obtain real wages. The real wage rates during the period 1989–2018 are shown in Figure 3. For the employment rates of high-skilled workers, we use the values of university graduates from the Labor Force Survey by the Statistics Bureau of Japan.

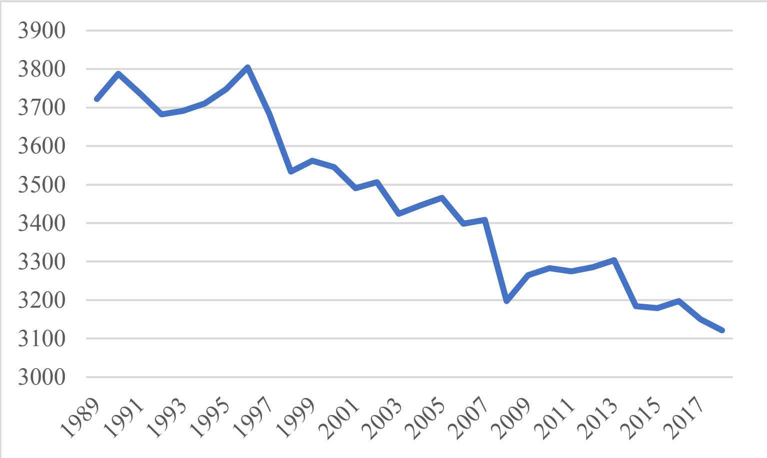

Figure 3: Time series of real wage rates of high-skilled workers. Source: Basic Survey on Wage Structure

We conduct the augmented Dickey–Fuller (ADF) test to check if the time series of real wage growth and employment rates have unit roots.[11] The results are shown in Tables 1 and 2. From these tables, we find that the null hypothesis that both time series have unit roots is rejected.

Table 1: Results of unit root test of growth rate of real wage

Null Hypothesis: Real wage growth has a unit root			
Exogenous: Constant, Linear Trend			
Lag Length: 0 (Automatic-based on SIC, maxlag=4)			
		t-Statisitc	Prob.*
Augmented Dickey-Fuller test statistic		-3.633797	0.0499
Test critical values:	1% level	-4.440739	
	5% level	-3.632896	
	10% level	-3.254671	
*Mackinnon (1996) one-sided p-values			

Table 2: Results of unit root test of employment rate

[11] We use EViews 11 for estimations.

Null Hypothesis: Employment rate has a unit root			
Exogenous: Constant, Linear Trend			
Lag Length: 3 (Automatic-based on SIC, maxlag=5)			
		t-Statisitc	Prob*
Augmented Dickey-Fuller test statistic		-3.471207	0.00702
Test critical values:	1% level	-4.498307	
	5% level	-3.658446	
	10% level	-3.268973	
*Mackinnon (1996) one-sided p-values			

Since we confirm that two variables do not have unit roots, we estimate the real wage Phillips curve by ordinary least squares (OLS). The results are presented in Table 3. We find that $\alpha = 0.301$ and $\beta = 0.805$. These values are consistent with the signs expected by our model.

Fourth, we obtain the wage gap between high- and low-skilled workers. We regard university graduates and high school graduates as high- and low-skilled workers, respectively. From the data of the "Basic Survey on Wage Structure," we find that $\gamma = 1.5$.

Fifth, based on the "National Accounts of Japan," the capital depreciation rate is set to $\delta = 0.04$.

Sixth, the population growth rate is set to $n = -0.0021$ from the data of "Demographics of Japan" by the Statistics Bureau of Japan. Japan is found to experience a population decline.

Seventh, we obtain the technology gap between high- and low-skilled workers. From the production function, we obtain $\bar{a} = E_L/E_H$. From the "2017 Labour Statistics Annual Report" by the Ministry of Health, Labour, and Welfare, we obtain $\bar{a} = 1.5$ by calculating the ratio of workers with high school education to those with at least university education.

Table 3: Estimates of real wage Phillips curve

Dependent Variable: Real wage growth				
Method: Least Squares				
Sample (adjusted): 2 24				
Included observations: 23 after adjustments				
Variable	Coefficient	Std.Error	t-Statistic	Prob.
Constant	-0.301641	0.022376	-13.48049	0.0000
Employment rate	0.805335	0.050542	15.93399	0.0000
R-squared	0.923606	Mean dependent var	0.051417	
Adjusted R-squared	0.919969	S.D. dependent var	0.052889	
S.E. of regression	0.014962	Akaike info criterion	-5.483652	
Sum squared resid	0.004701	Schwarz criterion	-5.384913	
Log likelihood	65.06200	Hannan-Quinn criterion	-5.458820	
F-statistic	253.8922	Durbin-Watson stat	2.070973	
Prob (F-statistic)	0.000000			

Eighth, for the growth rate of labor productivity, we use the data from the Japan Productivity Center. We use the growth rate of real labor productivity per worker during the period 1989–2017, which leads to $\phi = 0.01$.

These results are summarized in Table 4. These estimated parameter values satisfy all the parametric conditions given by equations (22)–(25).

Table 4: Estimates of 11 parameters

α	β	s_W^L	s_W^H	s_C	γ	δ	n	a	σ	ϕ
0.301	0.805	0.06	0.12	0.19	1.5	0.04	-0.0021	1.5	0.4	0.01

Table 5 presents the equilibrium values of the employment rates and the wage shares by using the values in Table 4.

Table 5: Estimated equilibrium values

	Average	Low-skilled workers	High-skilled workers
Employment rate	0.965839	0.579503	0.386335
Wage share	0.638636	0.319318	0.319318

Initial values are also required for numerical simulations. The initial values of the employment rates and the wage shares are calculated as follows. First, from the data of 2018, the employment rate and the wage share for the entire economy are $x_{2018} = 0.973$ and $y_{2018} = 0.65$, respectively, which are used for the initial values of the employment rate and the wage share. Numerical simulations are based on the dynamics of x_H and y_H, and the initial values of x_H and y_H are necessary. We then obtain $x_H(0) = x_{2018}/(1+\bar{a}) = 0.3892$ and $y_H(0) = y_{2018}\gamma/(\bar{a}+\gamma) = 0.325$ from equations (4) and (10), respectively. In addition, we obtain $x_L(0) = \bar{a}x_H(0) = 0.5838$ and $y_L(0) = (\bar{a}/\gamma)y_H(0) = 0.325$ from equations (5) and (11), respectively.

Using these estimated initial values and parameters, we conducted numerical simulations, the results of which are shown in Figures 4–6.[12] Figure 4 shows a closed orbit for the entire economy, Figure 5 shows one for high-skilled workers, and Figure 6 shows one for low-skilled workers. In these figures, the horizontal lines and the vertical lines correspond to employment rates and wage shares, respectively.

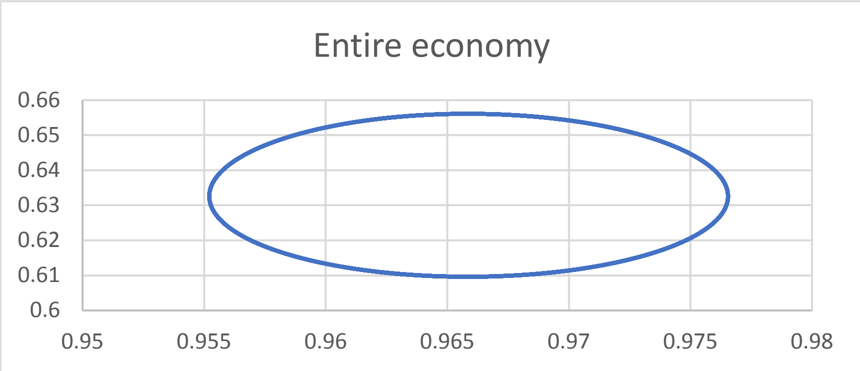

Figure 4: Estimated closed orbit for the entire economy

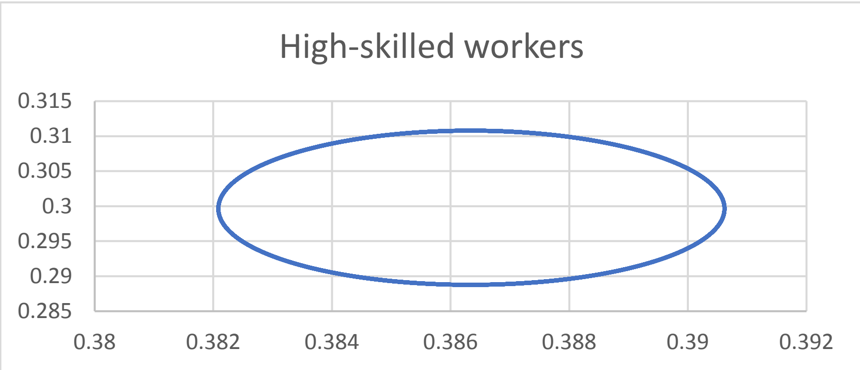

Figure 5: Estimated closed orbit for high-skilled workers

Figure 6: Estimated closed orbit for low-skilled workers

4. Introduction of minimum wage share

[12] Numerical analysis of the differential equations is conducted using Excel with the Runge-Kutta method.

This section introduces the minimum wage into our model and investigates its effect on business cycles. As stated in the introduction, Flaschel and Greiner (2009) theoretically show that the introduction of the minimum wage can diminish fluctuations of business cycles in the Goodwin model. Labor productivity is constant and does not grow and hence, minimum wage corresponds to minimum wage share, which leads to the restriction of business cycles. On the other hand, in our model, labor productivity grows at a constant rate and we therefore have to assume that the minimum wage continues to increase at the same rate as the growth rate of labor productivity for the minimum wage to correspond to the minimum wage share. Based on this assumption, the lower bound is set to the wage share. If we set the minimum wage share to a value lower than the steady-state wage share, the economy starting from point S (Figure 7) continues to move to the right from point P, reaches point Q, and then rides on a closed orbit that is smaller than the original. This means that the introduction of the minimum wage share can mitigate business cycles. However, if we set the minimum wage share to a value higher than the steady-state wage share, as shown in Figure 8, the economy finally converges to point Q, where the employment rate is zero. Therefore, the minimum wage share policy must be undertaken with due care.

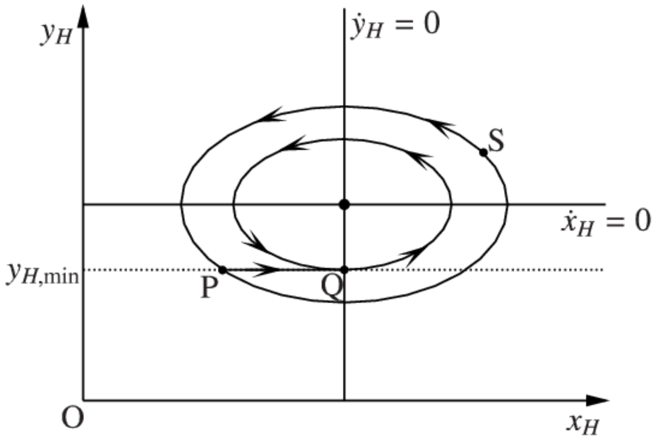

Figure 7: Dynamics of employment rate and wage share with appropriate minimum wage share

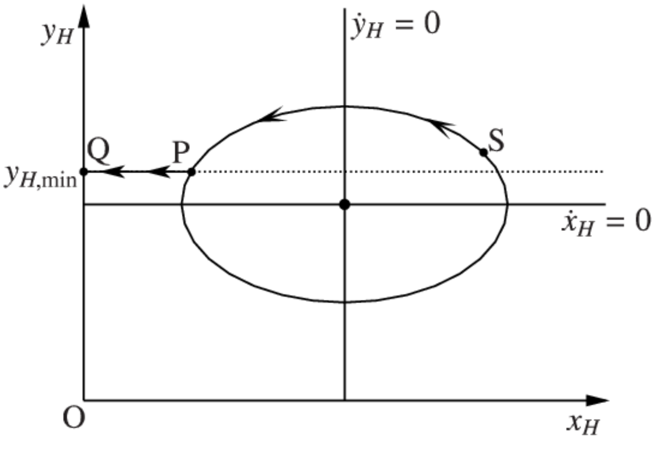

Figure 8: Dynamics of employment rate and wage share with inappropriate minimum wage share

We perform a regression analysis to convert a minimum wage into a minimum wage share. The data for wage shares and real wage rates are the same as those used above. First, we test whether both time series of wage shares and real wage rate shave unit roots. Performing the ADF test, we find that both wage shares and real wage rates have unit roots, as shown in Tables 6 and 7. When two time series have unit roots, there may be a spurious correlation between them. We test whether there is cointegration between the two variables; if there is cointegration, we can perform OLS for these two variables. Table 8 shows the results of the cointegration test.

Table 6: Unit root test for wage share

Null Hypothesis: Wage share has a unit root Exogenous: Constant Lag Length: 0 (Automatic-based on SIC, maxlag=7)			
		t-Statisitc	Prob*
Augmented Dickey-Fuller test statistic		-2.351669	0.15810
Test critical values:	1% level	-3.679322	
	5% level	-2.967767	
	10% level	-2.622989	
*Mackinnon (1996) one-sided p-values			

Table 7: Unit root test for real wage rates

Null Hypothesis: Real wage rate has a unit root Exogenous: Constant Lag Length: 0 (Automatic-based on SIC, maxlag=6)			
		t-Statisitc	Prob*
Augmented Dickey-Fuller test statistic		-0.563607	0.86350
Test critical values:	1% level	-3.689194	
	5% level	-2.971853	
	10% level	-2.625121	
*Mackinnon (1996) one-sided p-values			

Table 8: Cointegration test for wage share and real wage rates

Johansen Cointegration test				
Sample(adjusted): 3 29				
Included observations: 27 after adjustments				
Trend assumption: Linear determinisistic trend				
Series: Wage share, Real wage rate				
Lags interval (in first differences): 1 to 1				
Unrestricted Cointegration Rank Test (Trace)				
Hypothesized No. of CE(S)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None	0.424154	14.94124	15.49471	0.0604
At most 1	0.001463	0.039525	3.841465	0.8424
Trace test indicates no cointegration at the 0.05 level				
*denotes rejection of the hypothesis at the 0.005 level				
**MacKinnon-Haug-Michelis (1999) p-values				

Table 8 suggests that there is cointegration between wage shares and real wage rates. Hence, we perform OLS analysis such that the wage share is a dependent variable and the real wage rate is an independent variable, the results of which are shown in Table 9.[13] The results reveal that an increase in the real wage rate slightly increases the wage share.

Table 9: Relationship between wage share and real wage rate

Dependent Variable: Wage share				
Method: Least Squares				
Sample (adjusted): 1 29				
Included observations: 29 after adjustments				
Variable	Coefficient	Std.Error	t-Statistic	Prob.
Real wage rate	0.000198	2.42E-06	81.85784	0.0000
R-squared	-0.534718	Mean dependent var		0.688526
Adjusted R-squared	-0.534718	S.D. dependent var		0.036537
S.E. of regression	0.045263	Akaike info criterion		-3.318781
Sum squared resid	0.057365	Schwarz criterion		-3.271633
Log likelihood	49.12232	Hannan-Quinn criterion		-3.304015
Durbin-Watson stat	0.661104			

Using this result, we incorporate the minimum wage share into our model. In 2019, the weighted average of minimum wages across Japan is about 900 yen. If we increase it by 20%, we have 1100 yen. From the data of the “Handbook of Labour Statistics” by the Ministry of Health, Labour, and Welfare, we have average working hours to obtain nominal monthly salaries for workers who receive minimum wages. Next, by dividing this monthly salary by the consumer price index, we obtain the real wage rate for

[13] R-squared is negative because we conduct regression analysis through the origin with the interception being zero.

minimum wage workers. Finally, using the results shown in Table 9, we can convert the resultant minimum wage into the minimum wage share, which leads to $y_{H,\min} = 0.318$. We use this value for the minimum wage share of skilled workers. As previously stated, in our model, labor productivity grows at a constant rate and hence, we have to increase the level of the minimum wage at the same rate as the growth rate of labor productivity. Therefore, if the initial value of the level of the minimum wage is 1100 yen, we have to assume that the level of the minimum wage increases at the same rate (0.1%) as the growth rate of labor productivity to keep the minimum wage share 0.318.

We incorporate the minimum wage share into our model, the results of which are shown in Figures 9–11. Figure 9 shows the dynamics for the entire economy. With the minimum wage share for high-skilled workers set at 0.318, we use 0.636 for the minimum wage share for the entire economy because the wage share for the entire economy is twice that for high-skilled workers ($2 = (\bar{a} + \gamma)/\gamma$).

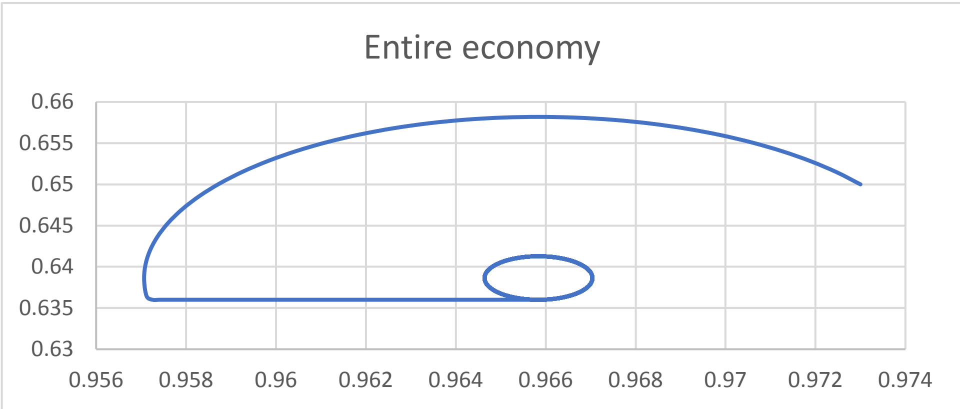

Figure 9: Dynamics for the entire economy with minimum wage share

Figure 10: Dynamics for high-skilled workers with minimum wage share

Figure 11: Dynamics for low-skilled workers with minimum wage share

From Figures 9–11, we find that the introduction of the minimum wage share considerably diminishes the extent of cyclical fluctuations. Nevertheless, it should be noted that the introduction of the minimum wage share requires a policymaker to increase the level of the minimum wage at the same rate as the growth rate of labor productivity, which in turn requires the policy maker to precisely measure the growth rate of labor productivity.

5. Reducing wage gap between the two types of workers

As we have seen in Section 4, the appropriate introduction of the minimum wage share diminishes fluctuations in business cycles. In this section, as an alternative policy, we investigate the effect of a reduction in the wage gap between high- and low-skilled workers on business cycles. A reduction in the wage gap is similar to "equal pay for equal work" that the Japanese government pursues. Figures 12–14 show business cycles when the wage gap is reduced from $\gamma = 1.5$ to $\gamma = 1.35$. With the reduction of the wage gap, from equations (4) and (10), the initial values are changed to $x_H(0) = 0.3892$ and $y_H(0) = 0.307895$, and from equations (5) and (11), to $x_L(0) = 0.5838$ and $y_L(0) = 0.342105$. In Figures 12–14, the blue lines correspond to pre-reduction business cycles, and the gray lines correspond to post-reduction business cycles.

Figure 12 shows the closed orbits for the entire economy. A reduction in the wage gap shifts the closed orbits downward and enlarges the business cycle. This suggests that the economy becomes more unstable and the wage share for the entire economy decreases overall.

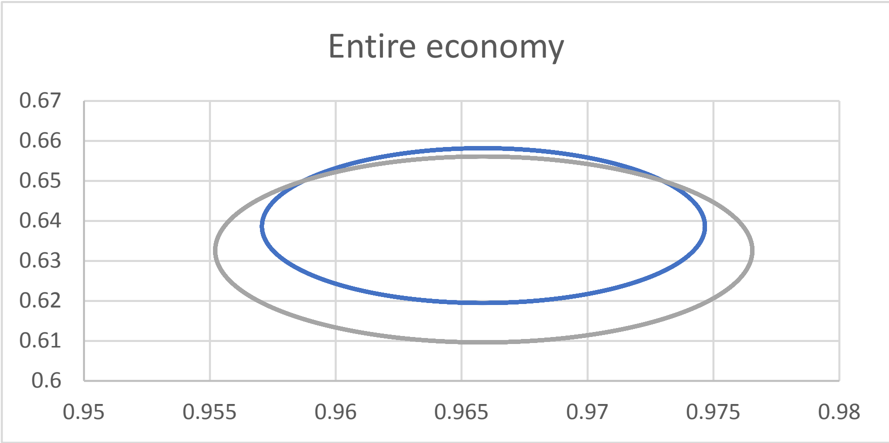

Figure 12: Effect of reduction in wage gap on the entire economy

Figure 13 shows the closed orbits for low-skilled workers. A reduction in the wage gap enlarges business cycles and shifts the closed orbit upward. This suggests that the wage share of low-skilled workers increases on average, while the economy becomes more unstable.

Figure 14 shows the closed orbits of high-skilled workers. A reduction in the wage gap intensifies business cycles and shifts the closed orbit downward. This suggests that the economy becomes more unstable and the wage share of high-skilled workers becomes lower on average.

A reduction in the wage gap seems desirable for wages of low-skilled workers, but not for the stability of the economy. The wage gap should be determined according to the ability of workers. The concept of equal pay for equal work is right as long as the job description is equal. However, excessively closing the wage gap between different job descriptions is likely to make the economy more unstable.

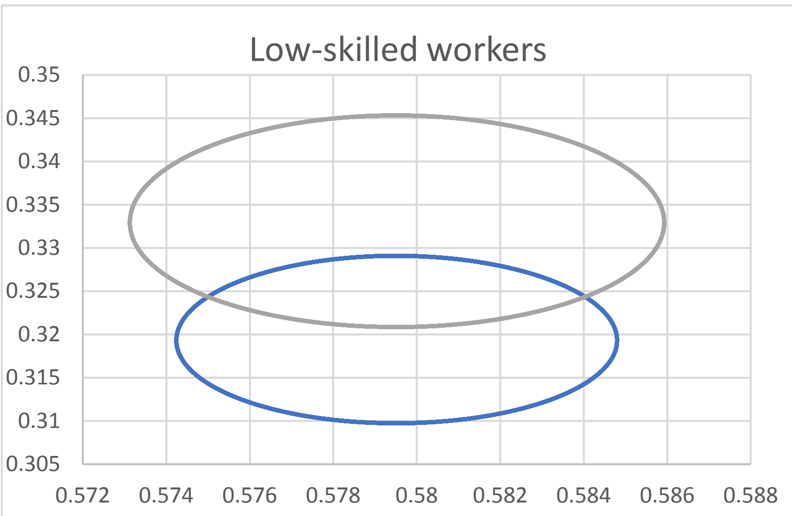

Figure 13: Effect of reduction in wage gap on low-skilled workers

Figure 14: Effect of reduction in wage gap on high-skilled workers

6. Conclusions

We presented an extended version of the Goodwin model that considers two types of workers and the workers' savings, and by estimating the parameters with data for Japan, we have reproduced Japanese business cycles. Moreover, using the results of the estimation, we investigated the effects of the minimum wage share and the reduction in the wage gap between the two types of workers on business cycles. Our results reveal that the introduction of the minimum wage share diminishes business cycles and that a reduction in the wage gap intensifies such cycles. Therefore, in terms of the stabilization of the economy, the introduction of the minimum wage share is a desirable policy compared to the reduction of the wage gap.

Although our study makes a certain contribution, there remain some research topics to be investigated both theoretically and empirically.

With regard to theoretical analysis, in our Goodwin model, the employment rate and the wage share can be more than unity along a closed orbit unless the initial values and parameters are chosen appropriately. For this issue, Desai *et al*. (2006) provide an extended Goodwin model in which both the employment rate and the wage share cannot exceed unity, which can be used for future research.

With regard to empirical analysis, we should investigate the possibility of a structural change during the period 1989–2018. During this period, the Japanese economy experienced the collapse of the bubble economy and the 2008 financial crisis. We used annual data due to data availability, but due to the restriction of the number of samples, we do not divide the sample period. However, if monthly or quarterly data are available, we can increase the number of samples and can conduct an empirical analysis that considers structural change.

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