A Mathematical Theory of Economic Growth: The Public Choice Growth Model

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A Mathematical Theory of Economic Growth

The Public Choice Growth Model

By

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Abstract:

The purpose of this paper is to offer a new theoretical framework in the field of development economics. This new theoretical framework has not yet been explored in development economics. Most economic theories seek to predict an outcome. The particularity of this theory that is being proposed in this paper, is not to predict a specific outcome about an economy. It is rather a methodology to explain an economic outcome. This new theory being introduced in the field of development economics is called the Public Choice Growth Model (PCGM), which is an economic theory that combines the principles of public choice theory and that of the Solow Growth Model. The goal of this theory though, is to demonstrate that our model is the adequate model to be used in a developing country in order to determine long-term economic growth.

Keywords: Economic Growth, Economic Development, Public Choice Theory, Economic Analysis, Neoclassical Economics, Economic Policy
Introduction

The theoretical framework that we seek to present in this paper is an economic theory that aims to be used as a tool to determine how economic growth can occur in a developing country or society. This theoretical framework that is being presented in this paper is the Public Choice Growth Model. It is noteworthy to emphasize that our model is specifically designed for economies or markets in developing countries. Our theory is not based on the economic foundations of developed countries because developed countries have already achieved such a level of development that our theory will not have a substantive effect on these economies if we use them as database to develop our framework. Our theory is specifically designed for developing countries because they are currently transitioning from an agricultural society to a more industrialized society due to the use of technology as the basic tool of economic development. What we seek to achieve in presenting this theory is to demonstrate that a developing country such as Côte d’Ivoire, Bangladesh, or Uganda; can achieve a long-term economic growth when it applies the precepts of a liberal economy, which are incorporated in our framework.

The Public Choice Growth Model (PCGM) is an economic tool which we conceived within the neoclassical framework in the field of development economics. This model has been generated from two significant elements in economic theory. One, from microeconomics, which encapsulates the principles of public choice theory; and the other, from macroeconomics, which encompasses the factors of the Solow Growth Model. In combining the two, we sought to determine how developing countries can increase their economic growth on a long-term basis. In a few words, the central argument of our theory is grounded on the fact that an economy prospers on a long-term basis when it maximizes its output. Yet in order to maximize output, the economy as whole or an economic sector ought to be substantially deregulated. It is preponderant to emphasize on the fact that our model is not an exhaustive theory. It evidently needs further development and contribution. We hope that other scholars will expand upon our framework to either validate or challenge our theory. Our principal goal though is, to at least, establish the theoretical foundations for our theory to make a substantial impact in the field of development economics. An impact that was not previously made. Our framework is interested in a set of fundamental ideas that will enhance development economics as a field of mainstream economics.

We are going to divide our analysis into three substantive parts. First, we shall endeavor to highlight the reasons for which we have chosen public choice as the fundamental tool for our model. The second part of our analysis will consist of analyzing the theoretical framework of our model, which is to explain the precise methodology to ascertain our theory. In elaborating our framework, we shall endeavor to elucidate the methodology of our theory in plain English then to summarize our argument mathematically. The third and last part of our analysis will consist of testing our theory with empirical evidence in order to validate the substance of our framework.
Part I

What is Public Choice Theory?

Public-choice theory is a branch of economics in which the theories and methods of economics are utilized to the analysis of political behavior, an area that was once the exclusive province of political scientists and sociologists.\(^1\) Political behavior is an extremely important factor in our analysis because it determines what the role of the government should be in economic affairs. The central analysis of public choice theory is in its reasoning on cost and benefits for all individuals whether they are in the private or public sector.\(^2\) James Buchanan, one of the founders of public choice theory, asseverated that public choice theory is “politics without romance”;\(^3\) which means that voters and politicians are both selfish and each pursue his own self-interests. Voters are self-interested. They vote because they expect politicians to make decisions based on their needs. On the other hand, politicians want to get elected because they want to have political power. Individuals in government, like those in the private economy, select options that represent the best set of cost and benefits.\(^4\) Their decisions are, in this sense, self-interested.\(^5\) Public choice theory holds that individual behavior within the political system is motivated by incentives similar to those motivating behavior in the private sector.\(^6\) Monetary rewards, to be true, play an important role in the decision of public officials; many of them do get rich by holding political office.\(^7\) Elected officials, like other people, also consider many other incentives, including family security, recognition, travel, access to information, and satisfaction derived from performing community service.\(^8\) Similarly, bureaucrats consider incentives such as expanding budgets for favored projects, gaining promotion, obtaining more and more high skilled stuff, and expanding influence with decisionmakers.\(^9\) In a few words, politicians and bureaucrats are ordinary individuals like those in the private sector who are making decisions based on what will benefit them. The great point about public choice theory is that its analysis allows us to predict how politicians, voters, and bureaucrats will behave when making a decision.

In our analysis though, our primary motive is not concerned with voters’ behavior, but we are concerned with the way in which the political process affects economic outcomes. We assume that voter’s behavior is already part of the concept of public choice theory, so we therefore do not need to give too much emphasis upon it. The public choice theory argues that an economy thrives if the role of the state is significantly minimized. Evidently, the public choice theory does not exclude the government at all in playing a role in the economy. Yet we believe that the role of the government should be minimized to its initial scope if we want to ensure that output is maximized.

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2. Shughart, Ibid.
3. Shughart, Ibid.
5. Schug & Fontanini, Ibid.
7. Schug & Fontanini, Ibid.
8. Schug & Fontanini, Ibid.
9. Schug & Fontanini, Ibid.
for economic growth to take place. In an economy, the government has two essential roles to play in order to ensure that the market allocates resources efficiently. The first role is to ensure that the government establishes a legal framework in which the actors of the market will access the tools in order to create the wealth. One of these tools is to implement the infrastructures that will facilitate the creation of wealth in a market economy. The second role of the government is to control the quantity of money supply in order to maintain inflation at its nominal rate, which is between 2 and 3 percent of the general price increase. Besides these two substantive roles, the government has no role to play in supplying goods and services to the citizenry. Evidently, there are exceptions. In our analysis based upon the public choice theory, we recognize that public goods such as national defense, the courts, the police and infrastructures; must be provided by the government. These are public goods that individuals cannot provide for themselves even if an economy was entirely deregulated. These public goods are incorporated into the government expenditures index.\(^\text{10}\) Under public choice theory, we assume that individuals are maximizing their utility as firms maximize profits. We argue that it is the maximization of utility and profit that generates economic output. The principle of our theory is embedded in the precept that the maximization of output leads to a sustainable economic growth.

The public choice theory argues that rent-seeking\(^\text{11}\) is an impediment to stimulate economic growth. Rent-seeking is the tool that the actors of the political process utilize to control the allocation of resources.\(^\text{12}\) Under the control of the state, the allocation of resources in a given industry is misallocated because those who have planned the economy did not take into account the laws of supply and demand when allocating these resources.\(^\text{13}\) Therefore, output is not maximized because the industry being subjected to rent-seeking is subsidized by government. Subsidization is a form of economic monopoly that directly impacts economic output. When an industry or market is subsidized, it necessarily decreases output over time because it weakens competition, efficiency and productivity.\(^\text{14}\) Our analysis aligns with the rationale of the public choice theory as to why economic growth occurs more steadfastly in a deregulated market rather than a regulated market.

\(^{10}\) *Government Spending* 2019 Index of Economic Freedom.

\(^{11}\) Rent-seeking is a theory developed by Gordon Tullock, who was one of the founders of public choice theory. Rent-seeking means seeking to increase one’s share of existing wealth without creating new wealth. Rent Seeking results in reduced economic efficiency through misallocation of resources, reduce wealth-creation, lost government revenue, and potential national decline. Source: Econlib.org by David R. Henderson.


In this part of our framework, the main objective is to incorporate the principles of public choice theory into the Solow Growth Model in order to determine the economic development of a developing society. Before we use the tools of public choice theory to into the Solow Growth Model, it is important to assess the Solow Growth model itself.

**A) The Basic Concept of the Solow Growth Model**

The Solow Growth Model is an economic tool in macroeconomics developed by Nobel Laureate economist, Robert Solow. It is an exogenous model that analyzes changes in the level of output in an economy over time as a result of changes in the population.\(^1\)\(^5\) The Solow Growth Model was inherently constructed within a neoclassical framework. That is one of the reasons why this model has been chosen as one of the principal tools to determine the economic development of regional place like sub-Saharan Africa, Southeast Asia or Central and South America. It is noteworthy to accentuate that the combination that is attempted in this analysis between the public choice tools and the Solow Growth Model, is not a combination of the very specific tools of public choice theory and the Solow Growth Model, but it is a combination of the general concept of public choice theory and Solow Growth Model. When Robert Solow designed his model, he did not give much emphasis on whether the economy is regulated or deregulated. He only designed his model based upon a market economy regardless of the level of regulation being implemented. In our analysis, we are going to compare the Public Choice Growth Model within a regulated market economy, and within a deregulated market economy. But beforehand, let’s first analyze the Solow Growth Model. The Solow Growth Model, from its inception, is based upon five factors or variables, which are the aggregate function product known as \((Y)\); capital accumulation or capital stock known as \((K)\); labor known as \((L)\), and technology or knowledge known as \((A)\); and time which is known as \((t)\). It follows then by this formal linear equation:

\[
Y(t) = K(t) \left[ A(t), L(t) \right]
\]

The rationale of that equation gravitates around the accumulation of physical capital. The accumulation of physical capital is based upon two pieces of information which are the saving function, and the equilibrium condition.\(^1\)\(^6\) The saving function principally deals with the question of “how much of output do people (population growth) in the modern economy save?”\(^1\)\(^7\) The assumption to the question is that people save a given fraction of output.\(^1\)\(^8\) The saving function


\(^{17}\) Ibid. p. 69

\(^{18}\) Ibid. p. 69.
could be considered also as the investment function known as $f(I)=s(k)$.\textsuperscript{19} The equilibrium condition is the most critical part of the Solow Growth Model. Robert Solow argued that if the economic growth of a country is solely based upon the accumulation of capital, logically this economy will reach the stage of the steady-state; which means that the savings accumulated are only sufficient to replace the depreciate capital stock.\textsuperscript{20} The depreciation of capital stock function is known as $f(d) = \partial(k)$. As capital accumulation increases, so is the depreciation of capital stock because the portion of the capital stock being used to create output must be replaced for that capital stock to remain sustainable.\textsuperscript{21} Yet, at some point, to constantly replacing the part of the capital stock that was used will reach a point of nullification where the depreciated capital stock will outpace the investment function.\textsuperscript{22} Let’s illustrate this argument into a concrete example issued by the Department of Economics at the University of Pittsburgh.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
Period & Capital & Output & Savings & Change in output \\
\hline
1     & 100    & 1000   & 250     & ---     \\
\hline
2     & 250    & 1581   & 395.3   & 581     \\
\hline
3     & 395.7  & 1988   & 497     & 407     \\
\hline
4     & 500    & 2229   & 557     & 241     \\
\hline
5     & 600    & 2360   & 590     & 131     \\
\hline
6     & 700    & 2429   & 607     & 69      \\
\hline
7     & 800    & 2464   & 616     & 35      \\
\hline
8     & 900    & 2500   & 600     & 26      \\
\hline
\end{tabular}
\caption{Source: University of Pittsburgh Department of Economics}
\end{table}

\textsuperscript{19} Ibid. p.69 \\
\textsuperscript{20} Ibid. p.70. \\
\textsuperscript{21} Ibid. p.70 \\
\textsuperscript{22} Ibid. p. 71
Figure 2 shows that over a period of time, the investment or saving curve known as $f(I)$ or $f(s)$ (depends on personal preference), and the depreciation curve also known as $f(\vartheta)$; intersect. This intersection marks the equilibrium condition of the steady-state in which depreciated capital stock nullifies the investment made in capital stock. It is important to fathom that the steady-state is the point that expresses that the accumulation of physical capital, as means of economic growth, has reached its limits and can no longer produce the output expected. Therefore, the function of technological progress known as $f(A)$ is introduced. In this twenty-first century, economic growth through physical capital accumulation is clearly outmoded. The utilization of technology has become the principal tool by which economic development occurs in developed and developing countries nowadays.

**B) Methodology of the Public Choice Growth Model**

In our framework, the model that we seek to introduce is the Public Choice Growth Model. As it was aforementioned, we seek to combine the principles of public choice and the elements of the Solow Growth Model in a single equation. The economic principles of public choice are embedded in a deregulated market whereby the government plays a very minimal role in the economy. It suggests that capital stock is unregulated. If capital stock is unregulated, therefore, we could have the following equation:

$$Y(t) = K(t) [A(t), L(t) - B(t), \pi(t)]$$
If we break our equation function by function without including the time variable, it comes to the following mathematical operation:

\[
f(Y) = f(K) \{[f(A) \times f(L)] - [f(B) \times f(\pi)]\}
\]

In this equation, in addition to having the variables of the Solow Growth Model, we have extended it by incorporating the variables of government-borrowing and inflation rate, which have been negated. The function \(f(B)\) represents government borrowing money from the central bank; and function \(f(\pi)\) represents the rate of inflation.

The rationale behind this equation is that \(f(B)\) and \(f(\pi)\) have been added primarily because in a regulated market economy, government borrowing increases over time due to state expenditures. Except for the military, the police and the courts, which seem a necessity to remain under state control, most government-programs are public goods that could be provided by the market more efficiently. Since the market can provide the same public goods that the government provides, except for the military, the judiciary, the police and some infrastructures; we have then negated the borrowing function as well the inflation function because, since both evolve simultaneously, their negation determines the extent to which the market could be deregulated. Evidently, the more government borrows money from central bank, the more the central bank increases the money supply, and therefore increases the rate of inflation. The more the money supply and inflation increase, the more it reduces economic output and efficiency and consequently impedes economic growth because the government imposes subsidies on programs, and these subsidies hamper competitiveness, innovation, and efficiency. A deregulated market is a system wherein the market mechanisms operate efficiently and to their full potential in order to deliver significant output. As technology has increased efficiency and reduced the physical effort of human labor, if the state regulates the use of technology as a tool to promulgate economic growth by borrowing from the central bank in order to impose more subsidization upon it, this regulation will simply lead to inefficiency and a decrease in aggregate output. The negation of \(f(B)\) and \(f(\pi)\) in our equation is then an indispensable factor in determining economic growth within a deregulated market economy. The central point of the rationale of this model is that the lack government borrowing constrains the ability of the state to regulate and plan the economy.

It is noteworthy to reiterate that our model is only an economic tool that could be used efficiently in a deregulated market economy. It would be judicious to empirically demonstrate the use of the Public Choice Growth Model in order to understand its mechanism. Let’s use a hypothetical scenario to demonstrate the mechanism of the Public Choice Growth Model. Let’s assume that the Ivorian government has decided to entirely deregulate its economy except for the military, the police, and the courts on a seven-year period. The general public goods that we know such as education, healthcare, public transportation, local and regional banks, and all other factors of the domestic market are totally deregulated, and a flat tax system is imposed as the main tax system. In other words, the state has decided to let the economy being managed by entrepreneurs and private investors instead of bureaucrats. The goal of this deregulation is to see if the lack of government intervention would promote economic growth faster. Let’s use the agricultural sector and let’s assume that it is totally deregulated, and let’s assume that the Ivorian government does not regulate any factor of production in this sector. All the factors of production are privately owned.
Compared to figure 2, which showed the Solow Growth under the framework of physical capital accumulation to determine economic output, in the Public Choice Growth Model though, output is significantly higher than in figure 2. The reason why output is much higher in figure 3 than in figure 2 is because we have replaced physical capital by technological capital. The substantial difference between figures 2 and 3 epitomizes the evolution of time. Indeed, as time evolved, technological progress also improved. Consequently, this improvement in technological progress led to an efficient allocation of resources and efficient economic output within the sector. Furthermore, as output increased, government-borrowing, and inflation rate decreased simultaneously. What we can deduce is that inflation rate and government-borrowing decrease in a deregulated market economy as long as the total factors of production (TFP) are not under government control. The public choice theory growth model could be represented as the following:
In this model, since technological capital is the main tool which has replaced the accumulation of physical capital, the curves \( f(s) \) and \( f(A) \) increase in parallel but never intersect because the economy does not reach the stage of steady-state since the depreciation factor is no longer relevant. The depreciation factor plays a role in an economy whereby the accumulation of physical capital is the primary source of economic growth. It is not the case in our model. Let us illustrate this argument by a concrete anecdotal example to make a parallel between our model and the Solow Growth Model. Let’s say an individual owns an iPhone 7 for more than two years and he has been using it for work purposes. After two years using the phone for various work activities, the battery of the phone began to decline steadily. In the time span of twenty minutes, for example, the battery of the phone can decrease from 92 percent to 31 percent. Nevertheless, the current phone on the market is the iPhone 11. The iPhone 7 is almost outmoded. With a deficient battery, the individual needs to purchase a more efficient phone in order to complete the assignments of his job. Under our model, we assume that since the individual seeks to maximize his utility, instead of using his savings to invest in a new battery to replace the old battery of the iPhone 7 that he is using, the individual will simply purchase a brand-new phone that is more efficient, faster, and with more functionalities than the phone he currently owns. Under the Solow Growth Model though, the individual would have had used his savings to purchase a new battery to replace the old one instead of purchasing a brand-new phone. Yet he will still be using the same phone although it is becoming rotten. As result, since the phone is his physical capital, the individual would have kept investing in a new battery every time in order to keep his phone performing. But such initiative would have not been rational in terms of maximizing his utility because, by spending time and resources fixing the issues of his phone, the individual would have lost a considerable amount of time in getting his work done. Therefore, there would have been a real loss of efficiency and productivity in producing output.
Our model is a post-steady-state model. It suggests that the savings will not be used to replace the capital stock being used but it will be used to invest in new technological tools to add upon the capital stock in order to increase output. It means that if the capital stock runs out of resources, the savings will be used to purchase a new capital stock rather than spending those savings on replacing that capital stock being used and which became rotten over time due to its excessive utilization. The output curve $f(Y)$ in our model is a straight line. It implies that output is being produced efficiently and at a higher speed. Based on the laws of supply and demand, entrepreneurs and private investors can efficiently coordinate the quantity of supply that needs to be produced, they can determine the method of production, and they can determine the timeframe necessary to increase output in a given period of time. In short, the use of technology in our model speeds up the process of production which logically increases output in a greater quantity over a long-time period. The curves $f(B)$ and $f(\pi)$ decrease as output increases. As it was expounded, as the market is deregulated, the government then has no substantive role to play in stimulating the economy since the means of production are privately controlled and utilized by the mechanism of the market process. Accordingly, government will borrow less money from the central bank over time, and this reduction in money borrowing will commonsensically reduce the rate of inflation.

Now, if we use our same example of the agricultural sector by using our Public Choice Growth Model; but this time in a regulated market economy whereby the agricultural sector is subsidized by the government, inflation will significantly increase because the government will borrow more money from the central bank in order to subsidize the sector. The outcome will be that, even though $f(s)$ and $f(A)$ will still not necessarily intersect in our model due to the post-steady-state stage in which we are in; both functions $f(A)$ and $f(s)$ will eventually intersect with $f(Y)$ because output will necessarily decrease over time. because the subsidization of the program will eliminate the competition, innovation, and efficiency while $f(B)$ and $f(\pi)$ will concurrently increase because government exerts a substantial control over the factors of production. Figure 5 shows how the agricultural sector will grow at a slower pace in our model if the market was regulated.

![Public Choice Growth Model in a regulated market](image)
In a regulated market, the mathematical interpretation of the PCGM could be written as the following equation:

\[ Y(t) = K(t) [A(t), L(t) + B(t), \pi(t)] \]

If we break the equation function by function, we will have the following:

\[ f(Y) = f(K) [f(A) \times f(L)] + [f(B) \times f(\pi)] \]

From analyzing figure 5, we clearly observe that output progressively declines. If economic growth is defined or embedded into the efficient increase of output over time, then our model is definitely inapplicable within a regulated market economy. The regulations imposed by the government over the factors of production do decrease output in the long-run because the government ends up misallocate the resources that should be used to stimulate production. This misallocation is due to the fact that the government spends more than it needs and saves less. The lack of savings impedes new investments in capital. In a regulated market, even with the use of technological capital, output can still decrease if the factors of production are malinvested or maladjusted. Our model here reflects the conventional theory that an economic sector that is regulated and subsidized by a central authority ends up producing low-output goods for consumption, and figure 5 evidently epitomizes that theory within the example of the agricultural sector being subsidized by the Ivorian government that we have assessed.

c) Summarization of the Public Choice Growth Model

Our model could be understood in very simple terms. We have built it from the assumption that individuals are fully rational in their decision-making process. Moreover, we assume that our model is based upon a perfect competitive market economy. In a competitive market economy where the government has an extensively limited role in deciding what to produce, what should be produced, how to produce, and how to allocate what ought to be produced; we assume that resources will be allocated efficiently and output will be maximized because individuals will have the ability to decide what is best for themselves. Individuals will seek to maximize utility and firms will seek to maximize profit. The maximization of utility and profits leads to the maximization of economic output and the maximization of output leads to economic growth in the long-run so long as individuals and firms are free to decide on what tools to use to create the wealth.

If the government has a limited role to play in the mechanism of a market or of an economy, it suggests then that government will have less money to borrow from the central banks, from financial institutions or from foreign governments. In the case of developing countries, their government usually borrow either from their central banks, from financial institutions, or from governments of developing countries. The more a government borrows, the more it increases the debt, and the more it increases inflation. Inflation is based on the increase of the quantity of money supply and the rise of prices. Of course, prices must rise once in a while, but they must not rise above the nominal rate (2-3 percent). When prices rise about that rate, it affects outputs and the factors of production which produce the goods or services intended for consumption. There are, indeed, two important facts that take place when the political process controls a market or an
industry. First, taxes increase because the money supply increase. For the fact of the matter, the government creates artificial growth by increasing the money supply. By creating artificial growth, the government then puts more money in the hands of consumers to spend more in order to create demand. But since the money supply upsurges, the purchasing power of the consumers increases accordingly because the goal is to make the consumer spending more. As the purchasing power of the consumer increases, the government raises taxes in order to tax accordingly to the level of the purchasing power of the consumer. The higher the purchasing power of an individual is, the more the state will tax him. The more the government taxes the individual, the more his purchasing power decreases over time. Consequently, the artificial growth that the state creates to stimulate demand, in fact, hampers the ability of the consumer to save in order to invest in ventures that could maximize his utility. Second, the control of the political process over the market decreases output because it negates competition, efficiency, and productivity as it was aforementioned throughout the methodology of our analysis. That is why in our model, under a deregulated market economy, output increases while inflation and government-borrowing decrease.

In a given market or in an economy that is regulated, the exact opposite happens as the linear equation above shows us and which is reiterated here again:

\[ Y(t) = K(t) [A(t), L(t) + B(t), \pi(t)] \]

Our model shows that the more an industry is regulated, the more subsidized it becomes. And the more subsidized it becomes, government-borrowing then keeps increasing because the government needs to keep borrowing money in order to keep subsidizing the industry it controls. Logically, as the government keeps borrowing, it increases the money supply, therefore it creates an enormous debt.

Overall, our model argues that economic growth occurs sustainably in a deregulated market economy then in a regulated one.

d) The Mathematical Analysis of the Public Choice Growth Model

In this part of our analysis, our objective is to express our model in mathematical language. We are going to dissect our equation, function by function and factor by factor. The purpose of dissecting our equation is to arrive at a logical conclusion. Before we commence to work on this equation, it is important to state what each letter or variable stands for. Our main equation is written as the following:

\[ Y(t) = K(t) [A(t), L(t) - B(t), \pi(t)] \]

Then we have separated our equation into two main blocks using the function symbols as the following:

\[ f(Y) = f(K) [f(A) \times f(L) - f(B) \times f(\pi)] \]

The first block of the equation is composed of the elements of the Solow Growth Model, which are \((K), (A), \text{and} (L)\). The second part of our equation is simply the political element of economic theory that we have added to our equation. Nevertheless, the first part of our equation can be seen as this:
\[ Y = K (A, L) \]

In our equation, \((Y)\) represents the answer, which is growth. Since our equation is built on the principles of the neoclassical model of economic growth, \((Y)\) therefore is mathematically written as:

\[ Y = F (K, L) \]

In our equation, \((K)\) represents the accumulation of capital. It is, once again, noteworthy to reiterate that the accumulation of capital is no longer based on physical capital but technological capital. Yet we still keep the same precept of the Solow Growth Model, but we only replace physical capital by technological capital. The formula can be written as the following: \(K \left( \frac{K}{L} \right) L\).

In the neoclassical model, \((A)\) represents technology. In our equation though, we will replace technology by capital output also known as output. It can be then mathematically written as \(\frac{K}{L}\).

In our equation, \((L)\) represent labor. Nonetheless, it is important to state that Labor, in our equation, means population growth. Since population is a variable that keeps increasing over time, it will be noted as \(\frac{1}{L} \times \frac{dL}{dt}\). Mathematically, labor can be written as \(\frac{1}{L} \times \frac{dL}{dt}\).

Now that we have identified the elements of the first part of our equation, it can then be written as the following:

\[
F (K, L) = f(K) \left[ \frac{K}{L} \times f \left( \frac{1}{L} \frac{dL}{dt} \right) \right]
\]

\[
F (K, L) = f(K) \left[ \frac{K}{L} \times f \left( \frac{1}{L} \frac{dL}{dt} \right) \right]
\]

\[
F (K, L) = f(K) \left[ \frac{K}{L} \times f \left( \frac{1}{L} \frac{dL}{dt} \right) \right]
\]

\[
F (K, L) = f(K) \left[ \frac{K}{L} \times f \left( \frac{1}{L} \frac{dL}{dt} \right) \right]
\]

\[
F (K, L) = f(K) \left[ \frac{K}{L} \right]
\]

Therefore, the first part of our equation can be written as this: \(Y = K \left( \frac{K}{L} \right)\)

The second part of our equation encapsulates the political element of economic theory. By political element, we mean the role that the government plays in the economy. It is mainly composed of government-borrowing and inflation rate. It can be assessed as the following:

\[ Y = K - (B, \pi) \]
(K), which is the capital accumulation factor, is included because it is the main variable that will be used as a multiplier once we have evaluated \( f(B) \) and \( f(\pi) \). Yet, in the assessment of second part of our equation, we will not take the substantive value of (K) into consideration until the second part [- (B, \pi)] is solved.

In our equation, government-borrowing is represented as (B). It is composed of three elements within the operation of government in our model. These three variables are government spending, which can be written as (G), taxes, which can be symbolized as (T), and public debt, which can be represented as (b). Therefore, Government-borrowing, in our equation, is the sum of government expenditures, taxes, and public debt. It could be mathematically formulated as the following:

\[
f(B) = G + T + b
\]

In our equation, inflation rate is represented as (\( \pi \)). The fundamental formula to calculate inflation rate is based upon the Consumer Price Index (CPI) or based on the GDP deflator. The rate of inflation formula measures the percentage change in purchasing power of a particular currency.\(^{23}\) As the cost of prices increase, the purchasing power of the currency decrease.\(^{24}\)

In our equation though, we chose to determine the inflation rate by using the GDP deflator, which goes by this formula:

\[
\text{GDP Deflator} = \frac{\text{Nominal GDP}}{\text{Real GDP}} \times 100
\]

Though, to simply our formula in order to calculate the inflation rate in our equation, we will assume that the Nominal GDP is represented as (GDP\(^{x+1}\)) and the Real GDP is represented as (GDP\(^{x-1}\)) and GDP Deflator will be symbolized as (y). Therefore, our inflation rate (\( \pi \)) formula will be written as:

\[
y = \frac{\text{GDP}^{x+1}}{\text{GDP}^{x-1}} \times 100
\]

This equation can be simplified as this:

\[
y = \left( \frac{\sqrt{x+1}}{\sqrt{x-1}} \right) 100
\]

Therefore, the inflation rate formula in our equation could be written as the following:

\[
f(\pi) = \left( \frac{\sqrt{x+1}}{\sqrt{x-1}} \right) 100
\]

Now that we have identified the variable of the political elements that are incorporated in our calculation, the second part of our equation gives us:

\(^{23}\) Rate of Inflation. Finance Formulas. (2019).
\(^{24}\) Finance Formulas, Ibid.
\[ [f(B) \times f(\pi)] \]

which could be translated to:
\[ [f(G + T + b) \times f\left(\frac{\sqrt{x+1}}{\sqrt{x-1}}\right)100] \]

Now that the second part of our equation has been assessed, the PCGM’s equation can be written in its entirety as the following, based on the deductions that we have made:
\[ Y= K \left[\frac{K}{L_t} - \{(G+T+b) \times \frac{\sqrt{x+1}}{\sqrt{x-1}}\}100}\right] \]

The formula of our equation could be then written as the following:
\[ Y= K [(A, L) – (B, \pi)] \text{ means } Y= K \left[\frac{K}{L_t} - \{(G+T+b) \times \frac{\sqrt{x+1}}{\sqrt{x-1}}\}100}\right] \]

Therefore:
\[ f(Y) = f(K) \left[\{f(A) \times f(L)\} - \{f(B) \times f(\pi)\}\right] \text{ is equivalent to} \]
\[ f(Y) = f(K) \left[\frac{K}{L_t} - \{(G+T+b) \times f\left(\frac{\sqrt{x+1}}{\sqrt{x-1}}\right)100}\right] \]

Now that the fundamental elements of our equation have been addressed and assessed, we can now proceed in elaborating the calculation of the whole equation with the variables available to us. We commence our equation from the assumption that:
\[ Y= K [(A, L) – (B, \pi)] \]

To now solve our equation, we are going to proceed by the same method as we did to establish the framework of the operation. \( Y= K (A, L) \) is the first part of our equation. Our calculation has led us to deduce that:
\[ Y= K \left(\frac{K}{L_t}\right) \]

Therefore, let’s assume that: (1) \( Y= K \left(\frac{K}{L_t}\right) \)

The second part of the equation is \( Y= K – (B, \pi) \). Our calculation has led us to deduce that:
\[ Y= K – \{(G+T+b) \times \frac{\sqrt{x+1}}{\sqrt{x-1}}\}100}\]

Therefore, let’s assume that: (2) \( Y= K – \{(G+T+b) \times \frac{\sqrt{x+1}}{\sqrt{x-1}}\}100}\]

Now that we have separated into two blocks our linear equation, we are going to solve it block by block. Let’s commence our operation with the first block.

(1) \( Y= K \left(\frac{K}{L_t}\right) \)

(1) \( Y= \frac{K^2}{L_t} \)
Let’s now proceed with the second block:

\[ Y = K - \left[ \left( G + T + b \right) \times \left( \frac{\sqrt{x + 1}}{\sqrt{x - 1}} \right) \right] 100 \]

\[ Y = K - (G + T + b) \times \left( \frac{\sqrt{x + 1}}{\sqrt{x - 1}} \right) 100 \]

\[ Y = - (KG + KT + Kb) \times \left( \frac{\sqrt{x + 1}}{\sqrt{x - 1}} \right) 100 \]

\[ Y = - (KG + KT - Kb) \times \left( K \left( \frac{\sqrt{x + 1}}{\sqrt{x - 1}} \right) \right) 100 \]

\[ Y = \left( -\frac{G + T + b}{K} \right) \times \left( \frac{K\sqrt{x + 1}}{\sqrt{x - 1}} \right) 100 \]

(2) \[ Y = \left( -\frac{G + T - b}{K} \right) \times \left( \frac{K\sqrt{x + 1}}{\sqrt{x - 1}} \right) 100 \]

Now that both blocks of the equation have been solved, here is the deductive mathematical formula of the PCGM in a deregulated market economy:

\[ Y = \left( \frac{K^2}{Lt} \right) - \left( \frac{G + T - b}{K\sqrt{x + 1}} \right) 100 \]

Consequently, our final equation can be reduced to this:

\[ Y = K \left[ (A, L) - (B, \pi) \right] \text{ which equates } Y = \left[ \frac{K^2}{Lt} \right] - \left( \frac{G + T - b}{K\sqrt{x + 1}} \right) 100 \]

At this stage of our analysis, we are not concerned with engaging in the same process for a regulated market economy or a subsidized industry because it is the exact same process except for the fact that we only need to positivize the political element instead of negating it as we did for a deregulated market. Lastly, our model is primarily designed for a deregulated market economy in a developing country, not for a regulated economy.

Our equation demonstrates that the negation of the political element in the market process is what enables the accumulation of capital to increase because economic output is maximized.
Part III
Empirical Evidence

In this part of our analysis, we are going to evaluate two countries in order to demonstrate the validity of our theory. These two countries are Bangladesh and Côte d’Ivoire. These two countries have been chosen because they are both developing countries which reflect a particular way of economically growing in their respective regions. In order to determine if our theory is consistent with the empirical evidence, we are going to focus our analysis on assessing inflation, the quantity of money supply, and economic output per sector in each of these two countries.

A) Bangladesh

The most recent data available to us in terms of measuring economic freedom is the 2019 Index of Economic Freedom. The data shows that Bangladesh ranks among the “mostly unfree” civil societies at the 121st place. Moreover, Bangladesh has a population of 170 million inhabitants, which is an increase of 2 million people compared to the year 2019. The question is why with a such a dense population, Bangladesh is one of the most economically unfree countries in the world? To answer this question; we are, first and foremost, going to evaluate the inflation rate in Bangladesh from 1984 to 2024. Then we are going to analyze how inflation rate has directly impacted economic output within these years.

Inflation in Bangladesh has dramatically fluctuated over the years from 1984 to 2016. Yet it has remained relatively constant since 2017 onward. The highest rate of inflation in Bangladesh was 11.46 percent in 2011 and the lowest ever been was 1.91 percent in 2001 as figure 6 illustrates it. We are going to give a much more rigorous emphasis on the impact of inflation on the Bangladeshi economy from 2001 to 2012.

Inflation Rate in Bangladesh from 1984 to 2024

![Inflation Rate in Bangladesh from 1984 to 2024](image)

Figure 6. Source: The World Bank

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There are, of course, internal and external factors that have led to the rise of inflation from 2001 to 2012. In 2001, inflation in Bangladesh was at 1.91 percent. In 2011, it has increased to 11.46 percent. This rate represents the generate inflation in the country. The prices of commodities have increased as well as the cost of living. The surge of the cost of living is a predicament for most Bangladeshi because the majority of the Bangladeshi population is unemployed and poor, which makes it very difficult for them to survive. Inflation in Bangladesh has been significantly rampant especially in the food commodity. If we take a close look at the agricultural sector of the Bangladeshi economy, we see that the rate of inflation is higher in the food (agriculture) sector than in the non-food sector as figure 7 shows.

The inflation rate in the food industry was higher than those of non-food commodities and that of general commodities. The increase of the food production in Bangladesh did not match consumers’ demand. Rice production became stagnant and the production of wheat has declined over the years. Moreover, the production of pulses and oilseeds has also declined significantly while the production of vegetables increased during that period. Furthermore, the market mechanism was highly distorted due to the government’s regulations upon the sector in regulating prices. Indeed, the subsidization of the agricultural sector has handicapped the overall production of the food industry. For the fact of the matter, the net domestic production of food was not sufficient to meet demand, the demand-supply gap of cereals, edible oil and other food items were imported from

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28 Ghosh, & Hossain, Ibid.
29 Ghosh, & Hossain, Ibid.
30 Ghosh & Hossain, Ibid.
31 Ghosh, & Hossain, Ibid.
32 Ghosh & Hossain, Ibid.
external markets. The subsidization of the agricultural sector by the Bangladeshi government has produced the inflationary pressures due to an excess of money supply in the market, which has increase the trend of government to borrow more. It is important to accentuate on the fact that agriculture plays a key role in Bangladesh’s economic growth. Over 87 percent rural people derive at least some income from agriculture. However, the subsidization of the agricultural sector by the Bangladeshi government during the 2000s (2000-2011) has led to an increase of the money supply as well of inflation, and a decrease in output in the agricultural sector the following decade (2010-2020). Figure 8 shows the GDP growth by economic sector. As we observe the data, we see that agricultural output decline over time because of the subsidization of the sector by the government, and because of the lack of the use of technological tools to make production more efficient. Yet the output of the manufacturing and services industries have produced a better output over time since the 2000s. As it was aforementioned, the control of the agricultural sector by the government led to an increase of the money supply in the 2010s. Figure 9 substantiates that increase.

GDP per economic sectors in Bangladesh from 2000 to 2018

![GDP per economic sectors in Bangladesh from 2000 to 2018](image)

Figure 8 Source: Trading Economics

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33 Ghosh, & Hossain, Ibid.
34 Ghosh, & Hossain, Ibid.
36 World Bank, Ibid.
The case of Bangladesh validates our theory in terms of highlighting the elements to create long-term economic growth according to our model. The fact that the agricultural sector was importantly regulated while the majority of Bangladeshi rely on it, shows that the production eventually dropped because the state makes decision that eventually leads to shortages of food supply. The state did not invest enough in technological assets to increase the food supply. Moreover, the example of Bangladesh confirms our theory because the subsidization of the agricultural industry led to tremendous increase in the money supply as well as the rate of inflation during the time of the subsidization and after. Today, the agricultural sector of Bangladesh is recovering yet some progress needs to be made if it seeks to maintain economic growth in that sector.

B) Côte d’Ivoire

The Republic of Côte d’Ivoire is one of the most advanced economies in West Africa. According to the 2019 Index of Economic Freedom, Côte d’Ivoire is ranked at the 78th place, which is within the “moderately free” category of the index. It is undeniable that Côte d’Ivoire is one of the fastest-growing economies in West Africa and on the African continent in general. The economy has expanded by an average of 8 percent per year since 2011.37 Yet, the country’s GDP growth has gradually declined from 10.1 percent in 2012 to 7.4 percent in 2017. Figure 10 illustrates the Ivorian GDP growth rate from 2010 to 2018. Furthermore, like Bangladesh, the agricultural sector plays a quintessential role in the economic development of Côte d’Ivoire.

What could potentially explain this slight drop in GDP growth over the years? We know that from 2010 to 2011, the GDP significantly decreased from 2 percent to -4.5 percent. This depreciation of output occurred because of the post-electoral crisis that took place between Alassane Ouattara and Laurent Gbagbo. That political crisis resulted in 3000 deaths and a considerable reduction of human capital. The political instability of the country disincentivized foreign investors to create investments, it discouraged foreign market to purchase our domestic products, and the lack of investment led to a significant increase in unemployment. However, from 2011 to 2012, economic output surged from -4.5 percent to 10.1 percent. This drastic outburst was, in fact, the product of liberal economic policies being implemented in order to recover the economy. President Ouattara deregulated major sectors of the economy to a considerable degree. This deregulation led to an increase in economic output between 2011 and 2012. The World Bank, the International Monetary Fund, The African Bank of Development, and many foreign governments from developed countries, have loaned money to the Ivorian government in order to relaunch its economy. In order to fully understand the slight decrease in GDP from 2012 to 2018, it is important to assess the inflation rate during these years. Figure 11 shows the inflation rate in Côte d’Ivoire from 2000 to 2024 according to the data of the World Bank. Inflation has relatively fluctuated before 2012; yet, it shows that the consumer price index was relatively stable in terms of the inflation nominal rate.\footnote{World Bank Data, Ibid.} However, from 2012 to 2018, inflation rate fluctuated between 1.3 percent in 2012 and 0.42 percent in 2018.\footnote{World Bank Data, Ibid.} It shows that there was a contraction in the economy during that time. The economic contraction that occurred during that time is the direct causation that led to a decrease in national output. Industrial production, retail sales, and real personal income
relatively plummeted during that period. It is important to reiterate that this slight decline in output did not undermine or overshadow the economic progress that has been made since the political crises of 2011. The deregulation of the economy overall makes of Côte d’Ivoire one of the economic leaders of the subregion of West Africa. Figure 12 substantiates the output in the three main sectors of the economy from 2008 to 2018.

Inflation Rate in Côte d'Ivoire from 2000 to 2024

Figure 11. Source: The Word Bank

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Figure 12 substantiates the rates of output in the three main sectors of the economy from 2008 to 2018. Evidently, each of these sectors is to some extent regulated because the Ivorian government is a shareholder in each of them. The industry sector, also known as the manufacturing sector, as well as the service sector; have provided higher outputs than the agricultural sector because these sectors are mainly privatized. More foreign investors invest in these two sectors because these sectors are those where technology is the most used as a tool to incentivize economic growth. Interestingly, only one-third of the population is employed in these two sectors compared to the agricultural sector. Like in Bangladesh, the agricultural sector is the most regulated among the three because two-thirds of the population are employed in the agricultural sector, which represents 30 percent of GDP and 70 percent of exports earnings. Yet output remained relatively below 30 percent because consumption failed in that sector. In addition to the evident contraction that was occurring in every sector to the Ivorian economy, the agricultural sector remained the most vulnerable because the food production is not adequate to the demand of Ivorian consumers. The economic contraction in the mid-2010s forced Ivorians to save more than they could spend, which considerably decrease private consumptions, yet public consumption increased. As figure 13 shows, public consumption slightly increased, yet remained significantly low because most of the economic activities are provided by market mechanism rather than the political process.

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The slight decrease of economic output did not prevent the money supply from increasing despite the contraction. It is important to clarify that the money supply that increase during the 2010s was based upon the government-borrowing following the political crisis of the presidential elections of 2010. Figure 14 shows the increase of the money supply during the 2010.
Inflation has remained low in Côte d’Ivoire because the government did not borrow much. It obviously did borrow in order to commence the economic recovery process in the early 2010. Compared to Bangladesh, which has a steadfast increase of money supply, the money supply of Côte d’Ivoire during the 2010s did not have a steadfast increase because of the economic contraction that affected prices as well as output. The money supply overall increased because of government expenditures in sectors that are directly under its control such as defense, the police, the maintenance of the courts, and other parts of the administrative sector that are owned by the Ivorian government.

The case of Côte d’Ivoire was a quiet delicate case to test our model and to determine its validity. Scholars who are in favor of a more regulated market overall, will assert that our theory may not be valid because we have contradicted ourselves by using the case of Côte d’Ivoire. At first glance, they may have a point, but if we clearly observe the details our analysis for the case of Côte d’Ivoire, it validates our theory because it aligns itself with the fundamental principles of the PCGM. Indeed, the sectors or market that are generally regulated and subsidized by the government have a decrease of output over time. The case of Côte d’Ivoire was delicate because there was an economic contraction that occurred. Contraction occurs whether an economy is regulated or unregulated. It is a natural process of the business cycle. Interestingly, the data shows that despite the contraction that occurred, unemployment; which was supposed to increase due to slow demand; decreased instead as figure 15 elucidates it. The slow demand that had taken place is therefore a temporary decline in the economy. Moreover, the decrease of unemployment despite economic contraction shows that output is mainly concentrated in the manufacturing and services industries, which are the two industries that mainly use technological tools to determine output. In both industries, which are more deregulated than the agricultural sector, output is considerably higher than in the agricultural sector while two-thirds of the Ivorian labor force is concentrated in the agricultural sector. If the major part of the Ivorian labor force was concentrated in the service and manufacturing sectors, the overall output would have been significantly higher than it is today. Therefore, the case of Côte d’Ivoire does also validate our theory.

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42 *Ivory Coast Profile, OECD. (2015)*
C) Conclusion

Both, Bangladesh and Côte d’Ivoire, are countries that are still in the developing process of their economic condition. Both countries still do heavily rely on agriculture as the main resource to stimulate economic growth. Yet, with overgrowing populations in both countries, the food production does not suffice to everyone. People living in the urban areas of both countries have priority of food consumption over those who live in the rural areas of both countries.

In both countries, the agricultural sector remains importantly subjected to government control. In both countries, the government decided over the production of food because of the impact of imports and exports. Our analysis has led us to observe that in both countries, the service industry is the most efficient with an output of more than 50 percent of the national production yet, the most undermined in terms of labor force participation. Furthermore, the service industry is the industry wherein technology is utilized the most as the basic tool for production. With the use of technology, economic efficiency is heightened because it is cost-efficient for the workers. In the agricultural sector of both countries, an important portion of the labor force is employed in that sector, yet production remains insufficient. It is primarily insufficient because the lack of technological tools to enhance production is a serious conundrum to stimulate economic growth, especially when the entire economy chiefly relies on agriculture.
Conclusion

We have tested our theory with the empirical evidence available to us in order to determine if it was valid or not. Our results show that the Public Choice Growth Model is a valid economic model. As it was enunciated in the introduction, the chief goal of this model is not to predict a particular outcome but to determine the process of an outcome. Our model is more concerned with the process rather than the outcome. We are well aware that attempting to predict outcome is somewhat futile and even impossible because human nature changes constantly. Economic activities are not motionless like objects are. They are conducted and conveyed by human action.

Our model is clearly not exhaustive. But our goal was to, at least, established the fundamental elements of our model. We are expecting this theoretical framework to be challenged by other development economists. We perceive the challenge as a new way to improve the model, and therefore as a new way to expand on the branch of development economics itself. We believe that the economic growth of a society is not principally based upon its natural resources but upon its human capital in addition to exogenous factors such as technology. Our framework does not stipulate, in no way, that a deregulated market is a flawless economic environment where economic growth perpetually happened. Every economic system is subjected to the expansion-recession phenomenon of the business cycle. That is exactly why we have chosen Côte d’Ivoire as a case study to elaborate on our theory. The formula (the equation) and model we have designed was to ensconce a fundamental principle of economic theory in the field of development economics. This principle is that economic growth is more likely to occur on a long-term basis in a perfect competitive market economy if the economy or a sector of the economy is substantively deregulated. We have departed from the assumption that deregulation means freedom because in a deregulated market, individuals are free to decide on how they want to produce a commodity. Since individuals are free to decide on how they want to produce and use a commodity, therefore, this freedom will lead them to be more efficient about the way they want to increase their production. In other words, deregulation stimulate people to maximize output because individuals seek to maximize utility and firms seek to maximize profits. Firms will produce accordingly to the level of demand of the consumer.

In our analysis, we did not include spillovers as part of our initial equation because we fathomed that spillovers are part of what lead to the regulation of the market. Regulations come from the assumption that markets fail, and it is to prevent spillovers of the market that the government regulates. Spillovers are mainly included in our equation under a regulated market economy. Our framework has substantiated that economic output eventually decreases in a regulated economy or regulated sector of the economy because the subsidization of the industry negates competition, innovation, and efficiency. The lack of efficiency in a competitive market inevitably leads to a reduction of economy output. The political process is the element that distorts the mechanism of the factors of production because it misallocates resources when it tries to supply goods and services to the general public. Consequently, this misallocation leads generally to a shortage of production. We do not believe that it is the role of the state to rectify spillovers when they occur. Our model assumes that the market eventually regulates itself based on the laws of supply and demand.
References


2. Shughart, Ibid.

3. Shughart, Ibid.


5. Schug & Fontanini, Ibid.


7. Schug & Fontanini, Ibid.

8. Schug & Fontanini, Ibid.

9. Schug & Fontanini, Ibid.


11. Rent-seeking is a theory developed by Gordon Tullock, who was one of the founders of public choice theory. Rent-seeking means seeking to increase one’s share of existing wealth without creating new wealth. Rent Seeking results in reduced economic efficiency through misallocation of resources, reduce wealth-creation, lost government revenue, and potential national decline. Source: Econlib.org by David R. Henderson.


17. Ibid. p. 69

18. Ibid. p. 69.

19. Ibid. p69

20. Ibid. p.70.

21. Ibid. p70

22. Ibid. p. 71


24. Finance Formulas, Ibid.


29. Ghosh, & Hossain, Ibid.

30. Ghosh & Hossain, Ibid.


32. Ghosh & Hossain, Ibid.

33. Ghosh, & Hossain, Ibid.

34. Ghosh, & Hossain, Ibid.

36. World Bank, Ibid.


38. World Bank Data, Ibid.

39. World Bank Data, Ibid.


42. *Ivory Coast Profile*, OECD. (2015)