Linking Educational Loan Subsidies to Pay-as-you-go Pension Reforms

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Abstract

This study examines public pension reform in a small open economy model where households fully finance education. Departing from previous studies that assume fully publicly funded education, we introduce loan interest subsidies on education in the presence of intergenerational transmission of human capital, which enables an earlier phase-out of pay-as-you-go (PAYG) pensions in a Pareto-improving way. We extend the analysis to a closed economy where wages and interest rates are endogenously determined. By incorporating general equilibrium effects through factor prices, we show that loan interest subsidies make a Pareto-improving, gradual reduction of PAYG pensions feasible even in closed economies. This result highlights the efficiency gains from linking pension reform with educational loan support, in contrast to prior studies that overlook private education spending or factor price adjustments.

JEL classifications: E62, H23, H55
Keywords: intergenerational transmission, human capital, loan interest subsidies, pay-as-you-go pension, pension reforms

1 Introduction

Aaron's (1966) seminal paper demonstrates that pay-as-you-go (PAYG) pensions are not welfare-justified in a dynamically efficient economy. In other words, support for introducing PAYG pensions is limited to situations of dynamic inefficiency. Dynamic inefficiency occurs when the interest rate, representing the return on capital, is lower than the population growth rate, representing the return on PAYG pensions, indicating an overaccumulation of capital. Only in such cases can the introduction of PAYG public pensions curb the overaccumulation of capital, enabling intergenerational Pareto improvements.

However, several recent studies support the existence of PAYG pensions even in dynamically efficient overlapping generations economies. One of the earliest studies is Boldrin and Montes (2005), who focus on the inefficient accumulation of human capital due to the absence of credit markets. In this setting, the introduction of PAYG pensions benefits the retired generation during the implementation period, while educational subsidy policies enhance human capital and benefit future generations, though the initial generations do not gain from these policies. Examining this trade-off, Boldrin and Montes (2005) propose a policy package in which the government finances

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educational subsidies through taxation on the middle-aged generation, with their tax burden offset by pension benefits in old age. This policy package ensures that utility levels remain unchanged from those before the policy's introduction. By implementing this approach, the utility of all generations equates to the level that would prevail with functioning credit markets, thereby providing a rationale for the introduction of PAYG pensions in dynamically efficient economies.

Andersen and Bhattacharya (2017) examine the role of a similar policy package, focusing on the intergenerational transmission of human capital. They show that, with the presence of intergenerational transmission of human capital, PAYG pensions could be gradually reduced and eventually phased out in a Pareto-improving manner. The mechanism behind this result is that, through educational subsidy policies, human capital improves across generations, leading to income increases that render pension benefits unnecessary. Unlike Boldrin and Montes (2005), who argue for a constant pension benefit to maintain laissez-faire utility, Andersen and Bhattacharya (2017) demonstrate that a Pareto-improving reduction of PAYG pensions is possible. This result has significant implications for designing pension reforms.

Bishnu et al. (2021) examine the feasibility of a Pareto-improving gradual reduction of PAYG pensions in an economy where credit markets are absent, as in Boldrin and Montes (2005). Similar to Andersen and Bhattacharya (2017), they demonstrate that such a reduction is achievable. However, their analysis differs from Andersen and Bhattacharya (2017) in two key aspects: first, they assume the absence of credit markets rather than intergenerational transmission of human capital, and second, they formulate the optimization problem of a utilitarian government seeking to maximize the welfare of all generations, thereby identifying the optimal trajectory of pensions and educational subsidies.

The studies by Andersen and Bhattacharya (2017) and Bishnu et al. (2021) contribute to the literature on pension reforms (see, e.g., Belan et al., 1998; Cremer and Pestieau, 2000; Gyárfás and Marquardt, 2001; Wigger, 2001; Damjanovic, 2006; Van Groezen et al., 2007; Adema et al., 2009; Roberts, 2013; Le Garrec, 2015).[1] Specifically, they propose pension reforms that achieve Pareto improvements in dynamically efficient economies. However, in their analyses, private household expenditures on education are fully crowded out by sufficiently large public educational subsidies, as they assume perfect substitutability between private and public educational expenditures. In other words, they focus on cases similar to those in many continental European countries, where education expenses are primarily covered by the government and private household contributions are minimal (Diris and Ooghe, 2018). Due to the absence of private educational investment in their models, their analyses ignore household adjustments in educational spending in response to policy changes—an essential factor in countries such as Japan and the United States, where households finance a large part of education expenditures, especially tertiary education, through borrowing in credit markets (Soares, 2003).

To address this limitation, this study abstracts from public educational investment and instead considers a setting where education is financed solely by households. Within this framework, we introduce a policy of interest subsidies on educational loans. These subsidies partially cover the interest on educational loans borrowed by households, who determine their borrowing amounts based on the subsidy rate. We demonstrate that the loan interest subsidy policy allows for an earlier phase-out of PAYG pensions compared to the educational subsidy policies proposed by Andersen and Bhattacharya (2017) and Bishnu et al. (2021). This constitutes the first contribution of this study.

The second contribution of this study is its analysis of a closed economy. Andersen and Bhat-

[1] For further recent studies on pension reforms over the past decade, see Bonenkamp et al. (2017); Lin et al. (2021); Wang (2021); Devriendt et al. (2023); Díaz-Saavedra (2023); Börsch-Supan et al. (2023); Gustafsson (2023a,b); Andersen et al. (2024); Grossmann et al. (2024); Nguyen (2024); Sánchez-Romero et al. (2024); Díaz-Giménez and Díaz-Saavedra (2025).

tacharya (2017) and Bishnu et al. (2021) assume a small open economy, thereby neglecting general equilibrium effects on factor prices such as wages and interest rates. An exception is Andersen and Bhattacharya (2020), who incorporate general equilibrium effects but, similar to Andersen and Bhattacharya (2017), abstract from private educational expenditures. Additionally, Amol et al. (2023) account for the positive externalities of fertility in PAYG pensions and characterize the trajectory of a gradual pension reduction; however, their model does not incorporate education. In contrast to these previous studies, this study develops a model that explicitly includes private educational expenditures. By considering general equilibrium effects, we demonstrate the feasibility of a Pareto-improving gradual reduction of PAYG pensions through loan interest subsidies. This constitutes the second contribution of this study.

In addition to the studies discussed earlier, this paper contributes to the literature on implementing educational subsidies alongside social security. Pecchenino and Pollard (2002), Rojas (2004), and Yew and Zhang (2013) demonstrate that expanded social security can crowd out educational subsidies and thereby reduce welfare. Omori (2009) further examines the effects of using income tax revenue for both educational subsidies and social security, focusing on how this balance influences fertility rates. While those works primarily seek the optimal policy mix of educational subsidies and social security, the present study moves beyond that framework by analyzing a Pareto-improving strategy: gradually phasing down PAYG pensions while introducing education loan interest subsidies.

The organization of this paper is as follows: Section 2 presents the model and undertakes the analysis in small open economies. Section 3 analyses the Pareto-improving pension reforms. Section 4 explores the investigation in closed economies. Section 5 provides concluding remarks.

2 Model

Consider a small open overlapping generations model in which individuals live for three periods (youth, middle age, and old age). In each period $t\,(=1,2,\cdots,\infty)$, the total population of the economy is assumed to be 1 and there is no population growth. Period-t middle-aged individuals are called generation t. The economy is assumed to be dynamically efficient and that $R>1$ holds where R is the gross interest rate which is constant across periods. In Section 4, we relax this assumption and consider a closed economy wherein R is endogenously determined to clear the market of physical capital.

2.1 Individuals

Individuals borrow to invest in education in youth and accumulate their human capital. In middle age, they are endowed with one unit of labor, inelastically supply it in the labor market and earn labor income. They allocate the income into education loan repayment, consumption and savings. In old age, they consume the returns from their savings. They obtain utility from consumption in middle and old age; they have no altruistic motives for their parents and children, so they transfer no income to their parents and leave no bequest to their children.

As a benchmark, consider a situation where no government intervention exists. In this situation, the budget constraint of an individual who reaches the middle age in period t is

$$0 \leq e_{t-1} \leq \frac{wH_t}{R}, \tag{1}$$

$$c_t^m = wH_t - Re_{t-1} - s_t, \tag{2}$$

$$c_{t+1}^o = Rs_t. \tag{3}$$

where e_{t-1} is borrowing for investment in education, c_t^m and c_{t+1}^o are consumption in middle and old age, s_t is savings, and w and R are the wages and gross interest rates, respectively. Here, H_t represents human capital of the period-t middle-aged individual. The subscripts under each variable denote the period.

Expression in (1) is a borrowing constraint indicating that borrowing is feasible up to an amount repayable with lifetime earnings. Under the standard assumption of the utility function described below, the individual's utility maximization yields an interior solution for investment in education, satisfying the constraint in (1). Equations (2) and (3) are budget constraints for middle and old age, respectively. In middle age, individuals earn wage income, part of which is allocated to repaying the debt incurred in youth and saving for old age, while the remainder is used for consumption. In old age, they receive returns on their savings, which are then used for consumption.

The utility function of generation t is expressed by the following additive-separable form:

$$U_t \equiv u(c_t^m) + \delta u(c_{t+1}^o), \tag{4}$$

where $u(\cdot)$ is the function representing utility of consumption and $\delta \in (0,1)$ is a discount factor. The function $u(\cdot)$ is assumed to be strictly increasing, strictly concave, and twice continuously differentiable, and satisfies the Inada condition.

Human capital is accumulated according to the following equation:

$$H_t = h(e_{t-1}, H_{t-1}), \tag{5}$$

where $h(\cdot,\cdot)$ is the human capital production function, H_{t-1} represents the human capital inherited from parents and e_{t-1} represents educational investment in youth. The function presumes that there is an intergenerational transmission of human capital. The following properties are assumed for the human capital production function $h(\cdot,\cdot)$.

Assumption 1. (i) $h(0,\cdot) = 0$, (ii) $\frac{\partial h}{\partial e} > 0$ and $\frac{\partial^2 h}{\partial e^2} < 0$, (iii) $\frac{\partial h}{\partial H} \in (0,1)$ and $\frac{\partial^2 h}{\partial H^2} < 0$, and (iv) $\frac{\partial^2 h}{\partial e \partial H} > 0$.

The first part of the assumption states that educational investment is essential for the formation of human capital. The second part indicates that the marginal product of educational investment is positive but diminishing, while the third part suggests that the marginal product of parental human capital is also positive but diminishing. The final part states that the marginal product of educational investment (parental human capital) increases as parental human capital (educational investment) increases.

The optimization problem for an individual who reaches middle age in period t is as follows.

$$\max_{s_t, e_{t-1}} u(wh(e_{t-1}, H_{t-1}) - Re_{t-1} - s_t) + \beta u(Rs_t) \text{ s.t. } (1). \tag{6}$$

In the expression of the optimization problem in (6), the budget constraints in (2) and (3) are substituted into the objective function. The individual chooses $\{s_t, e_{t-1}\}$ to maximize the lifetime utility in (6) subject to the borrowing constraint in (1) and non-negative constraints on consumption. Assuming an interior solution, we obtain the following first-order conditions with respect to s_t and e_{t-1}:

$$u'(c_t^m) = \beta R u'(c_{t+1}^o), \tag{7}$$
$$wh_e(e_{t-1}, H_{t-1}) = R. \tag{8}$$

Equation (7) is the Euler equation that determines an optimal allocation of consumption over the lifecycle. The Euler equation states that, in an optimal allocation, an individual equates the marginal utility of consumption in middle age to the expected discounted marginal utility of consumption in old age, adjusted by the real interest rate. Equation (8) determines investment in education to equate its marginal benefit (appearing on the left-hand side) and the marginal cost (appearing on the right-hand side).[2]

A *laissez-faire equilibrium allocation* $\{c_t^m, c_t^o, s_t, e_t, H_t\}_{t=1}^{\infty}$ is characterized by (2), (3), (5), (7), and (8) with the borrowing constraint in (1), given an initial condition H_1. The laissez-faire equilibrium path converges to a unique *steady state* where each variable is constant across periods. The laissez-faire equilibrium steady-state allocation is denoted as $\{\tilde{c}^m, \tilde{c}^o, \tilde{s}, \tilde{e}, \tilde{H}\}$.

2.2 Government

The government is represented by an infinitely-lived planner who attempts to implement a Pareto improvement from the laissez-faire equilibrium allocation using two types of policy: loan interest subsidy and pay-as-you-go (PAYG) pension. The government levies a lump-sum tax on middle-aged individuals to finance these policies. When middle-aged individuals repay their education loans, the government covers a portion of the interest payment. Old-aged individuals receive PAYG pension benefits and consume them.

The policies we consider here differ from those of Boldrin and Montes (2005) and Andersen and Bhattacharya (2017) in that direct government subsidies for education spending are replaced by loan interest subsidy. In Boldrin and Montes (2005) and Andersen and Bhattacharya (2017), individuals receive transfer from the government in youth, which they use to pay for education and therefore do not invest privately in education. In contrast, in the present framework, individuals decide how much to invest in education by comparing the post-subsidized interest rate with the marginal productivity of their educational investment. The loan interest subsidy policy provides a lens through which we can observe individual responses to the policy and assess the subsequent impact on the welfare enhancements that will be investigated below.

With loan interest subsidy and PAYG pension, the lifetime budget constraint for middle-aged individuals in period t is given by

$$c_t^m + \frac{c_{t+1}^o}{R} = Y_t \equiv wh(e_{t-1}, H_{t-1}) - (1-\sigma)Re_{t-1} - T_t + \frac{P_{t+1}}{R}, \tag{9}$$

where Y_t denotes the lifetime disposable income and $\sigma \in (0,1)$ is the subsidy rate on interest payment. The government's budget constraint is given by

$$\sigma Re_{t-1} + P_t = T_t, \tag{10}$$

where P_t is the PAYG pension benefit and T_t is a lump-sum tax.

Consider a policy pair (σ, P) designed to maximize lifetime disposable income in the steady state. Specifically, we aim to identify the pair that attains the maximum lifetime disposable income, denoted by Y^*, through the choice of σ alone, without relying on the provision of PAYG pensions. This pair, referred to as the *Golden Rule level* of lifetime disposable income, is formally defined below.

Definition 1. Given a policy pair (σ, P), the *Golden Rule level* is the lifetime disposable income that satisfies the following two conditions in a steady state:

[2] In practice, individuals may face borrowing constraints, resulting in the optimality condition for educational investment in (8) holding with strict inequality. Such cases can be represented by imposing an upper limit on borrowing at $\phi wH_t/R$, where $\phi \in (0,1]$ captures the degree of the borrowing constraint. The case of $\phi = 1$ corresponds to the absence of borrowing constraints, which is the focus of this study. The borrowing constraint becomes binding if ϕ is sufficiently low. Nevertheless, even in such cases, a similar analysis to the non-binding scenario can be conducted by incorporating ϕ into the expressions for private educational expenditure and the associated conditions.

1. $P = 0$,
2. $\frac{\partial Y}{\partial \sigma} = 0 \Longleftrightarrow w\frac{h_e e_\sigma}{1-h_H} - Re_\sigma = 0$.

The first condition requires the absence of pension provision in the steady state. If this condition is satisfied, reliance on the PAYG pension for maximizing lifetime disposable income is no longer necessary. The second condition requires that the loan interest subsidy rate be set to maximize households' disposable income in the steady state. When both conditions are met, it becomes possible to maximize disposable income—and thereby lifetime welfare—through the gradual elimination of PAYG pensions. The next section examines this policy reform, ensuring that no individual is made worse off.

3 Pareto-improving Pension Reforms

Consider the laissez-faire equilibrium allocation. Given the steady-state values of $\tilde{e}$ and $\tilde{H}$ defined in the previous section, the individual's lifetime income in the laissez-faire steady state equilibrium allocation, $\tilde{Y}$, is determined by the following equation:

$$\tilde{Y} = wh(\tilde{e}, \tilde{H}) - R\tilde{e} \tag{11}$$

Suppose that the government introduces a pair of loan interest subsidy and PAYG pension, $\{(\sigma, P_{t+j})\}_{j=0}^{\infty}$, from period t onwards. The loan interest subsidy rate is constant, while the pension benefits are set to satisfy Pareto criterion. It is assumed that individuals know in advance, in period $t-1$, that the policy will be implemented from period t onward. The objective is to characterize the combination of the loan interest subsidy and PAYG pension that leads to a Pareto improvement over the laissez-faire equilibrium allocation.

3.1 Analysis

The Pareto principle requires $Y_{t+j} \geq \tilde{Y}$ for any j, implying that the introduction of the loan interest subsidy and PAYG pension does not reduce lifetime disposable income. In other words, no generation is made worse off by the introduction of this policy pair.

Under the assumption of the small open economy, individuals can borrow freely at a constant interest rate, provided they adhere to the borrowing constraint in (1). Achieving identical utility before and after reallocation is possible when lifetime disposable income is equal in both cases. We examine the necessary pension P_{t+j} for a given loan interest subsidy rate σ to realize Pareto improvements over the laissez-faire equilibrium allocation.

To conduct the analysis, consider the period $t-1$ education spending determined by the following equation for a given loan interest subsidy rate σ:

$$wh_e(e_{t-1}, \tilde{H}) = (1-\sigma)R \tag{12}$$

This expression indicates that, given the individual's human capital inherited from the previous generation $\tilde{H}$, the interest rate R, and the wage w, the investment in education e_{t-1} is determined such that the marginal benefits and marginal costs of education investment coincide.

Next, we derive the period-t pension benefit that allows the individual to attain the same level of utility as before the policy implementation. The equal utility condition in period $t+j$ ($j = 0, 1, 2, \cdots$) is as follows:

$$Y_{t+j} = \tilde{Y} \Longleftrightarrow wh(e_{t+j-1}, H_{t+j-1}) - (1-\sigma)Re_{t+j-1} - T_{t+j} - P_{t+j} + \frac{P_{t+j+1}}{R} = wh(\tilde{e}, \tilde{H}) - R\tilde{e}. \tag{13}$$

Note that $P_t = 0$ and $H_{t-1} = \tilde{H}$. In the policy initiation period (i.e., period t), old-aged individuals maintain their education level at the laissez-faire steady-state equilibrium level and do not bear any tax burden. There is no change in their lifetime income before and after the policy implementation.

To solve for P_{t+1} in (13), we use the government's budget constraint in (10) and obtain

$$P_{t+1} = R[-(wh(e_{t-1}, H_{t-1}) - w\tilde{H}) + (1-\sigma)Re_{t-1} - R\tilde{e} + T_t]. \tag{14}$$

By continuing this calculation for subsequent periods, we can determine the period-$t+j$ pension P_{t+j} that satisfies equality utility condition in (13) as follows.

$$P_{t+j}/R = -\underbrace{(wh(e_{t+j-1}, H_{t+j-1}) - w\tilde{h})}_{\equiv A_{t+j}} + (1-\sigma)Re_{t+j-1} - R\tilde{e} + T_t, \tag{15}$$

where P_{t+j}/R is the present value of the pension benefit. The expression in (15) shows that the value is decomposed into the following three terms:

1. The term $A_{t+j} \equiv wh(e_{t+j-1}, H_{t+j-1}) - w\tilde{H}$ reflects the increase in human capital. As labor income increases, the pension required to achieve the same utility as in the laissez-faire becomes less.

2. The term $(1-\sigma)Re_{t+j-1} - R\tilde{e}$ represents the repayment of education loan. Individuals can borrow more funds due to loan interest subsidy.

3. The term T_{t+j} denotes the tax burden for the loan interest subsidy and PAYG pension. The higher the taxes, the lower the individual's lifetime income, requiring larger pensions for compensation.

To identify a feasible pension reform that adheres to the Pareto principle, we focus on the pension levels starting from period t onward. The pension required for period t is denoted as $P_t = 0$, as individuals in this period do not bear any additional tax burden for pension reform, thus necessitating no compensation for them. For period $t+1$, P_{t+1} must satisfy the equal utility condition in (15). Considering the government's budget constraint, we can express the disparity between successive-period pension levels, P_{t+1} and P_{t+2}, as follows. If $\sigma > 0$, then $e_{t-1} > \tilde{e}$, resulting in the term within the parentheses in the second term on the right-hand side of (15) being positive. Consequently, if the loan interest subsidy leads to an augmentation in human capital such that $A_{t+1} \geq (Re_{t-1} - R\tilde{e})$ holds, the pensions required for Pareto improvements decrease from period $t+1$ to $t+2$, as summarized in the following proposition.

Proposition 1. *Consider the pension benefit that satisfies the equal utility condition in (15). Given $\sigma > 0$, if $w(H_{t+1} - H_t) - R(e_t - e_{t-1}) > R\{w(H_t - \tilde{H}) - R(e_{t-1} - \tilde{e})\}$, then $P_{t+2} < P_{t+1}$ holds.*

Proof. See the appendix A. □

This proposition argues that if the increase in labor income surpasses the repayment of the loan, then the amount of pension required to satisfy the equal utility condition decreases from the period $t+1$ to period $t+2$. It is important to note that, given the pension reform starting from period t, a decline in pension from period $t+1$ to $t+2$ is not possible in the context of Andersen and Bhattacharya (2017), while it is feasible in the present framework. The difference arises as follows: In Andersen and Bhattacharya (2017), taxes paid by individuals in their middle age are directed towards education subsidies for the young, thereby increasing the burden on the middle-aged population. In the present framework, however, middle-aged individuals bear the tax burden but receive benefits through a subsidy on loan interest, resulting in a smaller necessary compensation for the tax burden via provision of PAYG pension.

The next proposition demonstrates that a combination of the loan interest subsidy and PAYG pension policies can accommodate both Pareto improvement and the phasing out of pensions.

Proposition 2. *Once pension increases cease, it is feasible to reduce pensions thereafter gradually. That is, if* $P_{t+j+2} - P_{t+j+1} \leq 0$*, then* $P_{t+j+3} - P_{t+j+2} < 0$ *for* $j = 1, 2, \cdots$.

Proof. See the appendix B. □

The implication of Proposition 2 is that the pension necessary to uphold the equal utility condition in (15) diminishes over time, eventually reaching a negative value at a certain future period. Assuming that upon reaching a negative pension threshold, the government can reset pension benefits (i.e., burden for transfer to the young) to zero for successive generations. Under such circumstances, middle-aged individuals within each generation would experience higher utility compared to the laissez-faire equilibrium allocation. This is due to the elimination of the pension-related burden on middle-aged individuals. Consequently, the earlier the pensions are terminated, the more favorable it becomes according to Pareto principle.

3.2 Numerical Examples

This section undertakes numerical analysis to assess the feasibility of achieving Pareto improvements as outlined in Section 3.1. Initially, we explore three scenarios of the loan interest subsidy, presenting a gradual reduction in pension provision that leads to Pareto improvements in each case. Following this, we conduct a comparative analysis between two scenarios – one involving subsidy for education spending, as discussed in Andersen and Bhattacharya (2017), and the other involving the loan interest subsidy, as explored in the present study. This comparison enables us to determine which approach is superior in terms of achieving Pareto improvements.

3.2.1 Benchmark Case

We quantitatively demonstrate the impact of the policies presented in Section 3.1. For analysis, the utility function and the human capital production function are specified as follows.

$$u(c^m_{t+j}, c^o_{t+j+1}) = \log c^m_{t+j} + \delta \log c^o_{t+j+1}, \tag{16}$$

$$H_{t+j} = h(e_{t+j-1}, H_{t+j-1}) = e^{\alpha}_{t+j-1} H^{\beta}_{t+j-1}, \tag{17}$$

where $\delta \in (0,1)$ is a discount factor, and $\alpha \in (0,1)$ and $\beta \in (0,1)$ with $\alpha + \beta < 1$ are elasticities of human capital with respect to investment and parental human capital, respectively. We set the parameters at $\alpha = 0.4$, $\beta = 0.4$, $w = 1$ and $R = 1.05$, and consider the three scenarios of the loan interest subsidy: $\sigma = 0.1$, 0.4, and 0.5. The loan interest subsidy is introduced in period t.

Figure 1 illustrates the path of pension benefits that satisfy the utility equality condition in (15). As shown, the pension benefit is set to zero in period t, implying that middle-aged individuals in period $t-1$ receive no pension benefits.

Under the current set of parameter values and specified functional forms, the Golden Rule level, as determined by Definition 1, is attained in the long run when the loan interest subsidy rate is set at $\sigma = 0.4$. For this scenario, the pension provision becomes necessary for periods $t+1$ and $t+2$ to maintain utility at the same level as the laissez-faire equilibrium. However, from period $t+3$ onward, the required level of pension benefits turns negative, indicating that pensions could be phased out from the economy without adversely affecting any generation. In other words, achieving a Pareto improvement is possible by discontinuing the provision of pension benefits when the required level becomes negative and redistributing surplus resources appropriately.

The results differ in the other two scenarios where $\sigma = 0.1$ and 0.5. When the subsidy rate is low such that $\sigma = 0.1$, the level of pensions satisfying the equal utility condition in (15) becomes negative from period $t+1$ onwards. This implies that pensions could be terminated promptly by implementing the loan interest subsidy policy. Conversely, when the subsidy rate is high such that

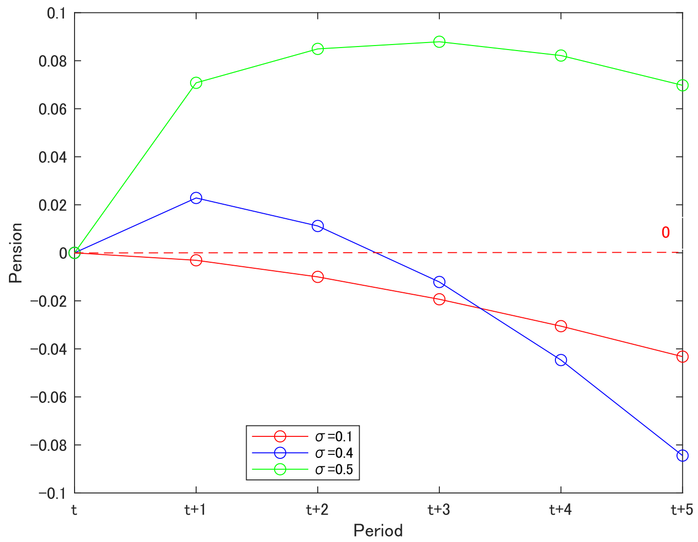

Fig. 1: Pension P_{t+j} $(j = 0, 1, \cdots, 5)$ that need to satisfy the equality utility condition for the three scenarios, $\sigma = 0.1$, 0.4, and 0.5.

$\sigma = 0.5$, a continuous provision of pensions from period $t+1$ onwards is required to maintain the utility level at the laissez-faire equilibrium, and the process of abolishing pensions takes longer compared to the other scenarios, $\sigma = 0.1$ and 0.4.

The mechanism behind this difference is as follows. A lower value of σ reduces the tax burden on middle-aged individuals and decreases the amount of pension benefits required to maintain the laissez-faire equilibrium utility level. Under Assumption 1, a decrease in σ accelerates the reduction in pension benefits due to the diminishing marginal productivity of human capital investment and the presence of intergenerational transmission of human capital. As a result, the scenario with $\sigma = 0.1$ achieves pension abolition at the fastest rate among the three scenarios. In all cases, educational subsidies promote human capital accumulation, which in turn raises labor income sufficiently. This leads to the emergence of a period that satisfies the condition in Proposition 1, from which point the Pareto-improving reduction in pension benefits begins.

3.2.2 Education Subsidy vs. Loan Interest Subsidy

The loan interest subsidy policy we have examined thus far represents an indirect means of supporting individuals by subsidizing education loan repayments during middle age. In contrast, the approach advocated by Andersen and Bhattacharya (2017) involves a policy wherein the government levies taxes on the middle-aged and subsequently transfers the tax revenue to the young, thereby supplementing their education spending with transfer. This section undertakes an evaluation to ascertain the comparative efficiency, in terms of Pareto improvements, between the education subsidy policy proposed by Andersen and Bhattacharya (2017) and the loan interest subsidy policy proposed in the present study.

For the purpose of comparison, the education subsidy is established to mirror private education spending under the loan interest subsidy policy. More precisely, the government aims to align the education subsidy with the private education spending level achieved under the loan interest subsidy policy. Initially, the economy is presumed to be in the laissez-faire steady state equilibrium

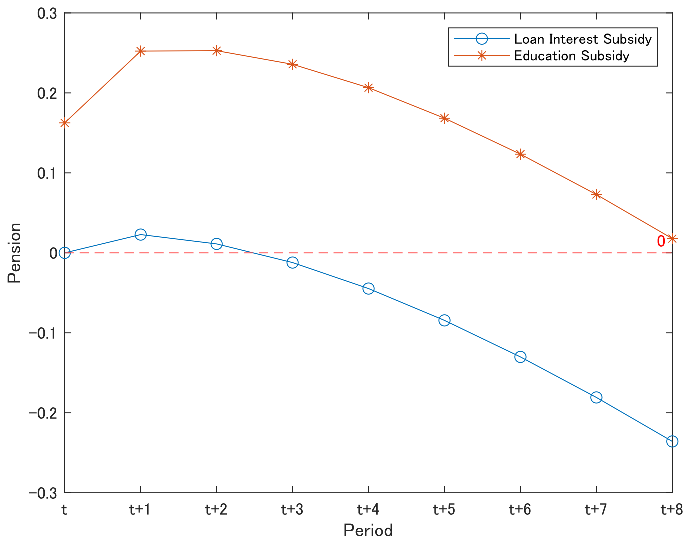

Fig. 2: Pension P_{t+j} ($j = 0, 1, 2, \cdots, 8$) that needs to satisfy the equal utility condition under the loan interest subsidy and education subsidy policies.

allocation, and starting from period $t-1$, the government introduces the education subsidy.[3]

After the introduction of the education subsidy, the government's budget constraint formula is:

$$T_{t+j} = e_{t+j} + P_{t+j}, \; j = -1, 0, 1, 2, \cdots \tag{18}$$

The inequality $e_{t+j} > \tilde{e}$ holds true across all periods. This signifies that, under the education subsidy policy, the young do not borrow privately for investment in education. Consequently, the investment level in education remains consistent between the loan interest subsidy policy and the education subsidy policy, resulting in an identical level of human capital in each period.

The budget constraints in middle age under the education subsidy policy are as follows:

$$c^m_{t-1} = wh(\tilde{e}, \tilde{H}) - R\tilde{e} - T_{t-1} - s_{t-1}, \tag{19}$$

$$c^m_{t+j} = wh(e_{t+j-1}, H_{t+j-1}) - T_{t+j} - s_{t+j} \quad j \geq 0. \tag{20}$$

Equation (19) represents the budget constraint for middle-aged individuals at the inception of the education subsidy policy (i.e., in period $t-1$). During this period, they are obliged to repay their own education loans acquired in period $t-1$, along with covering the costs associated with transferring resources to the succeeding generation. Equation (20) expresses the budget constraint applicable to middle-aged individuals in the subsequent period. In this phase, they are relieved of the obligation to repay education loans, given the absence of private borrowing within the framework of the education subsidy policy.

Figure 2 illustrates a numerical comparison between the loan interest subsidy policy and the education subsidy policy, showing pension benefit levels necessary to satisfy the equal utility condition. The parameters utilized, namely $\alpha = 0.4$, $\beta = 0.4$, $w = 1$, and $R = 1.05$, align with those specified in the benchmark of Section 3.2.1. Given these parameter settings, the government opts for $\sigma = 0.4$ under the loan interest subsidy policy, maximizing disposable income in the steady

[3]The education subsidy is announced from period $t-1$ rather than period t. In the case of the loan interest subsidy, private education expenditure in period $t-1$ changes in anticipation of its introduction in period t. Consequently, the education subsidy must be provided from period $t-1$ to maintain the same sequence of private education expenditure as in the loan interest subsidy case from period $t-1$ onwards.

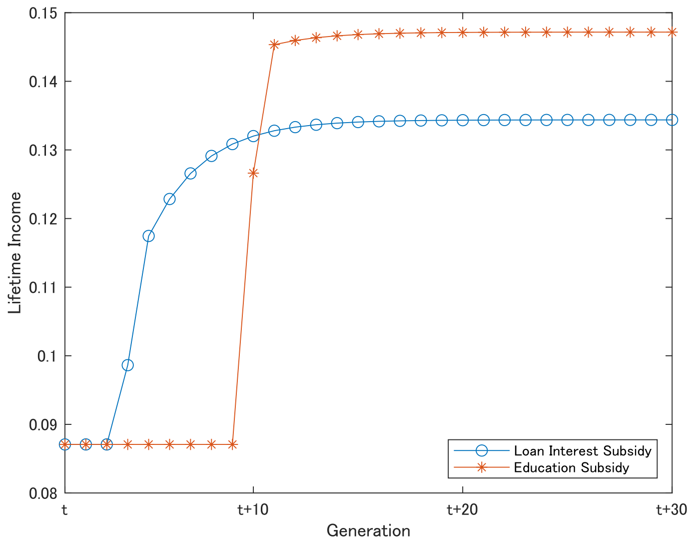

Fig. 3: Lifetime income Y_{t+j} $(j = 0, 1, 2, \cdots, 30)$ under the loan interest subsidy and education subsidy policies.

state post the abolition of pensions. Within the education subsidy policy, the government offers individuals an equivalent subsidy to private education expenses under the loan interest subsidy at $\sigma = 0.4$.

Under the education subsidy policy, pension benefits provided to the period-t old-aged are essential to maintain the same lifetime utility level as in the laissez-faire equilibrium allocation. This is because the period-$t-1$ middle-aged individuals not only receive no direct benefit from the education subsidy but also bear the cost of funding the next generation's education through tax payments. Although the pension benefits required to satisfy the utility equality condition decline over time under the education subsidy policy, the phase-out of pensions is more gradual compared to the loan interest subsidy scenario. This slower phase-out is due to the broader scope of the education subsidy policy, which covers all education expenditures and imposes a heavier burden on early middle-aged individuals.

As illustrated in Figure 2, the pension level satisfying the equal utility condition is lower for the loan interest subsidy policy than for the education subsidy policy from period t to period $t+8$. This indicates that the loan interest subsidy policy is superior to the education subsidy policy in the Pareto sense, as long as we consider generations up to $t+7$. However, this Pareto dominance of the loan interest subsidy policy does not persist in the long run. To demonstrate this, we focus on the steady-state when pensions are abolished. We denote the level of education expenditure and human capital in the steady state by e^* and H^*, respectively. Note that in the steady state the education expenditures and the associated human capital levels are equal under both policies.

Under the loan interest subsidy policy, the lifetime income in the steady state after the pension is abolished is $Y = wH^* - Re^*$, where Re^* is the loan repayment. In contrast, under the education subsidy policy, the lifetime income in the steady state after the abolition of the pension is $Y = wH^* - e^*$, where e^* is the tax burden for the education subsidy. Given the assumption of $R > 1$, we find that lifetime income under the education subsidy policy is higher than that under the loan interest subsidy policy in the steady state. This implies that, in the long run, the education subsidy policy dominates the loan interest subsidy policy in terms of utility.

Based on the numerical analysis outlined above, Figure 3 compares the lifetime income of each generation under the loan interest subsidy policy and the education subsidy policy. Until generation $t+10$, lifetime earnings are higher under the loan interest subsidy policy than under the education subsidy policy. However, from generation $t+11$ onward, the education subsidy policy achieves higher lifetime income. Thus, the choice between loan interest subsidy and education subsidy policies leads to an intergenerational lifetime income (i.e., lifetime utility) trade-off.

4 Closed Economy

In the model of a small open economy examined thus far, taxation does not have a general equilibrium effect through the factor prices such as wage and interest rate because factor prices are assumed to be fixed. However, in the real world, the wage and interest rates are endogenous and determined to clear the labor and physical capital markets, respectively, and thus depend on policy choices through saving. To illustrate such a general equilibrium effect, we now shift our focus to a closed economy where the wage and interest rates are endogenously determined in equilibrium.

In a closed economy, the decrease in private savings due to taxation raises the interest rate and lowers the wage rate. These changes, in turn, affect the required level of pension benefits needed to achieve Pareto improvements. Additionally, a higher interest rate increases the repayment burden of education loans, hindering human capital accumulation. As a result, higher pension benefits become necessary to offset this burden. At the same time, the higher interest rate benefits older adults by increasing the return on their savings, potentially allowing for a reduction in pension benefits.

Fluctuations in factor prices impose burdens on the younger generation through two channels. First, an increase in the interest rate raises education loan repayments, thereby discouraging human capital accumulation. Second, a decrease in the wage rate not only directly reduces consumption but also lowers the return on educational investment, further discouraging human capital accumulation. In contrast, a higher interest rate benefits older adults by increasing the return on their savings, leading to higher consumption. If the latter effect outweighs the former, it becomes possible to achieve a Pareto-improving pension reform even in a closed economy. The following analysis explores this possibility.

The expression of $K_{t+1} = s_t - e_t$ shows the equilibrium condition for the physical capital market in the closed economy. The e_t is the education loan young individuals take in period t. The savings s_t of middle-aged individuals are used to finance investment in physical capital K_{t+1} and loans to younger generations e_t. Using the equilibrium condition for the physical capital market, the ratio of physical to human capital, $k_{t+1} \equiv K_{t+1}/H_{t+1}$, is reformulated as

$$k_{t+1} = \frac{s_t - e_t}{H_{t+1}}. \tag{21}$$

For analytical tractability, we assume that the firm's production function is of a standard Cobb-Douglas type $F(K_t, H_t) = B(K_t)^a (H_t)^{1-a}$, with constant $B > 0$ and $\alpha \in (0,1)$. Factor prices are determined by the following first-order conditions for profit maximization:

$$R_t = \alpha B (K_t)^{a-1} (H_t)^{1-a}, \tag{22}$$

$$w_t = (1-\alpha) B (K_t)^a (H_t)^{-a}, \tag{23}$$

where R_t is the rental price of capital and w_t is the wage rate. Physical capital is assumed to depreciate fully within each period.

The laissez-faire steady-state equilibrium allocation before the policy implementation is denoted by $\{\tilde{R}, \tilde{w}, \tilde{s}, \tilde{e}, \tilde{H}, \tilde{k}\}$. The allocation is characterized by the human capital production function in (5), firms' profit maximization conditions in (22) and (23), and the following three conditions:

$$\tilde{w}h(\tilde{e},\tilde{H}) = \tilde{R}, \tag{24}$$

$$\tilde{s} = s(\tilde{Y},\tilde{R}) \text{ with } \tilde{Y} \equiv \tilde{w}\tilde{H} - \tilde{R}\tilde{e}, \tag{25}$$

$$\tilde{k} = \frac{s(\tilde{Y},\tilde{R}) - \tilde{e}}{\tilde{H}}. \tag{26}$$

Equation (24) is the optimality condition for investment in education, corresponding to (8) for the small open economy. Equation (25) is the savings function, and Equation (26) is the equilibrium condition for the physical capital market.

For numerical analysis, we follow the specification of the utility function and the human capital production function in Section 3.2 and assume the functions as follows:

$$u(c_t^m, c_{t+1}^o) = \ln c_t^m + \delta \ln c_{t+1}^o, \tag{27}$$

$$H_t = h(e_{t-1}, H_{t-1}) = Ae_{t-1}^{\alpha} H_{t-1}^{\beta}, \tag{28}$$

where $\delta \in (0,1)$ is the discount factor, and $\alpha \in (0,1)$ and $\beta \in (0,1)$ with $\alpha + \beta < 1$ are the weight parameters for investment and inherited parental human capital, respectively. We incorporate the scale parameter A into the human capital production function to guarantee the presence of the steady-state equilibrium allocation.

Under the specification of each function, individuals' choice of investment in education, human capital, and savings become as follows

$$e_{t+j-1} = \left(\frac{A\alpha w_{t+j}}{(1-\sigma)R_{t+j}} \right)^{\frac{1}{1-\alpha}} H_{t+j-1}^{\frac{\beta}{1-\alpha}}, \tag{29}$$

$$H_{t+j} = \left(\frac{A\alpha w_{t+j}}{(1-\sigma)R_{t+j}} \right)^{\frac{\alpha}{1-\alpha}} H_{t+j-1}^{\frac{\beta}{1-\alpha}}, \tag{30}$$

$$s_{t+j} = \frac{\delta}{1+\delta}(w_{t+j}H_{t+j} - (1-\sigma)R_{t+j}e_{t+j-1} - T_{t+j}) - \frac{1}{1+\delta}\frac{P_{t+j+1}}{R_{t+j+1}}, \tag{31}$$

and the factor prices are:

$$R_{t+j} = aB\left(\frac{s_{t+j-1} - e_{t+j-1}}{H_{t+j}} \right)^{a-1}, \tag{32}$$

$$w_{t+j} = (1-a)B\left(\frac{s_{t+j-1} - e_{t+j-1}}{H_{t+j}} \right)^{a}, \tag{33}$$

where $j = 0, 1, 2, \cdots$ holds for all of the above expressions.

The policy is assumed to begin in period t and its start is announced in period $t-1$. The government's budget constraint in period $t+j$ is:

$$T_{t+j} = T_{t+j}^{\sigma} + T_{t+j}^{P} = \sigma R_{t+j} e_{t+j-1} + P_{t+j}, \; j = 0, 1, 2, \cdots. \tag{34}$$

The government determines the pension benefit P_{t+j} so that utility is equal to that before the policy implementation:

$$u(c_{t+j}^m, c_{t+j+1}^o) = \ln c_{t+j}^m + \delta \ln c_{t+j+1}^o = \tilde{u}, \tag{35}$$

where $\tilde{u} \equiv u(\tilde{c^m}, \tilde{c^o})$ denotes the steady-state utility level before the policy implementation. Unlike in the small open economy, due to the general equilibrium effect, the lifetime utility may change even if the lifetime income is the same as before the policy is implemented. Thus, it is necessary to determine pension benefits from the viewpoint of keeping the utility at the laissez-faire steady-state equilibrium level.

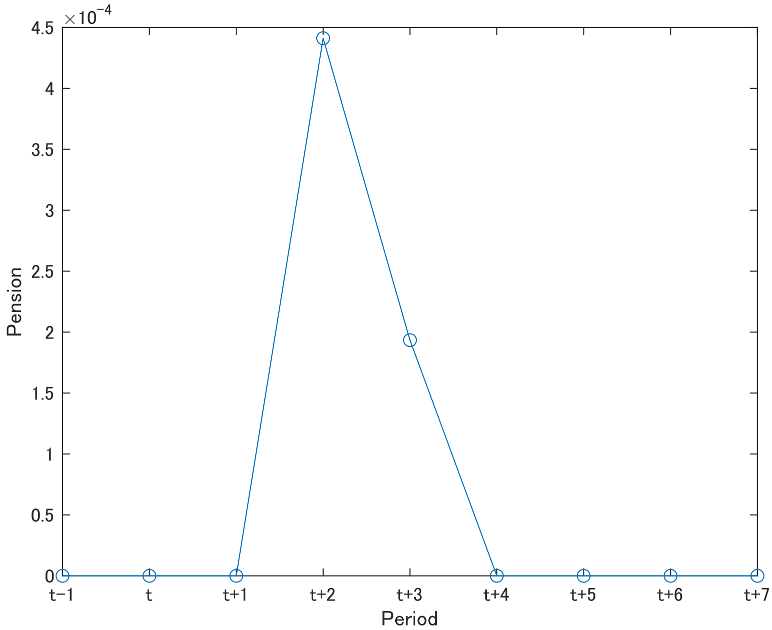

Fig. 4: Pension P_{t+j} ($j = -1, 0, 1, \cdots$) that needs to satisfy the equal utility condition under the loan interest subsidy policy.
Note. If $P_{t+j} < 0$, then it is set at $P_{t+j} = 0$.

4.1 Benchmark Case

For numerical analysis, we set each parameter at $a = 0.3$, $\alpha = 0.2$, $\beta = 0.75$, $\sigma = 0.715$, $\delta = 0.99$, $A = 0.5$, and $B = 5$. Figure 4 shows the path of pension that attains Pareto improvement upon the laissez-faire equilibrium. In period $t-1$, the economy is in the laissez-faire steady-state equilibrium. Remember that the loan interest subsidy policy is introduced in period t, and its implementation is announced in period $t-1$. The figure illustrates that in the current example, the pension benefit must be provided to the older adults in periods $t+2$ and $t+3$ to maintain the same utility as before the policy implementation. However, from period $t+4$ onwards, there is no need to provide public pension benefits to old-aged individuals. In other words, generations from $t+3$ onwards benefit from improved welfare without relying on pension benefits, as they can prioritize the augmented accumulation of human capital passed down by preceding generations.

The lifetime utility of each generation is depicted in Figure 5. As can be seen from Figures 4 and 5, the older adults in periods t and $t+1$ do not need pension benefits. Despite not receiving a pension, generation t (that is, middle-aged individuals during period t) experience an improved lifetime utility. This can be attributed to the implementation of the loan interest subsidy policy, which stimulates investment in education among the young in period t. As a result, this reduces capital available in period $t+1$, resulting in a rise in the interest rate during period $t+1$. Consequently, the return on savings received by generation t in period $t+1$ sees a noticeable increase.

Figures 4 and 5 also show that generations $t+1$ and $t+2$ need to be compensated by pension benefits in their old age to obtain the same level of lifetime utility as the laissez-faire equilibrium. This is because they have not been able to accumulate sufficient human capital due to rising interest rates and falling wages as illustrated in Figure 6. From period $t+3$ onwards, the factor price is almost constant, and human capital has increased sufficiently so that compensation through pensions is no longer necessary. Thus, generation $t+3$ and beyond can enjoy utility exceeding the laissez-faire equilibrium allocation even in the absence of compensation through pensions.

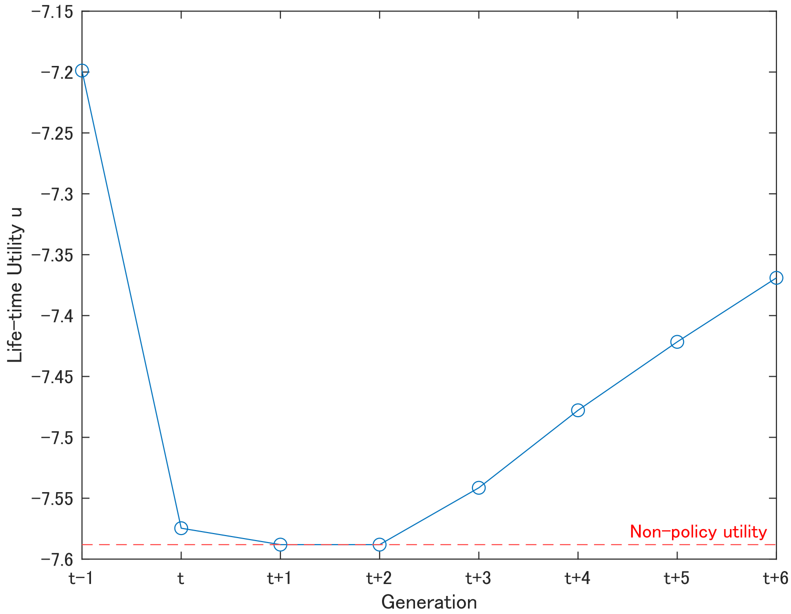

Fig. 5: Lifetime utility under the loan interest subsidy policy.

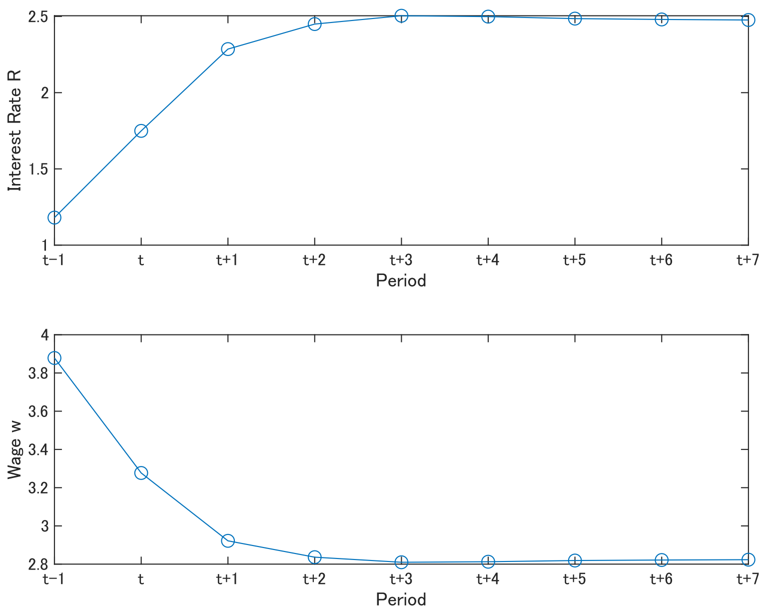

Fig. 6: Wages and interest rates under the loan interest subsidy policy.

4.2 Education Subsidy vs. Loan Interest Subsidy

In Section 3.2.2, we demonstrated that in a small open economy, loan interest subsidies are preferable to education subsidies in terms of utility for early generations, whereas education subsidies are more advantageous for distant future generations. In this section, we show that the same result holds in a closed economy.

For numerical analysis, the parameters are set at $a = 0.3$, $\alpha = 0.2$, $\beta = 0.75, \delta = 0.99, A = 0.5$, and $B = 5$ as in the benchmark case in Section 4.1. Under the education subsidy policy, the government directly covers the cost of investment in education e_{t+j}, where $j = -1, 0, 1, 2, \cdots$, as determined by equation (29). This subsidy is funded by a lump-sum tax on middle-aged individuals.

Under the education subsidy policy, the government budget constraint is formulated as follows:

$$T_{t+j} = e_{t+j} + P_{t+j}, j = -1, 0, 1, 2, \cdots.$$

Due to the substantial education subsidy received each period, individuals do not need to borrow privately for education during their youth. This implies that under the education subsidy policy, private education expenditure is zero. Thus, the level of human capital H_t under the education subsidy policy remains equivalent to that under the loan interest subsidy policy throughout all periods.

The budget constraints for middle-aged individuals under the education subsidy policy are as follows:

$$c^m_{t-1} = w_{t-1} h(\tilde{e}, \tilde{H}) - R_{t-1}\tilde{e} - T_{t-1} - s_{t-1}, \tag{36}$$

$$c^m_{t+j} = w_{t+j} h(e_{t+j-1}, H_{t+j-1}) - T_{t+j} - s_{t+j}; \; j \geq 0. \tag{37}$$

Equation (36) represents the budget constraint for middle-aged individuals in period $t-1$ when the education subsidy policy begins. These individuals face the dual responsibility of repaying education loans from period $t-2$ and covering taxes to finance the subsequent generation's education expenditure. Equation (37) is the budget constraint for middle-aged individuals after period t. As mentioned earlier, they do not invest privately in education because they are fully subsidized by the government.

Figure 7 illustrates the pension benefit level that satisfies the equal utility condition in (35) under the education subsidy policy. In period $t-2$, the economy is in the laissez-faire steady-state equilibrium, and the policy is introduced in period $t-1$. Under the education subsidy policy, no pension benefit is required for older adults in period $t-1$. This aligns with the scenario observed in the case of the loan interest subsidy policy. Middle-aged individuals in period $t-1$ bear the responsibility of repaying their borrowings from their youth (i.e., in period $t-2$), and they must also pay a lump-sum tax to finance educational expenditure for the next generation. To offset these burdens, pension benefits are necessary in period t when they reach old age.

In period $t+1$, the pension benefit level satisfying the equal utility condition in (35) is lower than the required pension benefit level in period t. Education subsidies eliminate the need for private borrowing to invest in education, thus alleviating the burden of loan repayment after period t. This absence of financial obligation leads to a reduction in pension benefits. Furthermore, in period $t+2$, the pension benefit level satisfying (35) is negative, indicating that pensions can be abolished in period $t+2$. The cessation of pension provision yields a utility surpassing the laissez-faire steady-state equilibrium utility level before the policy enactment.

The duration during which pension provision is necessary under the education subsidy policy spans the same two periods as those required under the loan interest subsidy policy (see Figures 4 and 7.). However, the two policies vary as to when pension benefits begin. Under the loan interest subsidy policy, pensions must be provided in periods $t+2$ and $t+3$ to meet the equal

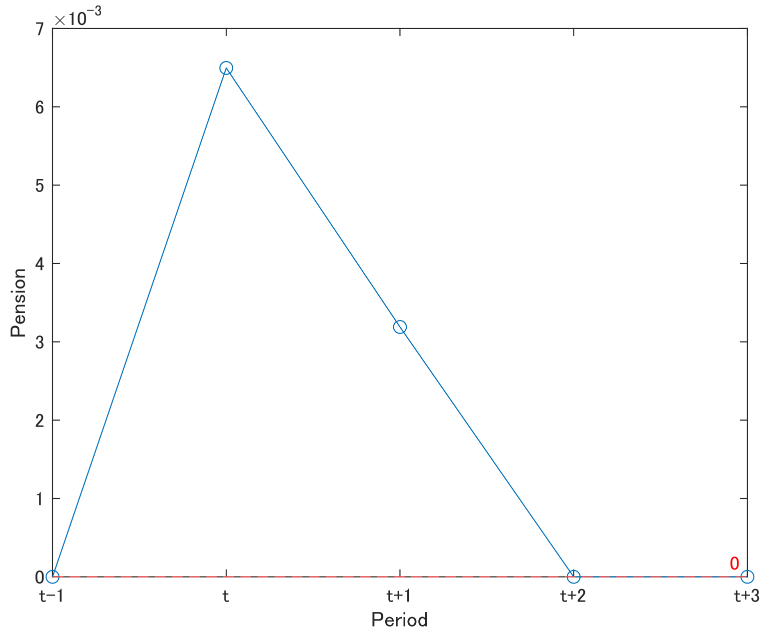

Fig. 7: Pension P_{t+j} ($j = -1, 0, 1, 2, 3$) that needs to satisfy equal utility condition under education subsidy policy.

utility condition in (35). Conversely, the education subsidy necessitates pensions to be supplied in periods t and $t+1$. Hence, the education subsidy calls for pension provision two periods earlier than the loan interest subsidy.

The difference of timing in pension provision between the two policies stems from their respective mechanisms. Firstly, consider the loan interest subsidy policy. The policy is implemented in period t, with their anticipation already known in period $t-1$. Young individuals in period $t-1$ can augment their borrowing, which translates to investment in human capital, foreseeing the subsidies awaiting them in period t. Given the current parameter setup, the enhancement of human capital resulting from increased investment proves substantial enough to preclude the requirement for pension benefits in period $t+1$. Nevertheless, in period $t+2$, pension provision becomes important due to a rise in interest repayment coupled with a decline in wages (see Figure 6), leading to a reduction in pension benefits compared to the previous period. This necessity persists in period $t+3$ for the same reasons. However, from period $t+4$ onwards, pension provision becomes unneeded owing to the cumulative effect of human capital accumulation.

Next, consider the education subsidy policy. Similar to the loan interest subsidy policy, education subsidies are initiated in period $t-1$. Middle-aged individuals in period $t-1$ do not directly benefit from these subsidies but are responsible for covering the educational expenses for the next generation. Consequently, the education subsidy policy necessitates pension benefits in period t to alleviate the tax burden of the period-$t-1$ middle-aged individuals.

For middle-aged individuals in periods $t-1$ and t (referred to as generations $t-1$ and t, respectively), the loan interest subsidy policy yields greater utility compared to the education subsidy policy (see Figure 8). This arises because the former policy imposes less tax burden on middle-aged individuals. However, beyond generation $t+1$, utility becomes higher under the education subsidy policy. This shift is attributed to a substantial rise in the interest rate and a substantial decline in the wage rate post period t under the loan interest subsidy policy (see Figure 6). Hence, as in the scenario of the small open economy, the loan interest subsidy policy does

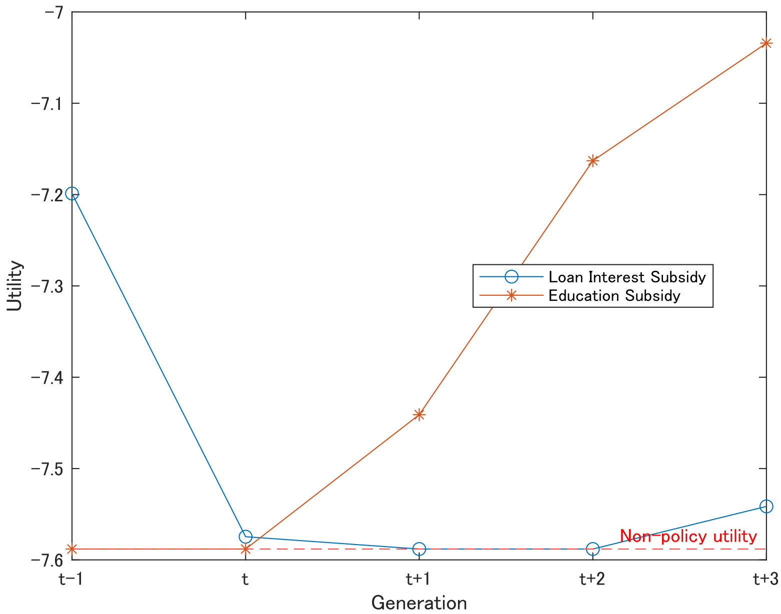

Fig. 8: Lifetime utility under the loan interest subsidy and education subsidy policies.

not dominate the education subsidy policy from a Pareto perspective in the case of the closed economy. However, from a perspective of policy implementation, the loan interest subsidy policy is more likely to garner support during the introductory phase of the policy since as depicted in Figure 8, it is preferred by old-aged and middle-aged individuals over education subsidies during that period.

5 Conclusion

This study analyzed a Pareto-improving policy designed to internalize intergenerational externalities in human capital within a decentralized economy through the use of interest subsidies on education loans. The government finances both the loan interest subsidies and PAYG pensions with a lump-sum tax imposed on the middle-aged generation. The welfare gains generated by the intergenerational transmission of human capital are used to gradually phase out pension benefits. The analysis demonstrated that in small open economies, where wages and interest rates are fixed, a Pareto-improving phase-out of PAYG pensions is achievable when combined with loan interest subsidies.

In contrast, in closed economies, the combination of loan interest subsidies and PAYG pensions affects lifetime utility through changes in factor prices. Specifically, an increase in the interest rate is detrimental to young and middle-aged individuals, as it raises the burden of repaying education loans. Conversely, it benefits older adults by increasing the return on savings. As a result, the impact of these two policy instruments in closed economies is more complex, with general equilibrium effects influencing the level of pension benefits.

The main contribution of this study is to demonstrate, through numerical analysis, that Pareto improvements and the gradual phase-out of pensions are feasible using loan interest subsidies even in closed economies. This finding suggests that it may be possible to implement policies that enhance the welfare of future generations without compromising the welfare of current generations, thereby helping to overcome intergenerational conflicts in real-world economies. Moreover, the methods developed in this study are expected to be applicable to the analysis of various policies involving intergenerational conflict, such as infrastructure investment and environmental preservation.

A Proof of Proposition 1

Assume a small open economy. For any given subsidy rate σ,

$$\begin{aligned}
P_{t+2} - P_{t+1} &= R[-w(H_{t+1} - H_t) + (1-\sigma)R(e_t - e_{t-1}) + T_{t+1} - T_t] \\
&= R[-w(H_{t+1} - H_t) + R(e_t - e_{t-1}) + P_{t+1}] \\
&= R[-w(H_{t+1} - H_t) + R(e_t - e_{t-1}) + R\{-w(H_t - \tilde{H}) + (1-\sigma)Re_{t-1} - R\tilde{e} + T_t\}] \\
&= R[-w(H_{t+1} - H_t) + R(e_t - e_{t-1}) - R\{w(H_t - \tilde{H}) - R(e_{t-1} - \tilde{e})\}].
\end{aligned}$$

Therefore, the condition that $P_{t+2} < P_{t+1}$ from the $t+1$ to the $t+2$ is obtained as follows.

$$P_{t+2} - P_{t+1} < 0 \quad \Longleftrightarrow \quad w(H_{t+1} - H_t) - R(e_t - e_{t-1}) > R\{w(H_t - \tilde{H}) - R(e_{t-1} - \tilde{e})\}.$$

B Proof of Proposition 2

We show that once the increase in pension benefits stops or the benefits begin to decline, the pension benefit level continues to decrease thereafter. As the most stringent case satisfying this condition, we focus on the scenario in which $P_{t+2} = P_{t+1}$ holds. If the proposition is proven for this case, it can be similarly established for other cases where $P_{t+2} < P_{t+1}$ also holds.

As the minimum pension decline starts in period $t+2$, we assume that the pension increment was 0 from period $t+1$ to period $t+2$, i.e. $P_{t+2} - P_{t+1} = 0$. In this case, $P_{t+3} - P_{t+2} < 0$ holds. $P_{t+3} - P_{t+2}$ is obtained as follows.

$$P_{t+3} - P_{t+2} = R[-w(H_{t+2} - H_{t+1}) + R(e_{t+1} - e_t) + P_{t+2} - P_{t+1}].$$

In other words, for $P_{t+3} - P_{t+2} < 0$, the following conditions must be satisfied.

$$P_{t+3} - P_{t+2} < 0 \quad \Longleftrightarrow \quad w(H_{t+2} - H_{t+1}) > R(e_{t+1} - e_t). \tag{B1}$$

where $H_{t+2} = h(e_{t+1}, H_{t+1}) \simeq H_{t+1} + h_e(e_{t+1} - e_t) + h_H(H_{t+1} - H_t)$, from

$$\begin{aligned}
w(H_{t+2} - H_{t+1}) - R(e_{t+1} - e_t) &\simeq wh_e(e_{t+1} - e_t) + wh_H(H_{t+1} - H_t) - R(e_{t+1} - e_t), \\
&= (1-\sigma)R(e_{t+1} - e_t) + wh_H(H_{t+1} - H_t) - R(e_{t+1} - e_t), \\
&= wh_H(H_{t+1} - H_t) - \sigma R(e_{t+1} - e_t).
\end{aligned} \tag{B2}$$

Since R, w, σ are constant from t to $t+1$, $e_{t+1} \simeq e_t + e_H(H_{t+1} - H_t)$, we have

$$wh_H(H_{t+1} - H_t) - \sigma R(e_{t+1} - e_t) \simeq (wh_H - \sigma Re_H)(H_{t+1} - H_t). \tag{B3}$$

Since $P_{t+2} - P_{t+1} = 0$ was assumed, the following equation holds for the size of σ.

$$\begin{aligned}
& w(H_{t+1} - H_t) - R(e_t - e_{t-1}) = R[-w(H_t - \tilde{H}) + R(e_{t-1} - \tilde{e})], \\
\Longleftrightarrow\ & wH_t + wh_e(e_t - e_{t-1}) + wh_H(H_t - \tilde{H}) - Re_{t-1} - Re_H(H_t - \tilde{H}) = (1-R)(wH_t - Re_{t-1}) + R(w\tilde{H} - R\tilde{e}), \\
\Longleftrightarrow\ & (1-\sigma)R(e_t - e_{t-1}) = (-Rw - wh_H + Re_H)(H_t - \tilde{H}) + R^2(e_{t-1} - \tilde{e}).
\end{aligned}$$

Furthermore, since w, R are constant from $t-1$ to t and σ is announced and constant in $t-1$, then we use $e_t = e_{t-1} + e_H(H_t - \tilde{H})$, σ is obtained as follows.

$$\sigma = 1 - \frac{1}{Re_H}(-Rw - wh_H + Re_H) - \frac{R}{e_H}\frac{e_{t-1} - \tilde{e}}{H_t - \tilde{H}}.$$

Substituting this equation into $wh_H - \sigma Re_H$ and using $H_t \simeq \tilde{H} + H_e e_\sigma \sigma$, $e_{t-1} \simeq \tilde{e} + e_\sigma \sigma$, we obtain

$$\begin{aligned}
wh_H - \sigma Re_H &\simeq R\left(R\frac{e_{t-1} - \tilde{e}}{H_t - \tilde{H}} - w\right) \\
&\simeq R\left(R\frac{e_\sigma \sigma}{H_e e_\sigma \sigma} - w\right) \\
&= R\left(R\frac{1}{H_e} - w\right) \\
&= R\left(\frac{R}{H_e} - \frac{(1-\sigma)R}{H_e}\right) \quad > 0
\end{aligned}$$

With (B1), (B2) and (B3), the following relationship holds.

$$\begin{aligned}
wh_H - \sigma Re_H > 0 \quad &\Longleftrightarrow \quad w(H_{t+2} - H_{t+1}) - R(e_{t+1} - e_t) > 0, \\
&\Longleftrightarrow \quad P_{t+3} - P_{t+2} < 0.
\end{aligned}$$

Thus, once an increase has stopped, the pension will subsequently fall. It can be proved in the same way for the other periods.

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