An Optimal Policy for Social Resources Allocation: When Outbreak of Infectious Diseases

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Abstract This paper analyzes the optimal policy for social resources allocation when outbreak of infectious diseases like the coronavirus. Infectious diseases not only pose a threat to human health, but also have a great impact on society and economy. There remains considerable disagreement between economists' views and medical experts' views, for example, a gradual steps policy vs. an immediate lockdown policy. This paper grafts the epidemiological model (Susceptible-Infected-Recovered model or SIR model) for the spread of infectious diseases onto an economic optimization model in order to reconcile the epidemic control and economic activity. This paper considers that we can control the infection rate and recovery rate through our efforts which are parameters in the SIR model, for example, self-quarantine, lockdown, increasing the number of hospital and medical personnel, developing vaccines, etc. This paper calculated the optimal resources allocation and the timing for the infectious diseases. This paper concludes the preventive measures should be implemented before the number of infected people increases (preceding) and the treatment measures should be implemented according to the number of infected people (coinciding). The modified model can be seen as a persuasive model considering the compatibility between epidemic control and economic activity.

Keywords resource allocation · optimal policy · coronavirus · COVID-19 · SIR model · infectious diseases
JEL Classification Codes: C54 · C63 · I18

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1 Introduction

The coronavirus disease (later named COVID-19, COVID stands for COrona VIrus Disease), which occurred in Wuhan, Hubei Province, China at the end of 2019, has caused a global turmoil as of March 2020, and plagues many people worldwide. On March 12, the World Health Organization (WHO) declared a pandemic. As a result of the disinfection, sterilization, social distancing, quarantine, curfew, lockdown, etc. in each country, fortunately as of the end of March, the number of new infections has declined in some countries and as a result of the dedicated efforts of medical personnel, many affected people have recovered.

The epidemics of infectious diseases have had a huge impact on human history. The worst pandemics of all time are the Spanish flu and the Plague (the Black Death). The Spanish flu was an unusually deadly influenza pandemic that occurred in 1918. The Spanish flu affected 600 million people worldwide and the death toll is estimated at 20 to 100 million, making it one of the deadliest pandemics in human history. The plague occurred in Europe in 1346 and killed 20 to 30 million people in Europe. Other recent outbreaks and pandemics include SARS (severe acute respiratory syndrome) (2002-2003), swine influenza (2009-2010), MERS (middle east respiratory syndrome) (2012), COVID-19 (2019-2020). Hempei (2018) and WHO (2018) provided details of the survey on infectious diseases. Bourrier, et al. (2019) provided a collection of analytical studies about the managing epidemics through the global health governance. Looking at the epidemics of infectious diseases that occur occasionally in various parts of the world, we can find that the consequences of epidemics are terrible not only in terms of individual suffering, but also in terms of their social and economic effects.

The epidemic of infectious diseases is not only a medical and epidemiological problem, but also a social phenomenon that is highly dependent on how people communicate, how they interact, how they live, what demographic structure they have, etc.[1] Because there are various types of actual human behavior and virus characteristics (e.g. airborne, waterborne, insects and animals, human to human, etc.), it is difficult to deal with the spread process of infectious diseases only by experiment and observation in laboratory, so using mathematical models has been very effective to understand the process of transmission of infection. Mathematical studies on epidemics of infectious diseases have been conducted for a long time and a mathematical modelling of infectious diseases was founded about 100 years ago by Kermack and McKandrick (1927). The Kermack and McKandrick's infectious disease epidemic model is also known as SIR model (or Susceptible-Infected-Recovered model). The SIR model using differential equations is the basic model of infectious disease epidemic model in mathematical epidemiology.

Epidemiology is a field of medical science that studies the incidence, distribution, determinants of infectious diseases, etc. Epidemiologists who are interested in the public health condition may not be interested in economic activity. On the other hand, many economists naturally have little medical or epidemiological expertise. By watching television, we can find that politicians and economists put a

[1] The coronavirus is closely related to the demographic structure. Italy has the highest number of dead people in Europe with more than 10,000 cases followed by Spain. The average life expectancy of each of those countries is about 83 years old, which is one of the highest in the world. It has been known that the coronavirus infections are more likely to be more severe in elderly people and those with a history of diseases.

higher value on the economic activity while medical experts put a higher value on prevention of epidemics. They are at odds with each other over the coronavirus. We must consider both the economic activity and the spread of epidemic at the same time.

In this paper, we combined an infectious disease epidemic model and an economic model in order to solve the disagreement between the politicians' and economists' views and medical experts' views. Until now, there are no studies which have introduced the SIR model into the economic model although there are few papers that studied COVID-19 from an economic point of view, for example, Karnon (2020) which analyzed better or worse outcome between a gradual steps policy and an immediate lockdown policy using a decision tree. We grafted the SIR model into the economic model in order to reconcile the economic activity and epidemic control. This is one of the humble contributions of this research. Fortunately, economics deals with lots of differential equations models, so the method of analysis of the SIR model is not an entirely different matter and field. There is much common ground between both. We modified the SIR model and provided an optimal resources allocation policy for the infectious diseases based on the SIR model. We considered that we can control the infection rate and recovery rate through our efforts which are parameters in the basic SIR model. For example, disinfection, wearing a mask, restricting movement, lockdown, remote work from home, etc. are thought to be able to reduce the infection rate and are already in the process of being implemented, and increasing the number of hospital and medical personnel, developing new drugs and vaccines, etc. are also thought to be able to increase the recovery rate.

Countermeasures against epidemics may consist of two things which are preventive measures for the spread of the infectious disease in the first place and treatment measures after infection. In this paper, we considered how to optimally allocate social resources to the economic activity, the preventive measures against the infectious disease and curing infectious diseases. Of course, there is nothing more important than human life, then the optimal allocation would be to allocate all social resources to the prevention and treatment for infectious diseases in order to save lives. However, we cannot allocate all social resources to the prevention and treatment of epidemics and the production level to be 0. The outcomes from lockdown, quarantine, curfew, etc. due to the coronavirus, have had a serious impact on economic activity. In fact, some countries have severely restricted economic activities, as a result, they have even voiced the complaint that people would die from the lack of food because people had no income even before the coronavirus. Since March 13, 2020, the Philippines has prohibited the use of public transportation to reduce the movement of people. On March 16, at Ortigas Manila, there was a demonstration by taxi drivers to guarantee their work. On April 15, demonstrators in Michigan, North Carolina, Ohio, etc. called for relaxation of restrictions on lockdown and resuming of the economy.

In this paper, we assumed that because life is invaluable and priceless, people never die by the infectious diseases even though it may be possible to assign value to human life in the insurance field. We calculated the optimal resources allocation and the timing for the infectious diseases. The results we obtained are summarized below.

1. The preventive measures for the spread of the infectious diseases should be implemented before the number of infected people increases. (preceding)
2. The treatment measures should be implemented according to the number of infected people. (coinciding)

Depending on the country, there are considerable differences in coronavirus measures. While some countries have strict restrictions like lockdown, others take a gradual step policy like self-quarantine. Of course, the timing of the outbreak and the speed at which the infection spreads are different. However, our conclusion which is the preventive measures and treatment measures should precede and coincide with the number of infected people, respectively, would be a necessary rule for making management policies on infectious diseases in all countries.

This paper is organized as follows: Section 2 outlines the basic SIR model. Section 3 modifies the basic SIR model, solves it, and interprets the results of the analysis. Section 4 offers conclusions on this research. Finally, the detailed calculation can be found in the Appendix.

2 The basic SIR Model

In this section, we outline the SIR model proposed by Kermack and McKandrick (1927), which is the basis of the mathematical model for predicting infectious diseases. The SIR model is named after the variables of the model: S for the number of susceptible, I for the number of infectious, and R for the number of recovered or deceased (or immune) individuals. The SIR model is well known in the field of epidemiology, but not so well known in the field of economics. The SIR model is a model that describes the epidemic process of infectious diseases using three linear differential equations. Even though we made some arrangements for the purpose of this study, Section 2 has no authors' contributions.

The basic SIR Model assumes that no one dies from the infectious diseases and that the population is also isolated and stable because the epidemics of infectious diseases are rapid and short-term. Although, in reality, the infected person may die, the population is assumed to be constant as Eq. (1).[2]

$$\bar{N} = S(t) + I(t) + R(t) \tag{1}$$

where $S(t)$, $I(t)$, $R(t)$ and $\bar{N}$ are the stock of susceptible population, the stock of infected population, the stock of recovered population, and the sum of these three which is the number of total population, respectively.

The basic SIR system can be expressed by the following set of differential equations:

$$\begin{cases} \dfrac{d}{dt}S(t) = -\beta S(t)I(t) & \text{(2a)} \\ \dfrac{d}{dt}I(t) = \beta S(t)I(t) - \gamma I(t) & \text{(2b)} \\ \dfrac{d}{dt}R(t) = rI(t) & \text{(2c)} \end{cases}$$

[2] As extended version of basic SIR model, there are models that introduce birth rate, death rate, population age structure, etc. in the model, e.g. Fredrickson (1971), Tudor (1985), Greenhalgh (1988), etc.

where β and γ are the infection rate (or contract rate) parameter and the recovery rate parameter, respectively. Eq. (2a) describes how the susceptible population changes over time. Eq. (2b) describes how the infected population changes over time. Eq. (2b) has two terms. The first term is the same as the right-hand side of Eq. (2a), except the sign is positive. It represents the increase in infected population. The second term is negative because it represents the decrease of infected cases. Eq. (2c) describes how the recovered population changes over time. The right-hand side is the same as the last term of Eq. (2b), except the sign is positive. It represents the transition of infected cases to recovered cases.

Solving the ordinary differential equation system, we can get the following solutions:

$$\begin{cases} R(t) = \dfrac{(\frac{\gamma}{\beta})^2}{S(0)} \left(\dfrac{S(0)}{\frac{\gamma}{\beta}} - 1 + \alpha \tanh\left(\dfrac{\alpha\gamma t}{2} - \phi \right) \right) & \text{(3a)} \\ S(t) = S(0) \exp\left(-\dfrac{R(t)}{\frac{\gamma}{\beta}} \right) & \text{(3b)} \\ I(t) = \bar{N} - S(t) + \dfrac{\gamma}{\beta} \log\left(\dfrac{S(t)}{S(0)} \right) & \text{(3c)} \end{cases}$$

where $\alpha = \left(\left(\frac{S(0)}{\frac{\gamma}{\beta}} - 1 \right)^2 + \frac{2S(0)\left(\bar{N} - S(0)\right)}{(\frac{\gamma}{\beta})^2} \right)^{\frac{1}{2}}$, $\phi = \frac{1}{\alpha} \tanh^{-1} \left(\frac{S(0)}{\frac{\gamma}{\beta}} - 1 \right)$. $R(t)$ in Eq. (3a) is function of t only. Substituting Eq. (3a) into Eq. (3b), we obtain $S(t)$ which is function of t only. Moreover, substituting $S(t)$ into Eq. (3c), we obtain $I(t)$ which is also function of t only. Then all of $R(t)$, $S(t)$ and $I(t)$ are expressed by t only. See Appendix for the detailed calculation. Section 2 is just an overview of the basic SIR Model so there is no originality of the authors. In the next section, we will introduce an economic optimization model into the basic SIR model.

3 Optimal resources allocation

Most of the studies in the field of epidemics have used SIR models to estimate the diffusion process and parameters of infectious diseases. We consider the optimal resources allocation for epidemics control by adding economic variables to the basic SIR model which is overviewed in the previous section.

3.1 Modified model

Let us explain the economic variables used in the modified model – production function, objective function, constraint for resource allocation, etc. First, we considered the production function of goods using one of Cobb-Douglas production functions as follows:

$$Y(t) = zX_Y(t)^{\theta} L(t)^{1-\theta} \tag{4}$$

where Y represents output in the production of goods, $X_Y(t)$ represents the resource used in the production of goods and $L(t)$ represents the amount of agents working at the sector of goods production. z and θ are constants. We assume that agents do not die by infectious diseases and that the number of population is constant ($\bar{N}$) as the basic SIR model. Those who can work in the goods production sector are the sum of those who have not been infected and those who have recovered. In other words, the infected person cannot work.

$$L(t) = S(t) + R(t) = \bar{N} - I(t). \tag{5}$$

Next, we consider the object function as follows:

$$\max \int_{t=0}^{T} e^{-\rho t} \log Y(t) dt \tag{6}$$

where ρ is discount rate. Eq. (6) means maximizing the sum of the present value of the goods produced. This is a finite-period maximization problem. In economics, a life-long utility function, which is the sum of the present value of contemporary utility consisting of consumption (c) and leisure (l), is often used as the objective function as Eq. (7).

$$\int_{t=0}^{\infty} e^{-\rho t} u\left(c(t), l(t)\right) dt \tag{7}$$

In order to evaluate the utility level, there are some things to be considered in addition to consumption and leisure. Karnon (2020) considers three outcomes – the number of COVID-19 cases, the effects on the economy and the effects of isolation on the well-being of the population – in order to make the decision to respond to the COVID-19 pandemic. In addition to these, the fear of infection, the pain when infected, the inconvenience due to restricted movement, etc. can be considered as factors that affect the utility level. However, these factors have not yet been considered in standard economics. The effects of these factors on the utility level are not clear. Therefore, in this paper, we omit the effects of these factors on the utility level without hesitation and suppose that the utility function relates only to the amount of production.

Next, we consider the constraint of social resources. In the economy, a certain amount of social resources ($\bar{X}$) is given each period, and the resources are used for the goods production, for the prevention of the spread of infectious diseases, and for the treatment of infectious diseases. We assume that there is no resource accumulation as the capital accumulation in standard economics. The constraint of social resource is Eq. (8).

$$\bar{X} = X_Y(t) + X_\beta(t) + X_\gamma(t) \tag{8}$$

where, $X_\beta(t)$ is an amount of resource which is used to prevent the spread of infectious diseases. $X_\beta(t)$ includes not only the resources which are directly used for disinfection and inspection for prevention but also the unused social overhead capital (SOC). Considering the opportunity cost, the unused SOC for production activities such as blocking roads due to lockdown, stopping public transportation, etc. can be interpreted as using SOC in order to prevent the spread of infectious diseases. And, $X_\gamma(t)$ is an amount of resource which is used to treat infectious diseases, for example, vaccine development, increase the number of hospital and medical personnel, etc.

The basic SIR model considers as constant the infection rate (β) and the recovery rate (γ). However, we consider these as control variables. We assume that β

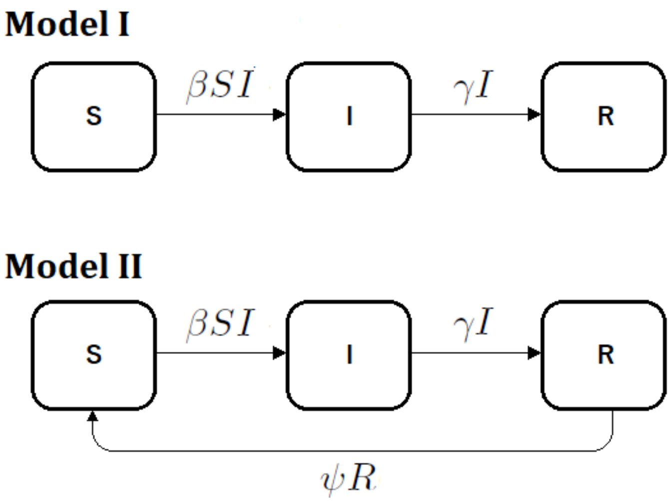

Fig. 1 Conceptual diagrams

and γ are functions of X_β and X_γ, respectively and that $\frac{\partial\beta(X_\beta)}{\partial X_\beta} < 0$, $\frac{\partial^2\beta(X_\beta)}{\partial X_\beta^2} > 0$, $\frac{\partial\gamma(X_\gamma)}{\partial X_\gamma} > 0$ and $\frac{\partial^2\gamma(X_\gamma)}{\partial X_\gamma^2} < 0$. As extension models of the basic SIR model, there are previous studies in which β is based on age structure (e.g. Tudor (1985), Greenhalgh (1988), Inaba (1990), etc.), however the β is a state variable and not a control variable in their models.

In this paper, we consider two types of models. In Model I, we assume that infected people recover from the infection at a constant rate γ, and will not be infected again after recovery because they develop an immunity to it. For example, the epidemics typically seen in children (e.g. measles, chicken pox, mumps, etc.) are once acquired and recovered, they obtain lifelong immunity and never get it again. In model II, we assume that infected people recover from the infection at a constant rate γ, and can be infected again after recovery at a constant rate ψ. The World Health Organization on 23 April 2020 said there was not enough evidence that a person who has recovered from COVID-19 is immune from a second infection. Fig. 1 depicts the images. Eq. (9) and Eq. (10) show the sets of differential equations of Model I and Model II, respectively.

1. Model I

$$\begin{cases} \frac{d}{dt}S(t) = -\beta\left(X_\beta(t)\right)S(t)I(t) & \text{(9a)} \\ \frac{d}{dt}I(t) = \beta\left(X_\beta(t)\right)S(t)I(t) - \gamma\left(X_\gamma(t)\right)I(t) & \text{(9b)} \\ \frac{d}{dt}R(t) = \gamma\left(X_\gamma(t)\right)I(t) & \text{(9c)} \end{cases}$$

2. Model II

$$\begin{cases} \dfrac{d}{dt}S(t) = -\beta\left(X_\beta(t)\right)S(t)I(t) + \psi R(t) & \text{(10a)} \\ \dfrac{d}{dt}I(t) = \beta\left(X_\beta(t)\right)S(t)I(t) - \gamma\left(X_\gamma(t)\right)I(t) & \text{(10b)} \\ \dfrac{d}{dt}R(t) = \gamma\left(X_\gamma(t)\right)I(t) - \psi R(t) & \text{(10c)} \end{cases}$$

Next, we assume a relationship between X_β and β as Eq. (11) and a relationship between X_γ and γ as Eq. (12).

$$\beta(t) = 1 - X_\beta(t)^{\eta_1},\ 0 < \eta_1 < 1,\ 0 < \beta(t) < 1 \tag{11}$$

$$\gamma(t) = X_\gamma(t)^{\eta_2},\ 0 < \eta_2 < 1,\ 0 < \gamma(t) < 1 \tag{12}$$

where η_1 and η_2 are constants.

It is difficult to solve the modified models – Model I and Model II – analytically. An alternative option is to provide the solutions numerically. We will carry out a numerical analysis. The parameter values for numerical calculation are set as follows, even though they are arbitrary: $z = 1$, $\rho = 0.111$, $\theta = \frac{1}{3}$, $\eta_1 = 0.5$, $\eta_2 = 0.5$, $\psi = 0.1$, $\bar{X} = 1$, $\bar{N} = 1$ and $T = 100$. We put the initial conditions of $S(0)$, $I(0)$ and $R(0)$ as 0.999, 0.001 and 0, respectively. To solve the system of nonlinear equations, we used Newton's method.

3.2 Results

3.2.1 Model I

Model I is a model in which people can be immunized once and never affected twice. We calculate the optimal problem from 0 period to 100 period and plot the results only from 0 period to 50 period in Fig. 2. In Fig. 2 (1), the blue line represents the susceptible population (S) over time, red line represents infected (I), and green line represents recovered (R). The red vertical line shows when I is the maximum (period 13). Fig. 2 (2) shows β, γ and I. The green vertical line represents when β is the minimum (period 7), the blue vertical line represents when γ is the maximum (period 13), and the red vertical line represents when I is the maximum (period 13). Because in Fig. 2 (2), the blue vertical line and the red vertical line are located at the same place, the blue vertical line is overlapped on the red vertical line so we can see only the blue vertical line. Comparing the place of the minimum of β and the place of the maximum of I, we can find that the bottom of β is located at the left of the peak of I.

To investigate the relationship between I and other variables, we calculate the time correction coefficients. Fig. 2 (3) shows the time correction coefficients between I and both β and γ. Fig. 2 (4) shows the time correction coefficients between I and all of X_Y, X_β, X_γ. The time correction coefficients are shown in Table 1. Fig. 2 (3) and (4) show the time correction coefficients from the -10th period to the 10th period, but Table 1 shows the time correction coefficients from the -7th period to the 7th period because of space constraints. The correlation coefficient between $I(t+6)$ and $\beta(t)$ is -0.93. It turns out that β precedes I.

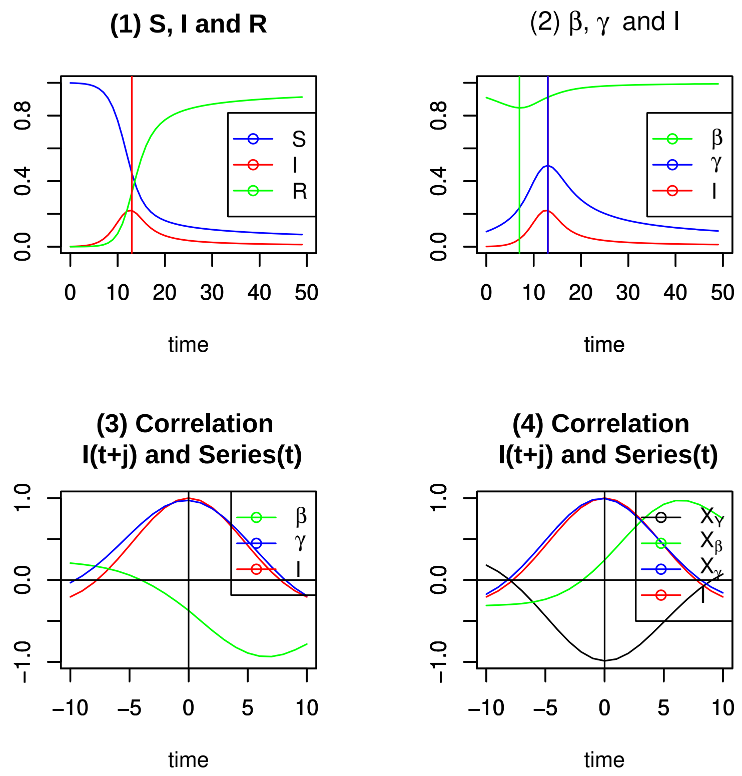

Fig. 2 Results of Model I

Table 1 Correlation ($I(t+j)$ and Series(t)): Model I

Variables	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7
I	0.10	0.25	0.42	0.59	0.75	0.88	0.97	1.00	0.97	0.88	0.75	0.59	0.42	0.25	0.10
β	0.14	0.11	0.06	0.00	-0.08	-0.16	-0.26	-0.37	-0.49	-0.62	-0.73	-0.82	-0.89	-0.93	-0.93
γ	0.28	0.41	0.55	0.69	0.81	0.90	0.96	0.97	0.94	0.87	0.76	0.63	0.48	0.32	0.16
X_Y	-0.13	-0.28	-0.44	-0.60	-0.75	-0.88	-0.96	-0.99	-0.96	-0.90	-0.79	-0.66	-0.51	-0.37	-0.23
X_β	-0.29	-0.27	-0.24	-0.18	-0.11	-0.02	0.10	0.24	0.40	0.56	0.71	0.83	0.92	0.97	0.97
X_γ	0.16	0.32	0.48	0.65	0.80	0.91	0.98	0.99	0.95	0.86	0.74	0.59	0.43	0.27	0.13

On the other hand, the correlation coefficient between $I(t)$ and $\gamma(t)$ is 0.97. It turns out that γ accompanies I. These results can also be confirmed from X_β and X_γ. The correlation coefficient between $I(t)$ and $X_\beta(t)$ is 0.24, but the correlation coefficient between $I(t+6)$ and $X_\beta(t)$ is 0.97. This means that $X_\beta(t)$ precedes rather than coincides with $I(t)$. This result suggests that to prevent the spread of

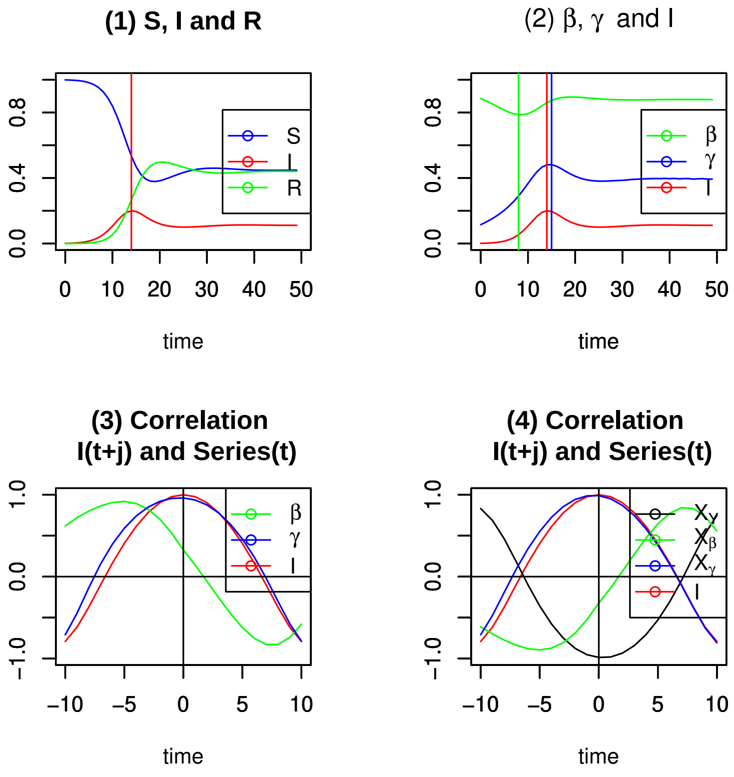

Fig. 3 Results of Model II

infectious diseases, the resource allocation (X_β) should be best preceded by the trend of the number of infected people (I).

On the other hand, the correlation coefficient between $I(t)$ and $X_\gamma(t)$ is 0.99. This result suggests that the resource allocation for recovery (X_γ) is best done along with the trend of the number of infected people (I).[3] These two findings obtained from the optimal problem seem quite in accordance with our intuition enough for saying a natural phenomenon. The modified model can be seen as a persuasive model.

3.2.2 Model II

Model II is a model in which people may be infected again even if people were infected once. The results are shown in Fig. 3 and Table 2. The way to read Fig. 3

[3] In this paper, we assumed that the input of resources for the treatments has effects on the recovery rate at the same period, that is, $\gamma(t)$ is the function of $X_\gamma(t)$. If the effects get delayed such as vaccine development, that is, if $\gamma(t)$ is the function of $X_\gamma(t-\Delta)$, the results obtained from this research may be changed. Analyzing the model in which the effects get delayed remains to be seen in our future research.

is the same with that of Fig. 2. In Fig. 3 (1) the blue line represents the susceptible population (S) over time, red line represents infected (I), and green line represents recovered (R). The red vertical line shows when I is the maximum (period 14). We calculate the basic reproduction number in order to confirm the convergence of the infectious disease, that is, whether the growth of I stops or not. The basic reproduction number $\frac{\beta S}{\gamma} = \frac{0.879 \times 0.448}{0.394} \approx 1$ which satisfies $\frac{d}{dt} I = 0$. Model II converges on the endemic steady state. Fig. 3 (2) shows β, γ and I. The green vertical line represents when β is the minimum (period 8), the blue vertical line represents when γ is the maximum (period 15), and the red vertical line represents when I is maximum (period 14). Comparing the place of the minimum of β and the place of the maximum of I, we can find that the bottom of β is located to the left of the peak of I. On the other hand, comparing the place of the maximum of γ and the place of maximum of I, we can find that both the peak of γ and I are located at almost the same place.

Table 2 Correlation ($I(t+j)$ and Series(t)): Model II

Variables	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7
I	-0.10	0.17	0.41	0.61	0.78	0.90	0.98	1.00	0.98	0.90	0.78	0.61	0.41	0.17	-0.10
β	0.86	0.91	0.92	0.89	0.82	0.70	0.53	0.33	0.14	-0.06	-0.26	-0.45	-0.62	-0.75	-0.83
γ	0.16	0.40	0.59	0.74	0.85	0.92	0.96	0.96	0.94	0.88	0.78	0.64	0.46	0.24	-0.02
X_Y	0.18	-0.10	-0.36	-0.58	-0.75	-0.88	-0.95	-0.99	-0.98	-0.93	-0.84	-0.71	-0.53	-0.31	-0.05
X_β	-0.84	-0.88	-0.89	-0.87	-0.81	-0.69	-0.52	-0.32	-0.14	0.07	0.27	0.47	0.64	0.77	0.84
X_γ	0.09	0.34	0.55	0.73	0.86	0.94	0.99	0.99	0.95	0.87	0.75	0.58	0.38	0.15	-0.11

The correlation coefficient between $I(t+7)$ and $\beta(t)$ is -0.83. It turns out that β precedes I. On the other hand, the correlation coefficient between $I(t)$ and $\gamma(t)$ is 0.96. It turns out that γ accompanies I. These results can also be confirmed from X_β and X_γ. The correlation coefficient between $I(t+7)$ and $X_\beta(t)$ is 0.84. This result suggests that the resource allocation to prevent the spread of infectious diseases (X_β) should be best preceded by the trend of the number of infected people (I). On the other hand, the correlation coefficient between $I(t)$ and $X_\gamma(t)$ is 0.99. This means that $X_\gamma(t)$ coincides with $I(t)$. This result suggests that it is optimal to allocate resources for recovery (X_γ) along with the trend of the number of infected people (I). The results of Model II are almost the same with those of Model I.

4 Conclusions

In recent years, several epidemics of infectious diseases have occurred suddenly, such as the global epidemic of COVID-19, and they not only pose a threat to human health, but also have a great impact on society and economy. In order to overcome the infectious diseases wisely, we considered the optimization of social resources allocation based on the SIR model for the infectious diseases. This research paper may be the first paper to graft the epidemiological model into the economic model. This paper is a welcoming addition to infectious diseases research.

The two major countermeasures against epidemics can be considered as preventive measures and treatment measures. We provided the modified the basic SIR model in which we allocate the limited social resources for the two major countermeasures and the production of goods. We calculated the optimal social resources allocation and the optimal timing of policy implementation. We found that it is best to carry out preventive measures before the increase of the number of infected people (preceding) and to carry out treatment measures with the trend of the number of infected people (coinciding). These results from the optimal problem are quite consistent with our intuition. The modified models can be seen as a persuasive model considering the compatibility between epidemic control and economic activity.

Finally, we would like to thank the medical professionals who are dedicated to the fight against coronavirus, wishing for an end to the COVID-19 pandemic as soon as possible.

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5 Appendix

Let us solve this ordinary differential equation system as mentioned in Section 2. Dividing Eq. (2b) by Eq. (2a), we get Eq. (13).

$$\frac{dI}{dS} = -1 + \frac{\gamma}{\beta S} \tag{13}$$

We integrate Eq. (13) to S, then we get Eq. (14).

$$I = -S + \frac{\gamma}{\beta} \log S + C \tag{14}$$

where C is a constant of integration. C can be obtained from substituting the initial condition which is $S(0) = \bar{N} - I(0)$ (i.e. $R(0) = 0$). We get C as follows: $C = \bar{N} - \frac{\gamma}{\beta} \log S(0)$. Substituting C into Eq. (14), we can get Eq. (15).

$$I(t) = \bar{N} - S(t) + \frac{\gamma}{\beta} \log \left(\frac{S(t)}{S(0)} \right) \tag{15}$$

Then, from Eq. (15) and Eq. (1), we get Eq. (16).

$$R(t) = -\frac{\gamma}{\beta} \log \left(\frac{S(t)}{S(0)} \right) \tag{16}$$

By re-arranging Eq. (16) into $S(t)$, the following can be obtained:

$$S(t) = S(0) \exp \left(-\frac{R(t)}{\frac{\gamma}{\beta}} \right) \tag{17}$$

From Eq. (2c) and Eq. (1), we can get Eq. (18).

$$\frac{d}{dt} R(t) = \gamma \left(\bar{N} - R(t) - S(t) \right) \tag{18}$$

Substituting Eq. (17) into Eq. (18), we obtain the following Eq. (19).

$$\frac{d}{dt} R(t) = \gamma \left(\bar{N} - R(t) - S(0) \exp \left(-\frac{R(t)}{\frac{\gamma}{\beta}} \right) \right) \tag{19}$$

By expanding the Maclaurin's series, $\exp\left(-\frac{R(t)}{\frac{\gamma}{\beta}}\right)$ can be approximated as follows:

$$\exp \left(-\frac{R(t)}{\frac{\gamma}{\beta}} \right) \approx 1 - \frac{R(t)}{\frac{\gamma}{\beta}} + \frac{1}{2} \left(\frac{R(t)}{\frac{\gamma}{\beta}} \right)^2 \tag{20}$$

Substituting Eq. (20) into Eq. (19), we obtain the following Eq. (21).

$$\frac{d}{dt} R(t) = r \left(\bar{N} - S(0) + \left(\frac{S(0)}{\frac{\gamma}{\beta}} - 1 \right) R(t) + \frac{S(0)}{2 \left(\frac{\gamma}{\beta} \right)^2} R(t)^2 \right) \tag{21}$$

Factoring the right-hand side quadratic in $R(t)$ and using Riccati-Gleichung, we can integrate Eq. (21) as follows:

$$R(t) = \frac{(\frac{\gamma}{\beta})^2}{S(0)} \left(\frac{S(0)}{\frac{\gamma}{\beta}} - 1 + \alpha \tanh \left(\frac{\alpha \gamma t}{2} - \phi \right) \right) \tag{22}$$

Now, $R(t)$ is function of t only. Substituting Eq. (22) into Eq. (17), we obtain $S(t)$ which is function of t only. Moreover, substituting $S(t)$ into Eq. (15), we obtain $I(t)$ which is also function of t only. Then all of $R(t)$, $S(t)$ and $I(t)$ are expressed by t only.